An EMM Approach to Derive an Energy Integral for the Direct Method of Stability Analysis in Power Systems

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Abstract

This paper presents a new approach to derive an energy integral based on an Equivalent Mechanical Model (EMM), which is developed by introducing imaginary springs for line resistances. The proposed EMM shows that phasor currents and voltages are directly analogous to the two-dimensional force and displacement vectors, respectively.

Through rigorous energy analysis of the proposed EMM, an exact energy integral expression is derived for multimachine systems, and several useful theorems are developed to derive an energy integral for power systems with detailed generator models. the energy integral exactly reflects the internal resistance, saliency and flux-decaying effects of the generator.

Finally, an illustrative example is given for a multimachine system adopting the Eq′-model for generators, which shows that the consideration of a detailed generator model does not aggravate the complicacy of the direct method of stability analysis in multimachine systems.

I. Introduction

This paper presents a new approach to derive an energy integral reflecting transmission-line resistances and flux-decaying effects on the basis of an Equivalent Mechanical Model (EMM) for stability analysis of multimachine power systems.

Many approaches have been presented to develop energy functions for power systems by using mechanical analogy and mathematical tools. Luders [1] suggested a mechanical analog system for a power system and derived an energy function of a power system. His mechanical analogy is developed with the strict restriction of constant voltage, which makes it impossible to take into account the effects of reactive bus powers and transmission-line resistances. However, most of the earlier energy functions have some limitations in view of the fact that the classical model is being adopted with the use of a lossless power system. With the introduction of a structural preserving energy concept [11] and center-of-mass energy analysis [6], considerable progress has been made recently in the development of energy functions to take into account the effects of reactive powers and transmission-line resistances. However, these approaches are still based on the classical model of the generator, which results in a drawback in the consideration of the flux decaying effects. Since this drawback was pointed out by Sasaki [2], many authors have devoted their time to the development of energy functions associated with detailed generator models [3-5, 12-14]. These approaches are mainly based on case-by-case analysis and fail to provide a general method to deal with various detailed models of the generator. On the other hand, a unified approach to the transient angular and voltage stability analyses has recently been posed by several authors, which gives another impetus to the understanding of transient behaviour of a power system with nonnegligible resistances.

In this paper, an exact equivalent EMM is systematically developed for multimachine systems by introducing imaginary springs for transmission-line resistances. The proposed EMM can allow bus voltage changes by taking reactive powers into account. By using the proposed EMM, it can be easily proven that phasor currents and voltages are directly analogous to the two-dimensional force and displacement vectors, respectively, and that the force balance condition in the EMM corresponds exactly to the Kirchhoff's current law.

On the basis of the proposed EMM, an exact energy integral expression is derived for lossy multibus systems

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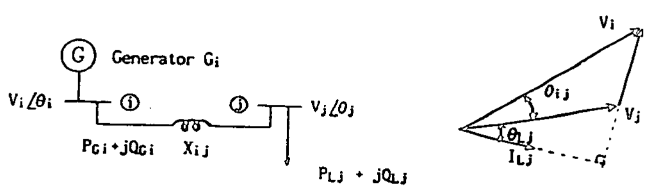
through rigorous energy analysis. By using the energy integral, an exact energy conservation theorem is developed to reflect the effects of resistances. This theorem can be utilized as a useful tool to show that an energy function has a seminegative time derivative. Several useful theorems are developed regarding the relationships between the generator mechanical input and electrical output by neglecting the stator/network transients. By using these theorems, a general approach is presented to derive an energy integral to deal with various kinds of generator models. This approach exactly reflects both the internal resistance and the saliency of the generator into the energy integral in a simple manner. Finally, an illustrative example is given for a multimachine system adopting the E_q'-model for generators, which shows that the consideration of the detailed generator model does not aggravate the complicacy of the direct method of stability analysis in multibus systems.

III. Development of Exact Mechanical Model for Power Systems

An exact EMM is derived systematically for multimachine systems with resistive and reactive transmission-lines. The EMM is developed first for a simple two-bus system, and later generalized to be applicable to multibus systems with the use of the classical generator model.

Derivation of EMM for Pure Reactive Systems

Consider the following two-bus system with a pure reactive line. The generator is assumed to be of round-rotor type with Xg=0. The system configuration and its phasor diagram are shown in Fig. 1.



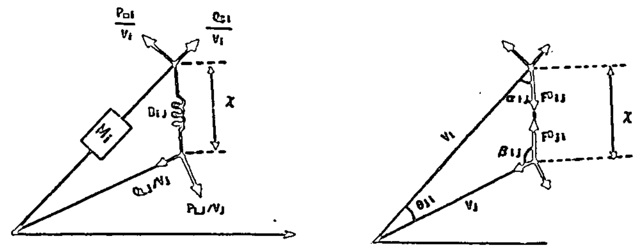
(a) System Configuration (b) V-I Phasor Diagram

Fig. 1. Pure Reactive two-bus System.

The following exact EMM can be developed by slightly modifying the conventional model[1] in order to take bus voltages as variables and to reflect reactive powers into the EMM.

The exactness of the above EMM can be verified by showing that the dynamic equations of EMM agree exactly with the power swing equations and load flow equations of the given power system.

In the above force diagram, the magnitudes of spring forces are given by



where Bij = 1./ Xij
Mi : Generator Inertia
(a) EMM

where $\chi \sin \alpha_{ij}$ = Vj sin θij
$\chi \cos \alpha_{ij}$ = Vi-Vj cos θij
(b) Force Diagram

Fig. 2. Equivalent Mechanical Model.

$$|F^B_{ij}| = |F^B_{ji}| = B_{ij}\chi \tag{1}$$

where B_{ij} is the spring constant
(Superscript B denotes "associated with susceptance B".)

Both of the force vectors can be represented with the directional unit vectors $\hat{\theta}$ and $\hat{r}$ as follows:

$$\begin{aligned} F^{B_{ij}} &= -B_{ij}\,x\cos\alpha_{ij}\,\hat{r} - B_{ij}\,x\sin\alpha_{ij}\,\hat{\theta} \\ &= -B_{ij}(V_i - V_j\cos\theta_{ij})\,\hat{r} - B_{ij}V_j\sin\theta_{ij}\,\hat{\theta} \end{aligned} \tag{2}$$

$$\begin{aligned} F^{B_{ji}} &= -B_{ij}\,x\cos\beta_{ij}\,\hat{r} + B_{ij}\,x\sin\beta_{ij}\,\hat{\theta} \\ &= B_{ij}(V_i\cos\theta_{ij} - V_j)\,\hat{r} + B_{ij}V_i\sin\theta_{ij}\,\hat{\theta} \\ &= -B_{ji}(V_j - V_i\cos\theta_{ji})\,\hat{r} - B_{ji}V_i\sin\theta_{ji}\,\hat{\theta} \end{aligned} \tag{3}$$

The above equations can be easily proven by using the trigonometric relations shown in Fig. 2(b).

The force balance conditions at both points in the EMM give the following dynamic equations:

$$\text{Generator : } \frac{1}{V_i}\left[-M_i\ddot{\theta}_i - D_i\dot{\theta}_i + P_{mi}\right] - B_{ij}V_j\sin\theta_{ij} = 0 \tag{4}$$

$$\frac{Q_{Gi}}{V_i} - B_{ij}(V_i - V_j\cos\theta_{ij}) = 0 \tag{5}$$

$$\text{Load : } -P_{Lj}/V_j - B_{ji}V_i\sin\theta_{ji} = 0 \tag{6}$$

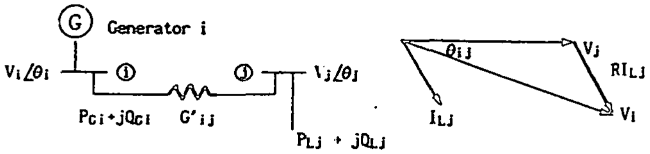
$$-Q_{Lj}/V_j - B_{ji}(V_j - V_i\cos\theta_{ji}) = 0 \tag{7}$$

Here, it can be easily checked that the first two equations describe exactly the power swing equations and reactive power contraints at Bus i, and the last two, the load flow equations at Bus j for the original power systems.

