Estimation of Channel States for Adaptive Code Rate Change in DS-SSMA Communication Systems: Part 1. Estimation of Effective Number of Users

Youngkwon Ryu, Iickho Song, Taejoo Chang, and Suk Chan Kim

Abstract

Adaptive code rate change schemes in DS-SSMA systems are proposed. In the proposed schemes, the error correcting code rate is changed according to the channel states. Two channel states having significant effects on the bit error probability are considered: one is the effective number of users, and the other is the fading environment. These channel states are estimated based on retransmission requests. The criterion for the change of the code rate is to maximize the throughput under given error bound.

I. Introduction

In FH-SSMA systems using Reed-Solomon error correcting codes, optimum code rate and optimum number of users for maximum throughput can be obtained when the number of hopping frequencies and packet error probability are given [1]. A simple retransmission control scheme for slotted FH-SSMA communication systems has been proposed in which the retransmission probability is changed adaptively so that the channel throughput can be maximized [2]. In DS-SSMA systems, however, neither optimum code rate nor optimum number of users has been obtained. In DS-SSMA systems, packet error probability and channel throughput are merely increasing functions of the code rate and number of users.

In this paper, a transmission scheme is proposed which controls the error correcting code rate adaptively to maximize throughput when the packet error probability is fixed in DS-SSMA communication systems. In this series of two papers, retransmission requests is used to estimate the channel states, *i.e.*, the number of users and fading environment. These two channel states have major effects on the bit error performance. Note that an adaptive error control technique for slowly varying channels has recently [3] been applied to the *single user communication environment*.

Two cases of channel state estimation are considered: one is the estimation of the effective number of users when the fading environment is known. Knowledge of the number of users is essential for the performance analysis in DS-SSMA systems. The other is the estimation of the fading environment when the effective number of users is known. The characteristics of a channel felt by mobile terminal change as the terminal moves, which result in different fading conditions over a period of communication.

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II. The System Model

The system considered here is shown in Figure 1. In Figure 1, $a_k(t)$, $b_k(t)$, $d_k(t)$, and θ_k are the random signature signal, encoded signal, data signal, and phase of the kth carrier, respectively. The common carrier (angular) frequency ω_c is known, and K is the number of users. The signals $a_k(t)$ and $b_k(t)$ can be written as

$$a_k(t)=\sum_{i=-\infty}^{\infty} a_i^{(k)} P_{T_c}(t-iT_c) \tag{1}$$

and

$$b_k(t) = \sum_{i=-\infty}^{\infty} b_i^{(k)} P_T(t-iT) \tag{2}$$

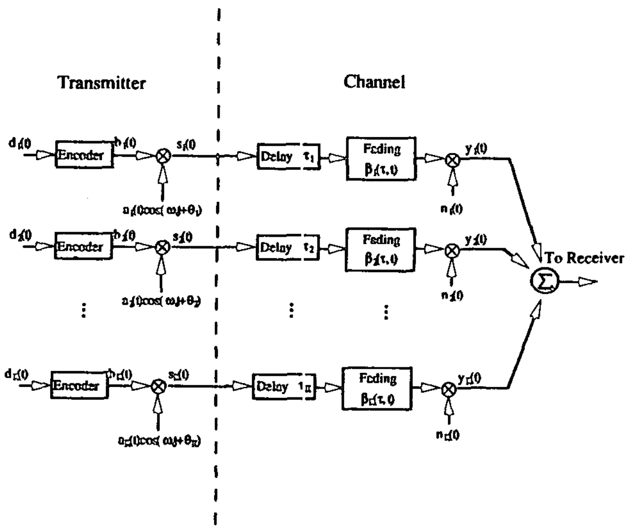

Fig. 1. The System Model.

where $\{a_i^{(k)}\}_{i=-\infty}^{\infty}$ is the signature sequence assigned to the kth user, $\{b_i^{(k)}\}_{i=-\infty}^{\infty}$ is the data bit stream, T is the bit duration, and T_c is the chip duration. In (1) and (2), $P_\tau(t)=1$ for $0 \le t \le \tau$, and $P_\tau(t)=0$ otherwise. We assume that each signature sequence has an integer period $N=T/T_c$. The transmitted signal for the kth user can be written as

$$s_{k(t)} = \sqrt{2P}\,\mathrm{Re}[\; a_k(t)\, b_k(t) \exp(j\omega_c t + j\theta_k)] \tag{3}$$

where P is the signal power.

