An IMM Approach for Tracking a Maneuvering Target with Kinematic Constraints Based on the Square Root Information Filter

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Abstract

An efficient interacting multiple model(IMM) approach for tracking a maneuvering target with kinematic constraints is described based on the square root information filter(SRIF). The SRIF is employed instead of the conventional Kalman filter since it exhibits more efficient features in handling the kinematic constraints and improved numerical characteristics. The kinematic constraints are considered in the filtering process as pseudomeasurements where the degree of uncertainty is represented by the magnitude of the pseudomeasurement noise variance. The Monte Carlo simulations for the constant speed, maneuvering target are provided to demonstrate the improved tracking performance of the proposed algorithm.

I. Introduction

An accurate mathematical model for the target dynamics is prerequisite in the target tracking problems. If the system model is not correct, track loss may occur easily in the tracking process. The system model of a target moving with constant velocity in a straight line is different from that of a target moving with acceleration or maneuver. A nonmaneuvering target can be modeled accurately with a constant velocity model. However, when the target maneuvers, the tracking performance of the constant velocity filter can be degraded significantly. A constant acceleration model can be utilized to track such a target, but the acceleration of maneuvering target are often time-varying. Also, the constant acceleration filter has worse tracking performance than that of the constant velocity filter when the target does not maneuver. There have been many approaches in the literature[1-4] to get around this dilemma of model mismatch problem, but it still remains a great deal of debate surrounding this problem. Among them, the IMM algorithm[5-8] may provide rather well tracking performance with comparatively efficient computation.

The IMM algorithm consists of a parallel Kalman filter for each model, a model probability evaluator, an estimate mixer at the input of each Kalman filter, and an estimate combiner at the output of the filters. The IMM algorithm for target tracking is implemented using models of different dimension: a second-order constant velocity model which is dominating when the target is nonmaneuvering state and one or several third-order acceleration models for the maneuvering state with different process noise levels.

In spite of the simplicity and the versatility on target tracking problems, the conventional Kalman filter mechanization is sensitive to computer roundoff and exhibits numerically unstable characteristics[9]. This is one of the Kalman filter's most notable weakness since the numerical accuracy sometimes degrades to the point where the results are meaningless. To counter such problems, more numerically stable and accurate algorithms such as U-D or SRIF formulations [10-12] are introduced as an alternative to the conventional Kalman filter. Although algebraically equivalent to the Kalman filter, the SRIF exhibits attractive numerical features, particularly in the ill-conditioned problems[11].

There are some tracking problems which are subject to kinematic constraint as well as dynamic constraint. When the trajectory of a target satisfies the kinematic constraint, the kinematic constraint can be used as additional information for the motion of the target to reduce the tracking uncertainty. One way to handle the kinematic constraint is to find a suitable set of state variables to incorporate it into the form of dynamic constraint. However, it may result in extremely complicated nonlinear model since most of the kinematic

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constraint for the maneuvering target are nonlinear. The more reasonable approach is to introduce the kinematic constraint into the tracking process as a additional pseudomeasurement[13,14]. In this approach, the degree of constraint adherence is represented by pseudomeasurement noise variance. This approach will allow any kinematic constraint to be incorporated without significantly increasing the computational cost.

In this paper, an efficient IMM algorithm is presented for tracking a maneuvering target with kinematic constraint based on the SRIF, which is called KCSRIF-IMM for brevity. The KCSRIF-IMM algorithm employed two different kinematically constrained SRIF(KCSRIF): one for the constant velocity dynamic model and the other for the constant acceleration dynamic model. The SRIF is selected over the conventional Kalman filter since it exhibits more efficient features in handling the kinematic constraint and improved numerical characteristics. The nonlinear kinematic constraint is linearized about time updated estimate by using Taylor series and put into the data array in the measurement update process. The Monte Carlo simulations for the constant speed, maneuvering target are provided to demonstrate the improved tracking performance of the proposed algorithm.