Derivation of EMM for a Pure Resistive System

Consider the following two-bus system with a pure resistive line.

In this case, a mechanical equivalent model can be developed by introducing an imaginary spring. An imaginary spring will be assumed to yield two forces proportional to its displacement, and the directions of the forces are vertical to the spring and the rotational axis as shown in Fig. 4.



(a) System Configuration (b) V-I Phasor Diagram

Fig. 3. Pure Resistive Two-Bus System.

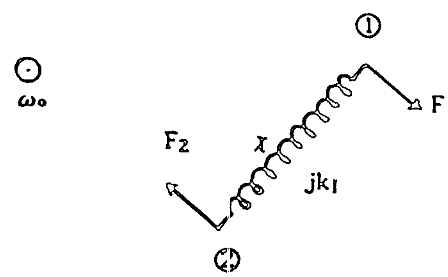
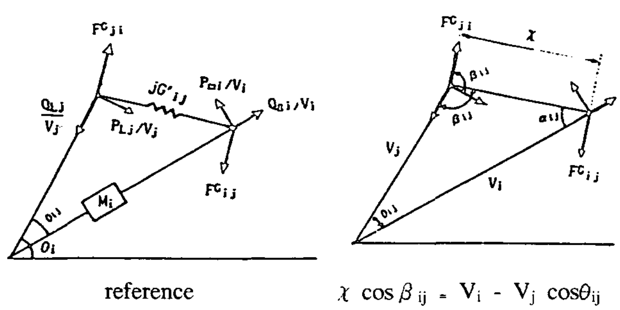


Fig. 4. Imaginary Spring.

The imaginary spring forces can be interpreted as some kinds of forces similar to Coriolis forces due to the rotating movement of the generator, and represented by vector cross-products as follows:

$$F_1 = k_I\, \chi_{12} \times \omega_o$$
$$F_2 = k_I\, \chi_{21} \times \omega_o \tag{8}$$
$$|F_1| = |F_2| = G'_{12}\, \chi$$
$$\text{where} \quad G'_{12} = |k_I| \times |\omega_o|$$



$\chi \cos\beta_{ij} = V_i - V_j \cos\theta_{ij}$
$\chi \sin\beta_{ij} = V_j \sin\theta_{ij}$

(a) EMM (b) Spring Force Diagram

Fig. 5. EMM for a two-Bus System with a Pure Resistive Line.

By using an imaginary spring, we can obtain the following equivalent mechanical model for the given system. In Fig. 5, the magnitudes of forces F^G_{ij} and F^G_{ji} are given by:

$$|F^G_{ij}| = |F^G_{ji}| = G'_{ij}\, \chi \tag{9}$$

Superscript G denotes "associated with Conductance G". By using the relationships given in the force diagram, we have the following vector representations:

$$F^G_{ij} = -G'_{ij}\, \chi \cos\alpha_{ij}\, \hat{\theta} - G'_{ij}\, \chi \sin\alpha_{ij}\, \hat{r}$$
$$= -G'_{ij}\, (V_i - V_j \cos\theta_{ij})\, \hat{\theta} - G'_{ij}\, V_j \sin\theta_{ij}\, \hat{r} \tag{10}$$

$$F^G_{ji} = -G'_{ij}\, \chi \cos\beta_{ij}\, \hat{\theta} - G'_{ij}\, \chi \sin\beta_{ij}\, \hat{r}$$
$$= -G'_{ij}\, (V_j - V_i \cos\theta_{ij})\, \hat{\theta} + G'_{ij}\, V_i \sin\theta_{ij}\, \hat{r}$$
$$= -G'_{ji}\, (V_j - V_i \cos\theta_{ji})\, \hat{\theta} - G'_{ji}\, V_i \sin\theta_{ji}\, \hat{r} \tag{11}$$
$$\text{where} \quad G'_{ji} = G'_{ij}$$

As a result, the force balance conditions at both ends of the imaginary spring yield the four equations similar to Eqs.(4)-(7).

For Point A, we have the following equations:

$$\text{Point A: } \frac{1}{V_i}[\, -M_i\, \ddot{\theta}_i + P_{mi}] - G'_{ij}(V_i - V_j \cos\theta_{ij}) = 0 \tag{12}$$

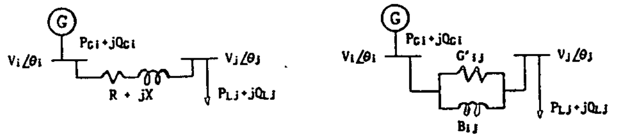
$$\frac{Q_{Gi}}{V_i} - G'_{ij}\, V_j \sin\theta_{ij} = 0 \tag{13}$$

The force balance equations for Point B are the same as obtained by exchanging Subscripts i and j in the above equations. In this case, generator parameters M_j, D_j, and P_{mj} should be zero since Bus j has no generator. Here, it is noted that multiplying Eqs.(12) and (13) by V_i gives the exact swing equation and reactive power equation when Bus i is a generator bus, or the exact real and reactive power flow equations when Bus i is a nongenerator bus. This verifies the exactness of the EMM proposed for a resistive system.

Comment: The concept of imaginary spring may not be proper terminology. However, no appropriate mechanical analogy has been found yet to reflect the effects of transmission-line resistances. Instead of using the term "imaginary spring", one may think that there exists some rotating field with intensity G'_{ij} which produces Force F^G_{ij} and F^G_{ji} proportional to distance χ. In this case, it is impossible to avoid the overlapping of such fields in a multibus system. In order to present the EMM in a simple manner, the imaginary spring can be used to indicate only the existence of a rotating field.

Generalized EMM for Multimachine Systems with Resistive and Reactive Transmission Lines

To begin with, consider a two-bus system with a resistive and reactive transmission line in Fig.6 (a). This system can be changed to an equivalent system with two parallel lines: one with pure susceptance B_{ij} and the other with pure conductance G_{ij}, as shown in Fig.6 (b).



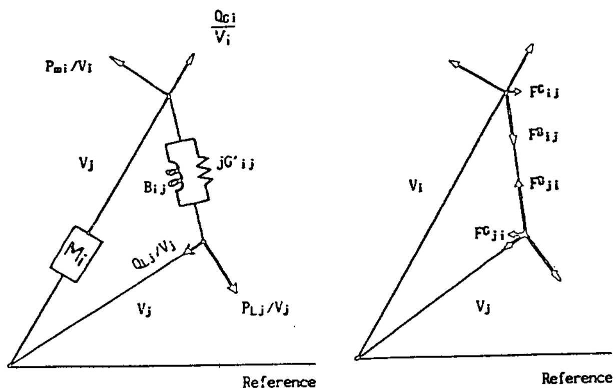
where $G'_{ij} = R_{ij} /(R_{ij}^2 + X_{ij}^2)$

$B_{ij} = X_{ij} /(R_{ij}^2 + X_{ij}^2)$

(a) two-Bus System with a Resistive and Reactive Line (b) Equivalent System

Fig. 6. Sample of Resistive and Reactive System.

For the system in Fig.6 (b), one can easily develop the following EMM with real and imaginary springs by using the former results.



(a) EMM (b) Spring Force Diagram

Fig. 7. MES for a two-Bus System with a Resistive and Reactive Line.