II. Problem Formulation

In general, it is better to represent the motion of the target in the Cartesian coordinate frame for the computational simplicity.

$$x_{k+1}^i = \Phi^i x_k^i + \psi^i w_k^i \qquad k \in [0, k-1] \tag{1}$$

where $x_k^i \in R^{n_i \times 1}$ is the state vector of the target at time k for model i. x^1 consists of position and velocity and consists of position, velocity and x^2 acceleration so that $n_1 = 6$ and $n_2 = 9$.

$$x^1 \equiv [x \; \dot{x} \; y \; \dot{y} \; z \; \dot{z}]^T \tag{2}$$

$$x^2 \equiv [x \; \dot{x} \; \ddot{x} \; y \; \dot{y} \; \ddot{y} \; z \; \dot{z} \; \ddot{z}]^T \tag{3}$$

The state transition matrix $\Phi^i \in R^{n_i \times n_i}$ and the noise gain matrix $\Psi^i \in R^{n_i \times 3}$ can be given as follows:

$$\Phi^i = \begin{bmatrix} F^i & 0 & 0 \\ 0 & F^i & 0 \\ 0 & 0 & F^i \end{bmatrix} \tag{4}$$

$$\Psi^i = \begin{bmatrix} G^i & 0 & 0 \\ 0 & G^i & 0 \\ 0 & 0 & G^i \end{bmatrix} \tag{5}$$

where

$$F^1 = \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix} \quad \text{and} \quad G^1 = \begin{bmatrix} ½T^2 \\ T \end{bmatrix} \tag{6}$$

for model 1 and

$$F^2 = \begin{bmatrix} 1 & T & ½T^2 \\ 0 & 1 & T \\ 0 & 0 & 1 \end{bmatrix} \quad \text{and} \quad G^2 = \begin{bmatrix} ½T^2 \\ T \\ 1 \end{bmatrix} \tag{7}$$

for model 2 and T is the sampling period.
The variable $w_k^i \in R^{3 \times 1}$ is a zero-mean white Gaussian process noise for model i, which has known covariance matrix such as

$$E[w_k^i (w_l^i)^T] = Q^i \delta_{kl} \tag{8}$$

where δ_{kl} is the Kronecker delta function which is equal to one if $k=l$, otherwise it is zero.

Since most sensors used for target tracking make measurements in polar coordinate frame, the measurement equation is usually nonlinear. The nonlinearity is generated from the transformation of the polar-to-Cartesian coordinate frame. Here, for convenience, we assume that the measurements provide only the position of the target in the Cartesian coordinate frame.

$$z_k^i = H^i x_k^i + v_k^i \tag{9}$$

where the measurement matrix $H^i \in R^{3 \times n_i}$ is given by

$$H^1 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \tag{10}$$

$$H^2 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \tag{11}$$

The variable $v_k^i \in R^{3 \times 1}$ is a zero-mean white Gaussian measurement noise for model i, which has known covariance matrix such as

$$E[w_k^i (v_l^i)^T] = R^i \delta_{kl} \tag{12}$$

The process noise w_k^i and measurement noise v_k^i are assumed to be uncorrelated so that

$$E[w_k^i (v_l^i)^T] = o \quad \textbf{for all k,l.} \tag{13}$$

In general, (1) is referred to as a dynamic constraint. There are, however, some practical problems which are subject to kinematic constraint as well as dynamic constraint. The kinematic constraints for model i can be expressed as

$$C^i(x_k^i) = \mu_k^i \tag{14}$$

where $\mu_k^i \in R^{m_i \times 1}$ represents the uncertainty of the constraints and assumed to be white Gaussian noise with known covariance

$$E[\mu_k^i (\mu_l^i)^T] = M^i \delta_{kl} \tag{15}$$

In addition, it is assumed that the covariance matrices Q^i, R^i, and M^i are symmetric and positive semi-definite so that their square-roots are defined as follows:

$$(Q^i)^{-1} \equiv (R^i_w)^T R^i_w \tag{16}$$

$$(R^i)^{-1} \equiv (R^i_v)^T R^i_V \tag{17}$$

$$(M^i)^{-1} \equiv (R^i_\mu)^T R^i_\mu \tag{18}$$

III. KCSRIF-IMM approach

A KCSRIF-IMM approach is developed here to track a target with kinematic constraint by employing two different KCSRIF's: a second-order CV filter for the quiescent mode and a third-order CA filter for the maneuvering mode. The approach consists of a KCSRIF for each model, a model probability evaluator, an estimate mixer at the input of the filters, an estimate combiner at the output of the filters and a pseudomeasurement calculator between time and measurement update process of each filter. The flow diagram of the KCSRIF-IMM algorithm is depicted in Figure 1, where $x_{k|k}$ is the state estimate which is obtained from a probabilistic sum of the each filter output, $x^i_{k|k}$ is the state estimate obtained from the ith filter, Λ^i_k is the model likelihood function for the ith model at time k, η^i_k is the model probability for the i-th model at time k, and $\overline{C}$ and $\overline{\overline{C}}$ are parameters for the pseudomeasurement, which is defined in (28) and (29), respectively. It is noted in Figure 1 that the KCSRIF is employed instead of the conventional Kalman filter and the pseudomeasurement calculator is added to the standard IMM approach to accommodate the kinematic constraints. An underlying Markov chain is assumed to govern the switching of each model.

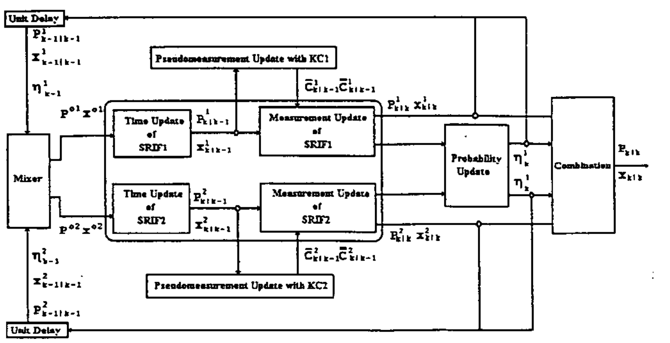


Fig. 1. The flow diagram of the KCSRIF-IMM algorithm.

The KCSRIF-IMM algorithm for tracking a maneuvering target with kinematic constraints can be outlined in the following 4 steps by considering the standard IMM algorithm and Chapter III. A detailed derivation and explanation about the standard IMM algorithm can be found in references[5-8].

STEP 1: Interaction Mixing

At the beginning of each cycle, *a priori* state estimate $x^i_{k-1|k-1}$, its error covariance $P^i_{k-1|k-1}$, and model probability η^i_{k-1} (i=1, 2) is needed for each model. The state estimate and its error covariance are mixed by each component as follows:

$$x^{oi}_{k-1|k-1} = \sum_{j=1}^{2} x^i_{k-1|k-1} \eta^{j|i}_{k-1|k-1} \tag{19}$$

$$P^{oi}_{k-1|k-1} = \sum_{j=1}^{2} \eta^{j|i}_{k-1|k-1} [P^i_{k-1|k-1} + (x^i_{k-1|k-1} - x^{oi}_{k-1|k-1})(x^i_{k-1|k-1} - x^{oi}_{k-1|k-1})^T] \tag{20}$$

where

$$\eta^{j|i}_{k-1|k-1} = \frac{1}{\overline{c}_i} p_{ji} \eta^j_{k-1} \tag{21}$$

$$\overline{c}_i = \sum_{j=1}^{2} p_{ji} \eta^j_{k-1} \tag{22}$$

where p_{ji} is the assumed transition probability matrix for the Markov chain, which imply for switching from model j to model i.

STEP 2: Filtering(The KCSRIF was derived in detail in reference [15].)

(1) Time Update

The two mixed state estimate and its error covariance are time updated in the KCSRIF as

$$\overline{\overline{T}}^i_{k|k-1^i} \begin{bmatrix} R^i_w & 0 & R^i_w w^i_{k-1} \\ -R^i_{k|k-1}(\Phi^i)^{-1}\Psi^i & R^i_{k|k-1}(\Phi^i)^{-1} & z^i_{k|k-1} \end{bmatrix} = \begin{bmatrix} R^i_w(k-1) & R^{i*}_{wx}(k-1) & (R^i_w \overline{w^i_{k-1}})^* \\ 0 & R^i_{k|k-1} & z^i_{k|k-1} \end{bmatrix} \tag{23}$$