In Fig. 7 (b), the total force by the real and reactive springs is given by

$$\mathbb{F}_{ij} = \mathbb{F}^G_{ij} + \mathbb{F}^B_{ij} = j(G_{ij} + jB_{ij})(\mathbb{V}_i - \mathbb{V}_j) = - j\mathbb{I}_{ij} \quad (14)$$

where $\mathbb{V}_i = V_i\angle\theta_i$, $\mathbb{V}_j = V_j\angle\theta_j$: Complex Phasor Voltages

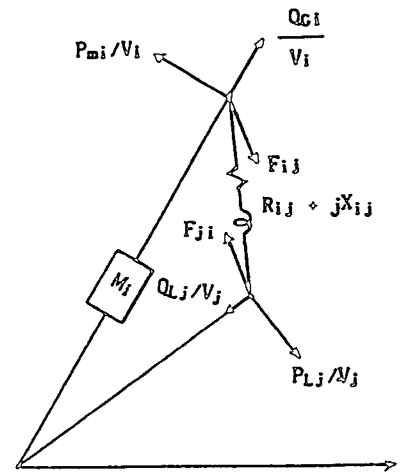
$$\mathbb{I}_{ij} = - (G_{ij} + jB_{ij}) (\mathbb{V}_i - \mathbb{V}_j) = (\mathbb{V}_i - \mathbb{V}_j)/(R_{ij} + jX_{ij}) \quad (15)$$

$$G_{ij} = -G'_{ij} \quad (i \neq j) \quad (16)$$

In the above equations, G'_{ij} is replaced by G_{ij} in order to avoid confusion with the real part of Y_{ij}, an element of the Y_{BUS} matrix. With this notation, G_{ij} and B_{ij} agree with the following conventional notation of the Y_{BUS} matrix:

$$Y_{BUS} = \begin{bmatrix} G_{ij} + jB_{ij} \end{bmatrix} \quad (17)$$

Here, it is noted that $\mathbb{I}_{ij}$, $\mathbb{V}_i$, $\mathbb{V}_j$ in the above equations can be considered simultaneously as either phasors or vectors without any conflict, and that $\mathbb{I}_{ij}$ is the same as the actual phasor current flowing from bus i to bus j through the transmission-line k. Since current $\mathbb{I}_{ij}$ can be calculated by using the line impedance, we will introduce the following impedance model for simplicity to replace the original model in Fig. 7 (a).

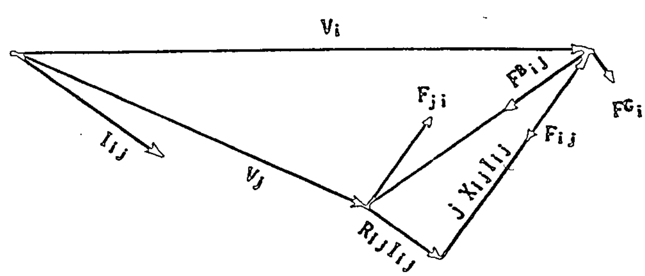


$$\mathbb{F}_{ij} = -j(\mathbb{V}_i - \mathbb{V}_j)/(R_{ij} + jX_{ij})$$
$$\mathbb{F}_{ji} = -j(\mathbb{V}_j - \mathbb{V}_i)/(R_{ij} + jX_{ij})$$

Fig. 8. EMM Using Line Impedances.

In the above figure, Force $\mathbb{F}_{ij}$ is represented with the directional unit vectors as follows:

$$\begin{aligned}\mathbb{F}_{ij} &= \mathbb{F}^G_{ij} + \mathbb{F}^B_{ij} \\ &= - [B_{ij}(V_i - V_j \cos\theta_{ij}) + G_{ij}V_j \sin \theta_{ij}] \hat{r} \\ &\quad - [B_{ij}V_j \sin\theta_{ij} - G_{ij} (V_i - V_j \cos\theta_{ij})] \hat{\theta}\end{aligned} \quad (18)$$



$\mathbb{V}_i = \mathbb{V}_j + R_{ij}\mathbb{I}_{ij} + j X_{ij}\mathbb{I}_{ij}$

$\mathbb{F}_{ij} = -j\mathbb{I}_{ij}$: reverse direction of $j X_{ij}\mathbb{I}_{ij}$

Fig. 9. Phasor Diagram of Voltages and Currents with Forces.

By using the conventional phasor representation, the relations of force $\mathbb{F}_{ij}$ with phasor voltages and currents can be represented as shown in Fig. 9. Here, it is noted that Force $\mathbb{F}_{ij}$ takes the direction opposite to vector $jX_{ij}\mathbb{I}_{ij}$. Here, the correspondence of the EMM and the original electric system can be summarized in the following Table 1.

Table 1. Correspondence Relations Between the Proposed EMM and the Electrical System.

EMM	Electric System
2-dimensional Displacement Vector	Voltage Phasor Vector
2-dimensional Force Vector	90° Rotation of Current Phasor Vector
Force Balance Condition	Kirchhoff's Current Law
Torque	Real Power
Torque Balance Equation	Real Power Balance Equation

The EMM developed for a general two-bus system can be easily generalized for multibus systems. For example, we will consider the following three-bus system, which is the smallest system, which includes all types of buses.

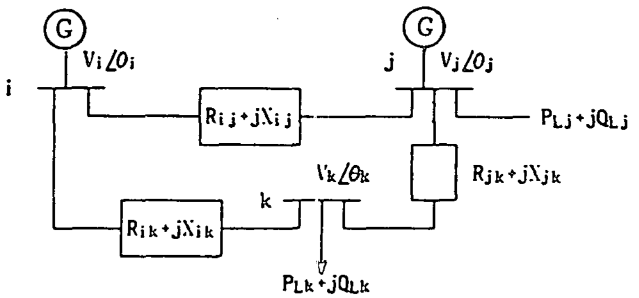


Fig. 10. three-Bus System.

By assuming $\theta_i > \theta_j > \theta_k$, we can obtain the following EMM for the above system with the impedance model.

By observing the above force diagram, it can be shown that the following force balance equation holds for arbitrary bus I:

$$\frac{M_i \ddot{\theta}_i}{V_i}\hat{\theta} + \frac{D_i \dot{\theta}_i}{V_i}\hat{\theta} = \frac{P_{mi}\hat{\theta} - P_{Li}\hat{\theta}}{V_i} + \sum_{j \neq i} \mathbb{F}_{ij} + \frac{Q_{Gi}+Q_{Ci}-Q_{Li}}{V_i}\hat{r}$$

where $i \in \{i, j, k\}$ (19)

Substitution of Eq. (18) for $\mathbb{F}_{ij}$ into the above equation yields the following equations.

$$(M_i\ddot{\theta}_i+D_i\dot{\theta}_i)/V_i-(P_{mi}-P_{Li})/V_i- \sum_{j \neq i}[G_{ij}(V_i-V_j\cos\theta_{ij})-B_{ij}V_j\sin\theta_{ij}] = 0 \quad (20)$$

$$\sum_{j \neq i} [G_{ij}V_j\sin\theta_{ij} + B_{ij}(V_i- V_j\cos\theta_{ij})] - [Q_{Gi}+Q_{Ci}-Q_{Li}]/V_i = 0 \quad (21)$$

From the above equations, it can be also easily checked that the force balance equations agree exactly with the power swing equations for a generator bus, and with real and reactive power balance equations for a load bus. The proposed EMM can be applied to any multimachine system if the classical model is adopted for all generators.

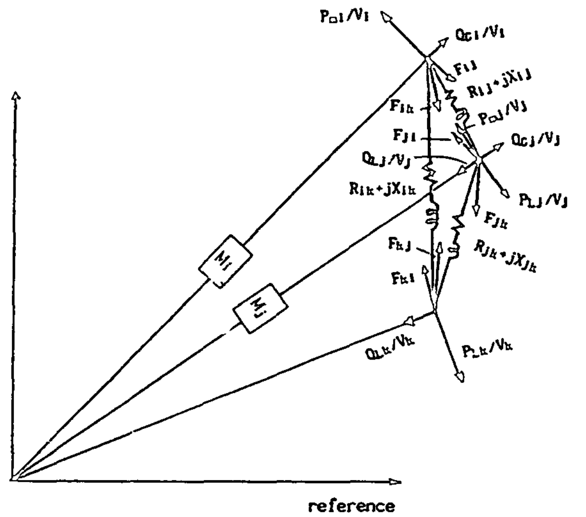


Fig. 11. EMM for Multibus System.

III. Derivation of Energy Integral for Power Systems Based on the Exact EMM

This section deals with derivation of an energy integral expression with the use of the exact EMM. The classical generator model is first adopted in the energy integral derivation, and later the energy integral is generalized to adapt to various detailed generator models.