where $R^i_{k|k-1}$ and $z^i_{k|k-1}$ are defined as

$$(P^{oi}_{k|k-1})^{-1} \equiv R^i_{k|k-1}(R^i_{k|k-1})^T \tag{24}$$

$$z^i_{k|k-1} \equiv R^i_{k|k-1} x^{oi}_{k|k-1} \tag{25}$$

The time updated state estimate can be obtained from (23 as

$$x^i_{k|k-1} = (R^i_{k|k-1})^{-1} z^i_{k|k-1} \tag{26}$$

(2) Measurement Update

The two time updated state estimate and its error covariance are measurement updated in the KCSRIF as

$$\overline{T}_{k|k-1^i} \begin{bmatrix} R^i_{k|k-1} & z^i_{k|k-1} \\ R^i_v H^i & R^i_v z^i_k \\ R^i_\mu \overline{C}^i(x_{k|k-1})^i & R^i_\mu \overline{\overline{C}}^i(x_{k|k-1})^i \end{bmatrix} = \begin{bmatrix} R^i_{k|k-1} & z^i_{k|k-1} \\ 0 & e^i_k \end{bmatrix} \tag{27}$$

where $\overline{C}^i(x^i_{k|k-1})$is the Jacobian which is defined by

$$\overline{C}_{k|k-1^i} = \left.\frac{\partial C^i}{\partial x^i}\right|_{x^i = x^i_{k|k-1}} \tag{28}$$

and $\overline{\overline{C}}^i(x^i_{k|k-1})$ can be considered as pseudomeasurement given by

$$\overline{\overline{C}}^i(x^i_{k|k-1}) \equiv \overline{C}^i(x^i_{k|k-1})x^i_{k|k-1} - C^i(x^i_{k|k-1}) \tag{29}$$

The filtered state estimate can be obtained from (27) as

$$x^i_{k|k} = (R^i_{k|k})^{-1} z^i_{k|k} \tag{30}$$

STEP 3: Model Likelihood Computation and Model Probability Update

The likelihood function of each model is computed as

$$\Lambda^i_k = \frac{1}{\sqrt{2\pi|S^i_k}} \exp[-0.5(\overline{z^i_k})^T (S^i_k)^{-1} \overline{z^i_k}] \tag{31}$$

where $\overline{z^i_k}$ and S^i_k are the residual and its covariance of the i-th filter, respectively. In the SRIF, it is important to note that [15]

$$|S^i_k| = \frac{|R^i_{k|k}|}{|R^i_{k|k-1}||R^i_v|} \tag{32}$$

$$(\overline{z^i_k})^T (S^i_k)^{-1} \overline{z^i_k} = (e^i_k)^T e^i_k \tag{33}$$

The model probability is updated by using (31) as

$$\eta^i_k = \frac{1}{c} \Lambda^i_k \overline{c}_i \tag{34}$$

where $\overline{c}$ is defined in (22) and c is another normalization constant which is defined by

$$c \equiv \sum_{j=1}^{2} \Lambda^i_k \overline{c}_j \tag{35}$$

STEP 4: Combination of State Estimate and Error Covariance

Using the updated model probability (34), the state estimate and error covariance are combined as

$$x_{k|k} = \sum_{i=1}^{2} x^i_{k|k} \eta^i_k \tag{36}$$

$$P_{k|k} = \sum_{i=1}^{2} \eta^i_k [p^i_{k|k} + (x^i_{k|k} - x_{k|k})(x^i_{k|k} x_{k|k})^T] \tag{37}$$

IV. Simulation Results

To demonstrate the tracking performance of the KCSRIF-IMM algorithm, a target which has the trajectories consisting of a quiescent and maneuvering mode is selected for use in a simulation study. The target moves in a straight line with constant velocity for the first 70 samples and then it performs "C" curve maneuver with constant speed for the next 60 samples and finally it comes back to the constant velocity motion for the last 70 samples. Thus the number of samples K is 200. The maneuvering target with constant speed can be described as

$$\frac{dS}{dt} = 0 \tag{38}$$

where S is the speed of the target given by

$$S = (\dot{x}^2 + \dot{y}^2 + \dot{z}^2)^{\frac{1}{2}} \tag{39}$$

In view of (38) and (39), we can obtain the following kinematic constraint for the constant speed, maneuvering target:

$$V \cdot A + \mu = 0 \tag{40}$$

where the target velocity vector V and acceleration vector A are defined by

$$V \equiv [\dot{x}\ \dot{y}\ \dot{z}]^T \tag{41}$$

$$A \equiv [\ddot{x}\ \ddot{y}\ \ddot{z}]^T \tag{42}$$

Two-dimensional Cartesian coordinate profile of the true position trajectory with initial state $x_0 = [1000\ 150\ 0\ 2000\ 100\ 0]^T$ is depicted in Figure 2.