The energy integral in a system can be obtained by integrating the differential energy dW due to its differential displacement. In this section, an energy integral expression for power systems is derived based on rigorous analysis of differential energy changes in the EMM.

Consider the differential energy dW_i due to the differential changes in voltage magnitudes and phase angles, i.e., dV_i and $d\theta_i$, for a EMM of multibus system in Fig. 11. The energy integral of the whole system can be calculated by integrating the total sum of the differential energy dW_i for each bus i as follows:

$$W = \int_c (\sum_i dW_i) \quad (22)$$

where c is a integral path along the solution path.

In the above equation, the differential energy dW_i is given by the scalar product of the total force acting on Bus i and the differential distance vector. That is

$$dW_i = -\mathbb{F}_{i,ttl}\cdot d\chi_i = -\mathbb{F}_{i,ttl}\cdot(V_i\, d\theta_i\hat{\theta} + dV_i\hat{r}) \qquad (23)$$

where

$$\mathbb{F}_{i,ttl} = -\left[\frac{M_i\,\ddot{\theta}_i}{V_i} + \frac{D_i\,\dot{\theta}}{V_i}\right]\hat{\theta} + \frac{(P_{mi}-P_{Li})}{V_i}\hat{\theta} + \sum_{j\neq i}\mathbb{F}_{ij} + \frac{QGi+QCi-QLi}{V_i}\hat{r} \qquad (24)$$

In the above equations, force $\mathbb{F}_{ij}$ is given in Eq.(18). Substitution of Eq. (24) into Eq. (23) yields

$$dW_i = \sum\{[-G_{ij}(V_i-V_j\cos\theta_{ij}) + B_{ij}V_j\sin\theta_{ij}]\,V_i d\theta_i + [G_{ij}V_j\sin\theta_{ij} + B_{ij}(V_i-V_j\cos\theta_{ij})]\,dV_i\}$$
$$-(P_{mi}-P_{Li})d\theta_i - \left[\frac{Q_{Gi}+Q_{Ci}-Q_{Li}}{V_i}\right]dV_i + M_i\omega_i d\omega_i + D_i\omega_i d\delta_i = 0$$

Here it is noted that the differential energy dW_i is always zero, which comes from the energy conservation law. The above equation can be rewritten as follows:

$$dW_i = G_{ii}V_i^2\,d\theta_i - B_{ii}V_i\,dV_i + \sum_{j\neq i}\Big[V_i\,V_j(B_{ij}\sin\theta_{ij}+G_{ij}\cos\theta_{ij})d\theta_i + V_j(G_{ij}\sin\theta_{ij} - B_{ij}\cos\theta_{ij})dV_i\Big] - P_{mi}d\theta_i + P_{Li}d\theta_i$$
$$-\left[\frac{Q_{Gi}+Q_{Ci}-Q_{Li}}{V_i}\right]dV_i + M_i\omega_i d\omega_i + D_i\omega_i d\delta_i = 0 \qquad (25)$$

where $G_{ii} = -\sum_{j\neq i}G_{ij} = \sum_{j\neq i}g_{ij}$ (26)

$B_{ii} = \sum_{j\neq i}B_{ij} = -\sum_{j\neq i}b_{ij}$ (27)

(Note that $Y_{bus} = [\,G_{ij} + jB_{ij}\,]$)

In a similar way, it can be shown that the differential energy dW_j for bus j is given by

$$dW_j = G_{jj}V_j^2\,d\theta_j - B_{jj}V_j\,dV_j + \sum\Big[V_j\,V_i(B_{ji}\sin\theta_{ji}+G_{ji}\cos\theta_{ji})d\theta_j + V_i(-g_{ji}\sin\theta_{ji}-b_{ji}\cos\theta_{ji})dV_j\Big] - P_{mj}d\theta_j + P_{Lj}d\theta_j$$
$$-\left[\frac{Q_{Gj}+Q_{Cj}-Q_{Lj}}{V_j}\right]dV_j + M_j\omega_j d\omega_j + D_j\omega_j d\delta_j \qquad (28)$$

The total differential energy due to all of the differential displacements of $d\theta_i$, dV_i $(i=1,2,\cdots,N)$ can be calculated by either the following:

$dW = \sum_i dW_i$ or

$dW = \sum_j dW_j$

For the simplicity of manipulation, the following calculation is taken rather than either of the above.

$$dW = \frac{1}{2}\Big(\sum_i dW_i + \sum_j dW_j\Big) \qquad (29)$$

After some manipulation by substituting Eqs. (25) and (28) into Eq.(29), the following differential energy is obtained.

$$dW = -\sum_i d\Big(\frac{1}{2}B_{ii}V_i^2\Big) + \frac{1}{2}\sum_i\sum_{j\neq i} d(-B_{ij}(V_iV_j\cos\theta_{ij})) + \sum_i G_{ii}V_i^2 d\theta_i$$
$$+\frac{1}{2}\sum_i\sum_{j\neq i}G_{ij}(V_iV_j\cos\theta_{ij}d\theta_i + V_j\sin\theta_{ij}dV_i + V_iV_j\cos\theta_{ij}d\theta_j - V_i\sin\theta_{ij}dV_j)$$
$$-\sum P_{mi}\,d\theta_i + \sum P_{Li}\,d\theta_i - \sum_i\left[\frac{Q_{Gi}+Q_{Ci}-Q_{Li}}{V_i}\right]dV_i$$
$$+\sum D_i\omega_i d\delta_i + \sum_i d\Big(\frac{1}{2}M_i\omega_i^2\Big) = 0 \qquad (30)$$

In the above equation, the first two and the last terms are expressed in the total differential form. Consequently, integration of each side of Eq. (30) from the initial state to a certain state yields the following energy integral.

$$\Delta W = W_1 - W_O = {}_c\!\int \sum dW_{Ti}$$
$$= \sum_i\Bigg[\Big[-\frac{1}{2}B_{ii}V_i^2 - \frac{1}{2}\sum_{j\neq i}V_iV_jB_{ij}\cos\theta_{ij}\Big]\Bigg|_{(\theta_o,V_o)}^{(\theta,V)}$$
$$+\frac{1}{2}\sum_{j\neq i}G_{ij}\;{}_c\!\int_{(\theta_o,V_o)}^{(\theta,V)}(V_iV_j\cos\theta_{ij}d\theta_i + V_j\sin\theta_{ij}dV_i + V_iV_j\cos\theta_{ij}d\theta_j - V_i\sin\theta_{ij}dV_j)$$
$$+{}_c\!\int_{\theta_o}^{\theta}G_{ii}V_i^2 d\theta_i + \frac{1}{2}M_i(\omega_i^2-\omega_{io}^2) - {}_c\!\int_{\theta_o}^{\theta}P_{mi}d\theta_i$$
$$+{}_c\!\int_{\theta_o}^{\theta}P_{Li}d\theta_i - {}_c\!\int_{V_o}^{V}\left[\frac{Q_{Gi}+Q_{Ci}-Q_{Li}}{V_i}\right]dV_i + {}_c\!\int D_i\omega_i^2 dt\Bigg] = 0 \qquad (31)$$

with $\theta_i = \delta_i$ for generator bus i

$P_{mi}=0,\ M_i=0,\ D_i=0,\ Q_{Gi}=0$ for load bus i

$\theta = [\theta_1, \theta_2, \cdots, \theta_N]^T$

$\mathbb{V} = [V_1, V_2, \cdots, V_N]^T$

In the above equation, it is noted that all of the integral terms are path dependent, which are difficult to deal with without appropriate assumptions. For practical applications, one can eliminate the path dependency by assuming constant mechanical inputs and constant loads, and neglecting line resistances. However, we will keep the path dependency in the energy integral for a rigorous energy analysis from the theoretical point of view.