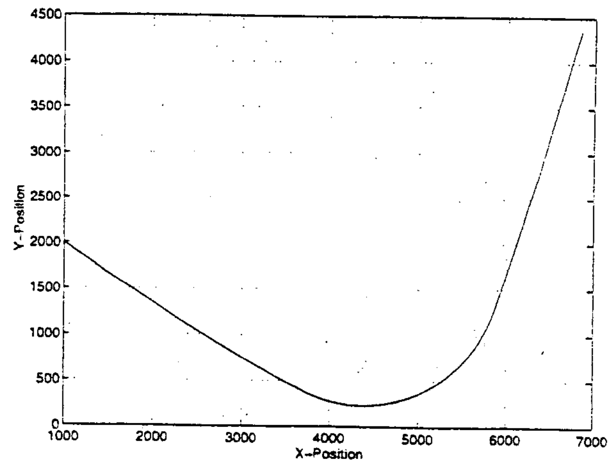


Fig. 2. Target trajectory in Cartesian coordinate frame.

The SRIF-IMM and KCSRIF-IMM algorithms are used to track the target given in Figure 2. The KCSRIF-IMM algorithm consists of a second-order constant velocity SRIF with process noise variance $Q^1 = 50I_2$ and measurement noise variance $R^1 = I_2$ and a third-order constant acceleration KCSRIF with $Q^2 = 50I_3$, $R^2 = I_3$, and pseudomeasurement noise variance $M=1$. The same parameter values are used for both filters except the kinematic constraint. The sampling period is chosen as so that the simulation was performed for 50 seconds. The initial estimated state and the initial error covariance matrix were chosen as $x^i_{0|-1} = x^i_o$ (for I=1,2), $P^1_{0|-1} = 100I_4$ and

$P^2_{0|-1} = 100I_6$, respectively . In general, the choice of initial covariance matrix is not important as its effect quickly decays with increasing time. The assumed model switching probabilities(Markov transition probabilities) are 0.05 for p_{12} and p_{21} and 0.95 for p_{11} and p_{22}.

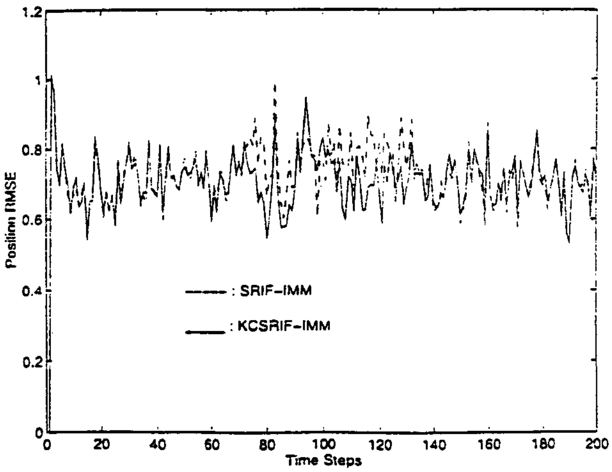


Fig. 3. Average RMSE for position.

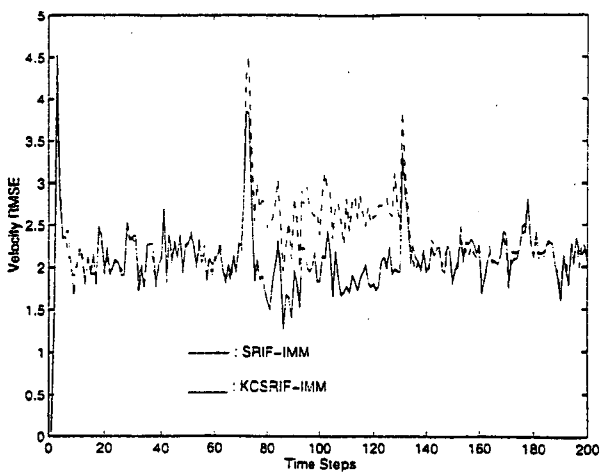


Fig. 4. Average RMSE for velocity.