As mentioned earlier, the fact that the above energy integral should always be zero implies the energy conservation law in the EMM for a power system. It can be easily checked by rearranging Eq.(31) into the following form:

$$E_K + E_P = E_{IN} - E_{OUT} - E_{loss} \qquad (32)$$

where

$$E_K = \tfrac{1}{2}\sum M_i\,(\omega_i^2 - \omega_{io}^2) \qquad (33.a)$$

$$E_P = \sum_i \Big[\Big[-\tfrac{1}{2} B_{ii}V_i^2 - \tfrac{1}{2} \sum_{j \neq i} V_iV_jB_{ij}\cos\theta_{ij} \Big]\Big|_{(\theta_o,V_o)}^{(\theta,V)} + {}_c\!\int_{(\theta_o,V_o)}^{(\theta,V)} G_{ii}V_i^2 d\theta_i + \tfrac{1}{2} \sum_{j \neq i} G_{ij}\ {}_c\!\int_{(\theta_o,V_o)}^{(\theta,V)} (V_iV_j\cos\theta_{ij}d\theta_i + V_j\sin\theta_{ij}dV_i + V_iV_j\cos\theta_{ij}d\theta_j - V_i\sin\theta_{ij}dV_j) \Big] \tag{33.b}$$

$$E_{IN} = \sum_i \Big[{}_c\!\int_{\theta_o}^{\theta} P_{mi}d\theta_i + {}_c\!\int_{(\theta_o,V_o)}^{(\theta,V)} \Big[\frac{Q_{Gi} + Q_{Ci}}{V_i} \Big] dV_i \Big] \tag{33.c}$$

$$E_{OUT} = \sum_i \Big[{}_c\!\int_{\theta_o}^{\theta} P_{Li}d\theta_i + {}_c\!\int_{V_o}^{V} \frac{Q_{Li}}{V_i} dV_i \Big] \tag{33.d}$$

$$E_{loss} = \sum_i \Big[{}_c\!\int D_i\omega_i^2 dt \Big] \tag{33.e}$$

The potential energy asssociated with the conductances can be interpreted as the potential energy of the imaginary spring which is stretched out to support the line loss power. As a result, the PE associated with line conductances depends only on the system state. This makes it time independent when the system is in the steady state in the EMTP (Electro Magnetic Transient Phenomenon) sense.

The observation of Eq. (32) obviously shows that the actual energy stored in the power system can be represented as

$$\text{Actual Energy}: \ E_{actual} = E_K + E_P$$

However, the energy integral E is commonly defined as follows in order to be consistant with the conventional approaches.

$$E = E_K + E_P - E_{IN} + E_{OUT} \tag{34}$$

From Eq. (31), it can be shown that the energy integral has an alternative expression as follows:

$$E = \sum_i \Big[{}_c\!\int D_i\omega_i d\delta_i \Big] = \sum_i \int D_i\omega_i^2 dt \tag{35.a}$$

By using the above equation, the time derivative of the energy function can be directly evaluated as follows:

$$\frac{dE}{dt} = \sum_i \Big[\frac{\partial E}{\partial \omega_i}\frac{d\omega_i}{dt} + \frac{\partial E}{\partial \delta_i}\frac{d\delta_i}{dt} + \frac{\partial E}{\partial V}\frac{dV_i}{dt} + \frac{\partial E}{\partial \theta_i}\frac{d\theta_i}{dt} \Big] = -\sum D_i\omega_i^2 \tag{35.b}$$

This equation directly shows that the time derivative of the energy integral is nonpositive. This method can be utilized as a powerful tool to prove the nonpositiveness of the time derivative for an energy function under consideration, which conventionally required complicate procedures.

IV. The Derivation of Energy Integral with the Consideration of Detailed Generator Models

The generator model has two reactances for subtransient, transient and steady states. This makes it almost impossible to develop a precise EMM for the various detailed models of the generator. In this respect, the derivation of the energy integral will be pursued by considering the system energy.

The power system can be represented by a multibus network with bus admittance matrix Y_{BUS} as shown in Fig. 12. Each generator can be considered as a complex injection power supplier.

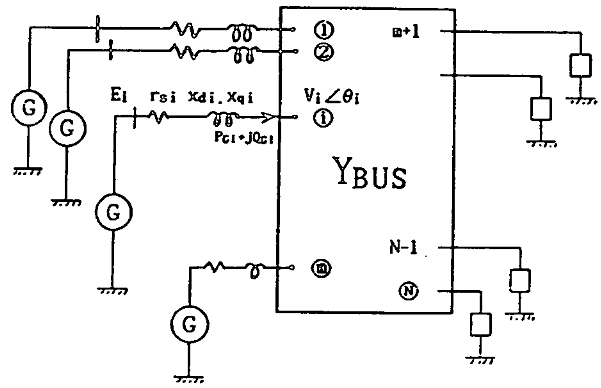


Fig. 12. Network Representation of Multibus System.

Now, consider the system energy stored in all transmission lines and loads except generators. With the use of the former results, the system energy is obtained directly as follows:

$$W = \sum_{i=1}^{N} \Big[\Big[-\frac{1}{2} B_{ii}V_i^2 - \frac{1}{2} \sum_{j \neq i} V_iV_jB_{ij}\cos\theta_{ij} \Big]\Big|_{(\theta_o,V_o)}^{(\theta,V)} + \frac{1}{2} \sum_{j \neq i}^{N} G_{ij}\ {}_c\!\int_{(\theta_o,V_o)}^{(\theta,V)} (V_iV_j\cos\theta_{ij}d\theta_i + V_j\sin\theta_{ij}dV_i + V_iV_j\cos\theta_{ij}d\theta_j - V_i\sin\theta_{ij}dV_j) + {}_c\!\int_{\theta_o}^{\theta} G_{ii}V_i^2 d\theta_i + {}_c\!\int_{\theta_o}^{\theta} P_{Li}d\theta_i + {}_c\!\int_{V_o}^{V} \Big[\frac{Q_{Li} - Q_{Ci}}{V_i} \Big] dV_i \Big] - \sum_{i=1}^{m} \Big[{}_c\!\int_{\theta_o}^{\theta} \Big(P_{Gi}d\theta_i + \frac{Q_{Gi}}{V_i}dV_i \Big) \Big] = 0 \tag{36}$$

with $Y_{BUS} = [\, G_{ij} + j\, B_{ij} \,]$

In the above equation, the energy terms associated with generator output powers P_{Gi} and Q_{Gi}, can be replaced by the energy terms associated with the input powers of generator i, which will be shown later. Regarding the generator modeling, the detailed model is adopted for all of the generators to keep generality. This study mainly follows the detailed model proposed by Sauer [7].

For a multibus system, we have two references, system and machine, for each generator. The two representations based on each reference have the following relationships.

$$E_{Gi} = E_{Di} + jE_{Qi} = (\, E_{di} + jE_{qi} \,)\, e^{j(\delta_i - \pi/2)} \tag{37.a}$$

$$V_{Gi} = V_{Di} + jV_{Qi} = (\, V_{di} + jV_{qi} \,)\, e^{j(\delta_i - \pi/2)} \tag{37.b}$$

$$\text{or} \quad V_{Gi} = V_i e^{j\theta_i} \tag{37.c}$$

$$I_{Gi} = I_{Di} + j\,I_{Qi} = (I_{di} + j\,I_{qi})\,e^{j(\delta_i - \pi/2)} \quad (37.d)$$

$$\Psi_{Gi} = \Psi_{Di} + j\Psi_{Qi} = (\Psi_{di} + j\Psi_{qi})\,e^{j(\delta_i - \pi/2)} \quad (37.e)$$

$$E_{gi} = E_{di} + j\,E_{qi} \qquad I_{gi} = I_{di} + j\,I_{qi}$$
$$V_{gi} = V_{di} + j\,V_{qi} \qquad \Psi_{gi} = \Psi_{di} + j\,\Psi_{qi} \quad (38)$$

$$V_i = \sqrt{V_{di}^2 + V_{qi}^2} = \sqrt{V_{Di}^2 + V_{Qi}^2} \quad (39)$$

where V, E, Ψ : Complex Phasors for Generator Terminal Voltage, Internal Voltage and Flux respectively,

δ_i : Angle between Generator Reference and System Reference, Capital Subscript : system-reference-based representations, Lower Case Subscript : machine-reference-based representations.

By using the above relations, the following theorems can be developed. The pu(per unit) unit system is adopted throughout the mathematical development.

Theorem 1

The energy integral of generator output power in Eq. (36), can be represented by the cross-product line integral of current and volt- age phasors as follows:

$$\int_{(V_o,\theta_o)}^{(V,\theta)} \left(P_{Gi}d\theta_i + \frac{Q_{Gi}}{V_i}dV_i\right) = \int_{V_{Gio}}^{V_{Gi}} Im\,(I_{Gi}\,d\,V_{Gi}^*) \quad (40)$$

Proof)

$$\int_{(V_o,\theta_o)}^{(V,\theta)} \left(P_{Gi}d\theta_i + \frac{Q_{Gi}}{V_i}dV_i\right) = \int_{(V_o,\theta_o)}^{(V,\theta)} Re\left[\frac{P_{Gi} + j\,Q_{Gi}}{V_i}(V_i d\theta_i - jdV_i)\right]$$

$$= \int_{(V_o,\theta_o)}^{(V,\theta)} Re\left[\frac{P_{Gi} + j\,Q_{Gi}}{V_i e^{j\theta_i}}\,d\left(\frac{V_i e^{j\theta_i}}{j}\right)\right]$$

$$= \int_{V_{Gio}}^{V_{Gi}} Re\left[\frac{P_{Gi} + j\,Q_{Gi}}{V_{Gi}}\,\frac{d\,V_{Gi}}{j}\right]$$

$$= \int_{V_{Gio}}^{V_{Gi}} Im\,(I_{Gi}^*\,d\,V_{Gi})$$

Q. E. D.

Theorem 2

If the stator/system transients are negligible for Generator i, it holds that

$$\int_{(V_o,\theta_o)}^{(V,\theta)} \left(P_{Gi}d\theta_i + \frac{Q_{Gi}}{V_i}dV_i\right) = \int_{\delta_{oi}}^{\delta_i} P_{ei}d\delta_i + \int_{E_{goi}}^{E_{gi}} (I_{qi}dE_{di} - I_{di}dE_{qi})$$
$$- \int_{\delta_{oi}}^{\delta_i} r_{si}(I_{di}^2 + I_{qi}^2)d\delta_i - \int_{I_{goi}}^{I_{gi}} r_{si}(I_{di}dI_{qi} - I_{qi}dI_{di}) \quad (41)$$

with $E_{qi} = -(\omega_s + \omega_i)\Psi_{di}$, $E_{di} = (\omega_s + \omega_i)\Psi_{qi}$

P_{ei} : internal electric power (electrically converted power)

ω_s : rated angular velocity in [pu]

r_{si} : generator internal resistance

(The proof is given in the Appendix.)

Here it is noted that the assumption in the above theorem imposes few restrictions on applications of the theorem, since most of detailed models are developed under this assumption [7,16].

On the other hand, we have the following generator power relation:

$$P_{ei} = P_{mi} - M_i\dot{\omega}_i - D_{wi}\omega_i \quad (42)$$

with D_{wi} : generator wind damping coefficient

By using these two theorems and Eq. (42), one can easily obtain

$$\int_{\theta_o}^{\theta} \left(P_{Gi}d\theta_i + \frac{Q_{Gi}}{V_i}dV_i\right) = \int_{\delta_{oi}}^{\delta_i} P_{mi}d\delta_i - \tfrac{1}{2}M_i\omega_i^2 - \int D_i\omega_i^2 dt$$
$$+ \int_{E_{goi}}^{E_{gi}} (I_{di}dE_{qi} - I_{qi}dE_{di}) - \int_{\delta_{oi}}^{\delta_i} r_{si}(I_{di}^2 + I_{qi}^2)d\delta_i$$
$$- \int_{I_{goi}}^{I_{gi}} r_{si}(I_{di}dI_{qi} - I_{qi}dI_{di}) \quad (43)$$

The substitution of Eq. (43) into Eq. (36) gives the following equation:

$$W = E_{kin} + E_{PB} + E_{PG} + E_{r,loss} - E_{INPUT} + E_{OUTPUT}$$
$$+ \sum_{i=1}^{m}\left[\int_{I_{goi}}^{I_{gi}} r_{si}(I_{di}dI_{qi} - I_{qi}dI_{di}) - \int_{E_{goi}}^{E_{gi}} (I_{di}dE_{qi} - I_{qi}dE_{di})\right.$$
$$\left. + \int D_i\omega_i^2 dt\right] = 0 \quad (44)$$

where

$$E_{kin} = \sum_{i=1}^{m}\left[\tfrac{1}{2}M_i\omega_i^2\right] \quad (45.a)$$

$$E_{PB} = \sum_{i=1}^{N}\left[-\tfrac{1}{2}B_{ii}V_i^2 - \tfrac{1}{2}\sum_{j\neq i}^{N} V_iV_jB_{ij}\cos\theta_{ij}\right]\Bigg|_{(\theta_o,V_o)}^{(\theta,V)} \quad (45.b)$$

$$E_{PG} = \sum_{i=1}^{N}\left[\int_{\theta_o}^{\theta} G_{ii}V_i^2 d\theta_i + \tfrac{1}{2}\sum_{j\neq i} G_{ij}\int_{(\theta_o,V_o)}^{(\theta,V)} (V_iV_j\cos\theta_{ij}d\theta_i\right.$$
$$\left. + V_j\sin\theta_{ij}dV_i + V_iV_j\cos\theta_{ij}d\theta_j - V_i\sin\theta_{ij}\,dV_j)\right] \quad (45.c)$$

$$E_{r,loss} = \sum_{i=1}^{m}\int_{\delta_{oi}}^{\delta_i} r_{si}(I_{di}^2 + I_{qi}^2)d\delta_i \quad (45.d)$$

$$E_{INPUT} = \sum_{i=1}^{m}\int_{\delta_{oi}}^{\delta_i} P_{mi}\,d\delta_i \quad (45.e)$$

$$E_{OUTPUT} = \sum_{i}^{N} \left[c\int_{\theta_o}^{\theta} P_{Li} d\theta_i + c\int_{V_o}^{V} \left[\frac{Q_{Li} - Q_{Ci}}{V_i} \right] dV_i \right] \quad (45.f)$$

From this equation, we can obtain the following general energy integral for a multimachine power system with detailed-model generators.

$$E = E_{kin} + E_{PB} + E_{PG} + E_{r,loss} - E_{INPUT} + E_{OUTPUT} + \sum_{i=1}^{m} \left[c\int_{I_{goi}}^{I_{gi}} r_{si}(I_{di}\, dI_{qi} - I_{qi}\, dI_{di}) - c\int_{E_{goi}}^{E_{gi}} (I_{di}\, dE_{qi} - I_{qi}\, dE_{di}) \right] \quad (46)$$

By using Eq.(45), the time derivative of the above energy integral can be easily calculated as follows:

$$\frac{dE}{dt} = -\sum_{i=1}^{m} D_i\omega_i^2 \leq 0$$

These results provide a general approach for the energy function derivation for power systems, which is applicable to any kind of detailed generator model which meets the assumption that the stator/system transients are negligible.