Monte Carlo simulations of 30 experiments are conducted to compare the tracking performance of the two algorithms. The average root-mean-square errors(RMSE's) for position, velocity, and acceleration are shown in Figure 3, 4, and 5, respectively. It is observed that the KCSRIF-IMM outperforms the SRIF-IMM during the maneuver in estimation accuracy, especially for velocity and acceleration. It is noted from (40), (41), and (42) that the kinematic constraint is composed of velocity and acceleration. Figure 6 (a) and (b) represent the probability of CV and CA model for the two algorithms, respectively. As can be expected, the probability of CV model dominates during the quiescent period and that of CA model dominates during the maneuvering period.

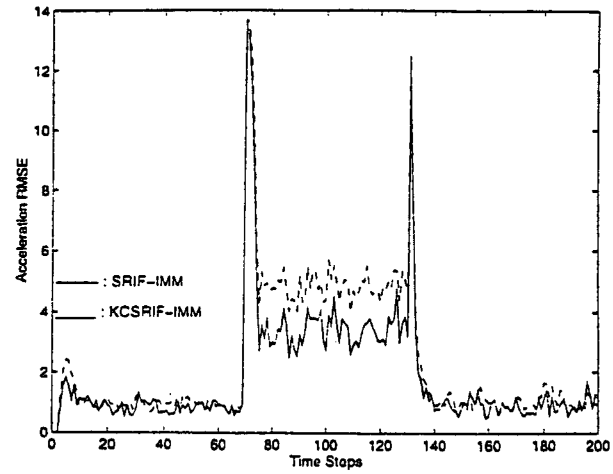
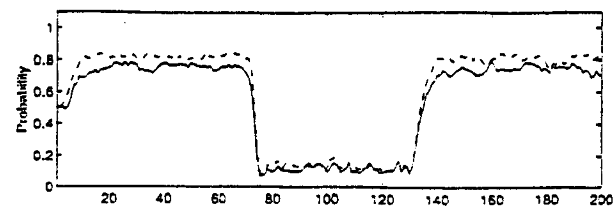
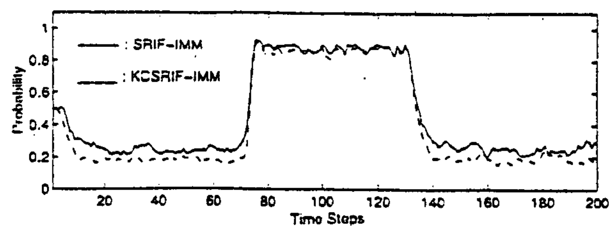


Fig. 5. Average RMSE for acceleration.



(a) Probability of CV model.



(b) Probability of CA model.

Fig. 6. Probability of the CV and CA model.

VI. Conclusions

A modified IMM algorithm has been formulated to track a maneuvering target with kinematic constraints based on the SRIF mechanization. The modified IMM algorithm employed two different kinematically constrained SRIF: one for the constant velocity dynamic model and the other for the constant acceleration dynamic model. The SRIF is selected over the conventional Kalman filter since it exhibits more efficient features in handling the kinematic constraint and improved numerical characteristics. The nonlinear kinematic constraint is linearized about time updated estimate by using Taylor series and put into the data array in the measurement update process. The simulation results for the constant speed, maneuvering target showed the improved tracking performance of the proposed algorithm.

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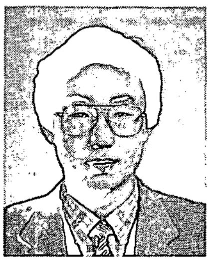

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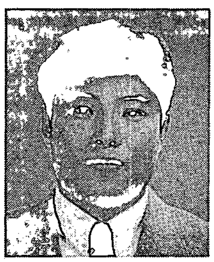

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