V. Illustrative Example with the Use of the Eq' Model

Consider the derivation of the energy integral for a multimachine system, where all of the generators are represented by the E_q' model described as follows:

$$T'_{doi} \frac{dE'_{qi}}{dt} = -E'_{qi} - (X_{di} - X'_{di}) I_{di} + E_{fdi} \quad (47)$$

$$M_i \frac{d\omega_i}{dt} = T_{mi} - E'_{qi} I_{qi} - (X_{di} - X'_{di}) I_{di} I_{qi} - D_{wi}\omega_i \quad (48)$$

$$I_{di} = (E'_{qi} - V_i \cos\delta_{Li}) / X'_{di} \quad (49.a)$$

$$I_{qi} = V_i \sin\delta_{Li} / X_{qi} \quad (49.b)$$

where $\delta_{Li} = \delta_i - \theta_i$ = Ang E'_{qi} - Ang V_i : Load Angle (49.c)

For the above E_q' model, we have the following relations between E_{di}', E_{qi}' and E_{di}, E_{qi}.

$$E'_{di} = (X_{qi} - X'_{qi}) I_{qi} \quad (50.a)$$

$$E_{di} = E'_{di} + X'_{qi} I_{qi} = X_{qi} I_{qi} \quad (50.b)$$

$$E_{qi} = E'_{qi} - X'_{di} I_{di} \quad (50.c)$$

By using the above equations, we can easily obtain the following equation:

$$c\int_{E_{goi}}^{E_{gi}} (I_{di}\, dE_{qi} - I_{qi}\, dE_{di}) = -\tfrac{1}{2}(X_{di} I_{di}^2 + X_{qi} I_{qi}^2) + c\int_{E'_{qo}}^{E'_{qi}} I_{di}\, dE'_{qi} \quad (51)$$

By using Eq.(47), the last term of the above equation can be rewritten as

$$c\int_{E'_{qo}}^{E'_{qi}} I_{di}\, dE'_{qi} = c\int_{E'_{qoi}}^{E'_{qi}} \frac{1}{(X_{di} - X'_{di})} \left(-T'_{doi} \frac{dE'_{qi}}{dt} - E'_{qi} + E_{fdi} \right) dE'_{qi}$$

$$= \frac{1}{(X_{di} - X'_{di})} \left[-T'_{doi} \int \left(\frac{dE'_{qi}}{dt} \right)^2 dt - \tfrac{1}{2} E'^2_{qi} + c\int_{E'_{qoi}}^{E'_{qi}} E_{fdi}\, dE'_{qi} \right] \quad (52)$$

Some manipulations of Eq. (45) by substituting Eqs.(51) and (52) yields the following energy integral expression:

$$W = E_{kin} + E_{PB} + E_{PG} + E_{r,loss} + E_{INPUT} + E_{OUTPUT} + \sum_{i=1}^{m} \left[\tfrac{1}{2} \left\{ X_{di} I_{di}^2 + X_{qi} I_{qi}^2 + \frac{1}{(X_{di} - X'_{di})} E'^2_{qi} \right\} + c\int_{I_{goi}}^{I_{gi}} r_{si}(I_{di}\, dI_{qi} - I_{qi}\, dI_{di}) + \frac{T_{doi}}{(X_{di} - X'_{di})} \int \left(\frac{dE'_{qi}}{dt} \right)^2 dt - \frac{1}{(X_{di} - X'_{di})} c\int_{E'_{qoi}}^{E'_{qi}} E_{fdi}\, dE'_{qi} + \int D_i\omega_i^2 dt \right] = 0 \quad (53)$$

Here, some appropriate assumptions are required to obtain a path-independent energy function from the above path-dependent energy integral. For instance, the following assumptions are adopted:

i) all resistances are negligible, and
ii) all governors and exciters are fixed, i.e., all P_{mi} and E_{fi} are constant, and
iii) real loads are constant, and reactive loads are dependent only on bus voltages.

Under these assumptions, we can eliminate all the path-dependent terms, and easily find an energy function E_1:

$$E_1 = \sum_{i=1}^{m} \left[\tfrac{1}{2} M_i(\omega_i^2 - \omega_{io}^2) - \sum_{i=1}^{N} \left[\frac{1}{2} B_{ii} V_i^2 + \frac{1}{2} \sum_{j \neq i} V_i V_j B_{ij} \cos\theta_{ij} \right] \right] \Bigg|_{(\theta_o, V_o)}^{(\theta, V)}$$
$$+ \sum_{i=1}^{m} \left[\tfrac{1}{2} \left[X_{di}(I_{di}^2 - I_{dio}^2) + X_{qi}(I_{qi}^2 - I_{qio}^2) + \frac{1}{(X_{di} - X'_{di})} E'^2_{qi} \right] - \frac{E_{fdi}(E'_{qi} - E'_{qio})}{(X_{di} - X'_{di})} \right] + \sum_{i=1}^{N} \left[-P_{mi}(\delta_i - \delta_{io}) + P_{Li}(\theta_i - \theta_{io}) + \int_{V_{io}}^{V_i} \left[\frac{Q_{Li}(V_i) - Q_{Ci}(V_i)}{V_i} \right] dV_i \right] \quad (54)$$

By the virtue of Eq.(53), the nonpositiveness of the time derivative of the above energy function can be easily proved as follows:

$$\frac{dE_1}{dt} = -\sum_{i=1}^{m} \left[D_i\omega_i^2 + \frac{T_{doi}}{(X_{di} - X'_{di})} \left(\frac{dE_{qi}}{dt} \right)^2 \right] \leq 0 \quad (55)$$

If the effects of resistances are considered, we can adopt the following path-dependent integral as an energy function E_2:

$$E_2 = E_{kin} + E_{PB} + E_{PG} + \sum_{i=1}^{m} \left[\tfrac{1}{2} \left\{ X_{di}(I_{di}^2 - I_{dio}^2) + X_{qi}(I_{qi}^2 - I_{qio}^2) + \frac{1}{(X_{di} - X'_{di})} E'^2_{qi} \right\} - \frac{E_{fdi}(E'_{qi} - E'_{qio})}{(X_{di} - X'_{di})} + c\int_{\delta_{oi}}^{\delta_i} r_{si}(I_{di}^2 + I_{qi}^2)\, d\delta_i \right.$$

$$+ c\int_{I_{goi}}^{I_{gi}} r_{si}(I_{di}\, dI_{qi} - I_{qi}\, dI_{di}) \Big]$$
$$+ \sum_{i=1}^{N} \left[-P_{mi}(\delta_i - \delta_{io}) + P_{Li}(\theta_i - \theta_{io}) + \int_{V_{io}}^{V_i} \left[\frac{Q_{Li}(V_i) - Q_{Ci}(V_i)}{V_i} \right] dV_i \right] \quad (56)$$

Here, it is noted that all of the path-dependent integral terms are related to resistances, and that the saliency of the generator does not aggravate the path dependency of energy integral. The above energy function has the same time derivative as given in Eq.(55).

Moreover, energy function E_2 has the same SEPs and UEPs near to those of energy functions E_1. This can be easily checked by observing the following equilibrium conditions:

$\frac{\partial E}{\partial \omega_i} = 0$ gives $\omega_i = 0$ (i= 1,⋯,m)

$\frac{\partial E}{\partial \delta_i} = 0$ gives the swing equation for generator i (i= 1,⋯,m)

$\frac{\partial E}{\partial E'_{qi}} = 0$ gives an equation to determine E'_{qi} (i= 1,⋯,m)

$\frac{\partial E}{\partial \theta_i} = 0$ gives the real power equation for bus i (i= 1,⋯,n)

$\frac{\partial E}{\partial V_i} = 0$ gives the reactive power equation for bus i (i= 1,⋯,n)

where E can be either of the energy functions E_1, E_2

By combining all of the above equations, we can see that the SEP and UEP are the same as the solutions of the load flow equations. Once SEP and UEPs are calculated from the load flow equations, the system stability can be immediately determined by comparing the system energy with the UEP energy. If the path dependencies can be eliminated somehow, the detailed model of generator can be considered with little increase in the computational burdens.

VII. Conclusions

In this paper, an exact equivalent EMM is systematically developed for multimachine systems to reflect line resistances and reactive powers as well by introducing imaginary springs for transmission line resistances. The proposed EMM has the following features:

- The effects of reactive powers are exactly reflected by allowing bus voltage changes with a structure-preserved model.
- Line resistances are directly taken into account as a part of EMM structure.
- The proposed EMM shows the correspondences of current-force and voltage-displacement between a real power system and its EMM.

On the basis of the proposed EMM, an exact energy integral expression is derived for lossy multibus systems through rigorous energy analysis. By using the energy integral, the exact energy conservation rule has been discussed. This rule can be utilized as a useful tool to show that an energy function has a semi-negative time derivative. Several useful theorems are presented regarding the relationships between generator mechanical input and electrical output. By using these theorems, a general approach is presented to derive an energy integral to deal with various kinds of generator models. The proposed approach exactly reflects both the internal resistance and the saliency of the generator into the proposed energy function. Finally, an illustrative example is given for a multimachine system adopting the E_q'-model for generators, which shows that the consideration of the detailed generator model does not aggravate the complicacy of the direct method of stability analysis in multibus systems.

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APPENDIX : Proof of Theorem 2

The full model of generator gives the following terminal voltage equations:

$$\frac{1}{\omega_s}\frac{d\Psi_d}{dt} = r_sI_d + \frac{\omega^+}{\omega_s}\Psi_q + V_d \qquad (A.1)$$

$$\frac{1}{\omega_s}\frac{d\Psi_q}{dt} = r_sI_q - \frac{\omega^+}{\omega_s}\Psi_d + V_q \qquad (A.2)$$

$$\frac{1}{\omega_s}\frac{d\Psi_o}{dt} = r_sI_o + V_o \qquad (A.3)$$

where $\omega^+ = \omega_s + \omega$ with ω_s : rated angular velocity in [pu]. By adding Eqs. (A.1) and j×(A.2), we get

$$\frac{1}{\omega_s}\frac{d}{dt}(\Psi_d+j\Psi_q) = r_s(I_d+jI_q) - \frac{\omega^+}{\omega_s}(\Psi_d+j\Psi_q) + (V_d+jV_q) \qquad (A.4)$$

With the use of phasor representations, the above equation can be rewritten as:

$$\frac{1}{\omega_s}\frac{d\Psi_g}{dt} = r_s I_g - \frac{\omega^+}{\omega_s} j\Psi_g + V_g \qquad (A.5)$$

For generators connected to a power system, in general, the electrical dynamics of the stator/network can be assumed to be very fast so that all flux linkages change instantaneously during transients. With the use of this assumption, Eqs. (A.1)- (A.3) can be rewritten as:

$$r_s I_g - \frac{\omega^+}{\omega_s} j\Psi_g + V_g \doteqdot 0 \qquad (A.6)$$

From Eqs.(37) and (38) regarding the terminal voltage, the following relation holds.

$$V_{Gi} = V_{gi}\, e^{j(\delta_i-\pi/2)} \qquad (A.7)$$

The substitution of Eq.(A.6) into the above equation gives

$$V_{Gi} \doteqdot \left[\frac{\omega^+}{\omega_s} j\Psi_{gi} - r_{si} I_{gi}\right] e^{j(\delta_i-\pi/2)} \qquad (A.8)$$

By using Eq.(A.8), we have the following equation:

$$\int_{V_{Gio}}^{V_{Gi}} Im\,(I_{Gi}^{*}\, dV_{Gi})$$

$$\doteq \int_{V_{Gio}}^{V_{Gi}} Im\left[I_{gi}^{*}\, e^{-j(\delta_i-\pi/2)}\, d\left[\left(\frac{\omega_i^{+}}{\omega_s} j\Psi_{gi} - r_{si} I_{gi}\right) e^{j(\delta_i-\pi/2)} \right]\right]$$

$$= \int_{\delta_{io}}^{\delta_i} Im\left(-I_{gi}^{*} \frac{\omega_i^{+}}{\omega_s}\Psi_{gi}\right) d\delta_i + \int_{\Psi_{qio}}^{\Psi_{qi}} Re\left[I_{gi}^{*}\, d\left(\frac{\omega_i^{+}}{\omega_s}\Psi_{gi}\right)\right]$$

$$- \int_{\delta_{io}}^{\delta_i} Re\,(I_{gi}^{*}\, r_{si} I_{gi})\, d\delta_i + \int_{I_{gio}}^{I_{gi}} Im\,(I_{gi}^{*}\, r_{si}\, dI_{gi})$$

$$= \int_{\delta_{io}}^{\delta_i} \frac{\omega_i^{+}}{\omega_s}(I_{qi}\Psi_{di} - I_{di}\Psi_{qi})\, d\delta_i + \int_{\Psi_{qio}}^{\Psi_{qi}} \left[I_{Di}\, d\left(\frac{\omega_i^{+}}{\omega_s}\Psi_{di}\right) + I_{qi}\, d\left(\frac{\omega_i^{+}}{\omega_s}\Psi_{qi}\right)\right]$$

$$- \int_{\delta_{io}}^{\delta_i} r_{si}\,(I_{di}^2 + I_{qi}^2)\, d\delta_i + \int_{I_{gio}}^{I_{gi}} r_{si}\,(I_{di}dI_{qi} - I_{qi}dI_{Di}) \quad (A.9)$$

On the other hand, the generator induced voltages are given by

$$E_{qi} = \frac{\omega_i^{+}}{\omega_s}\Psi_{di} = \left(1 + \frac{\omega_i}{\omega_s}\right)\Psi_{di}$$

$$E_{di} = -\frac{\omega_i^{+}}{\omega_s}\Psi_{qi} = -\left(1 + \frac{\omega_i}{\omega_s}\right)\Psi_{qi}$$

The electric output of the generator is also given by

$$P_{ei} = E_{qi}I_{qi} + E_{di}I_{di} = \frac{\omega_i^{+}}{\omega_s}(I_{qi}\Psi_{di} - I_{di}\Psi_{qi})$$

Consequently, the substitution of the above equations into Eq.(A.9) yields

$$\int_{(V_o,\theta_o)}^{(V,\theta)} \left(P_{Gi}d\theta_i + \frac{Q_{Gi}}{V_i}dV_i\right) = \int_{V_{Gio}}^{V_{Gi}} Im\,(I_{Gi}^{*}\, dV_{Gi})$$

$$= \int_{\delta_{oi}}^{\delta_i} P_{ei}d\delta_i + \int_{E_{goi}}^{E_{gi}} (I_{qi}\, dE_{di} - I_{di}\, dE_{qi})$$

$$- \int_{\delta_{oi}}^{\delta_i} r_{si}(I_{di}^2 + I_{qi}^2)\, d\delta_i - \int_{I_{goi}}^{I_{gi}} r_{si}(I_{di}\, dI_{qi} - I_{qi}\, dI_{di}) \quad \text{Q.E.D.}$$

On the other hand, we can derive several types of energy functions which have different energy dissipating ratios, from the energy conservation law in Eq.(53). We can obtain energy functions E_3 and E_4 by including the flux-related loss term and the damping loss term respectively, E_5 simultaneously in the energy function.

$$E_3 = E_2 - \frac{T_{doi}}{(X_{di} - X_{di}')}\int_{E_{qoi}'}^{E_{qi}'} E_{qi}\, dE_{qi}' \quad (57)$$

$$E_4 = E_2 + \sum_{i=1}^{m}\left[\int_{\delta_{oi}}^{\delta_i} D_i\omega_i d\delta_i\right] \quad (58)$$

$$E_5 = E_2 - \frac{T_{doi}}{(X_{di} - X_{di}')}\int_{E_{qoi}'}^{E_{qi}'} E_{qi}\, dE_{qi}' + \sum_{i=1}^{m}\left[\int_{\delta_{oi}}^{\delta_i} D_i\omega_i d\delta_i\right] \quad (59)$$

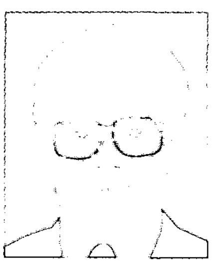
All of the above energy functions have nonpositive time derivatives as follows:

$$\frac{dE_3}{dt} = -\sum_{i=1}^{m}\left[D_i\omega_i^2\right] \leq 0 \quad (60)$$

$$\frac{dE_4}{dt} = -\sum_{i=1}^{m}\left[\frac{T_{doi}}{(X_{di} - X_{di}')}\left(\frac{dE_{qi}'}{dt}\right)^2\right] \leq 0 \quad (61)$$

$$\frac{dE_5}{dt} = 0 \quad \text{(identically zero)} \quad (62)$$

Here, it is noted that all of the above energy functions E_2-E_5 have the same SEPs and UEPs.



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