Assuming a Rician fading channel, the faded signal can be written as [4]

$$y_k(t) = \mathrm{Re}\{u_k(t-\tau_k)\} + n_{k(t)}, \tag{4}$$

where $n_{k(t)}$ is the additive white Gaussian noise (AWGN) term,

$$u_k(t) = \gamma \int_{-\infty}^{\infty} \beta_k(\tau, t) s_k(t-\tau) d\tau + s_k(t) \tag{5}$$

with γ the transmission coefficient (or, fading intensity) of the channel and $\beta_k(\tau, t)$ a zero-mean complex Gaussian random process. We assume that the channel states, the effective number of users (which results from near cell interference) and the fading environment, are varying slowly.

Figure 2 shows the receiver model, where it is assumed that the phase of the despreading signal is matched to the received signal, *i.e.* $\phi_k = \theta_k - \omega_c \tau_k$, and the selective repeat request strategy is employed in the ARQ [5]. For computational convenience, Reed-Solomon code is used for error control, although use of convolutional codes might be a more meaningful investigation.

There exist three ways to change the code rate. In this paper, we change the information bit length and fix the code word length. We assume that backloged packets, requested through an ideal backward channel, are retransmitted in the next packet transmission slot with probability 1. In any time slot, K packets are transmitted: if a user is requested for a retransmission the user is not allowed to send a new packet.

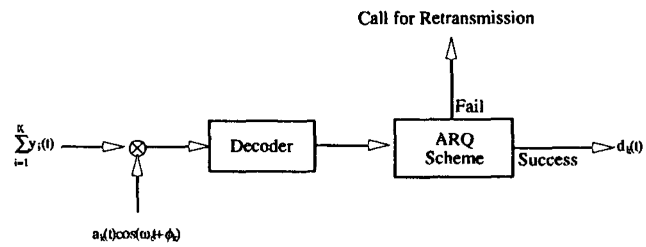

Fig. 2. The Receiver Model.

1. Bit Error Probability

Assuming the number of users is large enough to satisfy the central limit theorem, it has been shown in [4] that the approximation of multiple access interference (MAI) to AWGN results in quite close calculation of the bit error probability: the bit error probability is

$$P_b = Q[\;\overline{\mathrm{SNR}}\;] \tag{6}$$

where

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^{\infty} \exp(-u^2/2) du \tag{7}$$

and $\overline{\mathrm{SNR}}$ is the effective signal to noise ratio (SNR) with the approximation taken into account. The effective signal to noise ratio at the input of the decoder in Figure 2 has been calculated in [4], and is shown in [7]. The estimation procedures in Parts 1 and 2 are based on the observation that $\overline{\mathrm{SNR}}$ in (6) can be expressed as

$$\overline{\mathrm{SNR}} = (CK' + D)^{-1/2} \tag{8}$$

where K' is the effective number of users and C and D are determined by the fading types explained in [7]. The parameters C and D are functions of the fading intensity, fading range, channel covariance, random signature sequence length, and SNR of AWGN.

2. Packet Error Probability

Let P_c, P_d, and P_u denote the probabilities that a decoded packet contains no error, detectable errors, and undetectable errors, respectively. Then we have $P_c+P_d+P_u=1$. Since Reed-Solomon code can correct up to $\lfloor (n-k)/2 \rfloor$ errors, it is easy to see that

$$P_c = \sum_{i=0}^{\lfloor (n-k)/2 \rfloor} \binom{n}{i} P_b^i (1-P_b)^{n-i}. \tag{9}$$

where $\lfloor x \rfloor$ is the largest integer not greater than x, n is the codeword length, and k is the information bit length. We also have

$$P_u = \sum_{i=n-k+1}^{n} A_i P_b^i (1-P_b)^{n-i} \tag{10}$$

where A_i is the weight distribution [8]

$$A_i = \binom{n}{i} \sum_{j=0}^{i-n+k-1} (-1)^j \binom{i}{j} [(n+1)^{i-n+k-j} - 1] \tag{11}$$

The probability P_E that the receiver commits an error when the ARQ strategy is used is [9]

$$P_E = \sum_{i=1}^{\infty} P_u P_d^{i-1} = \frac{P_u}{1-P_d} = \frac{P_u}{P_c + P_u} \tag{12}$$

III. Adaptive Code Rate Change Algorithms

In general, the error performance of the DS-SSMA system is worse than that would be caused by K real users counted by the base station because of near cell interference. The added error performance degradation effects result in increase of the effective number of users K'. Note that P_E defined in (12) is now a function of the code rate k/n, the effective number of users K', fading environment, and the SNR.

1. Code Rate for Maximum Throughput

In this paper, the performance criterion is to maximize the throughput when P_E is limited to a fixed value, where the throughput is defined as

$$E = \frac{K}{N} \frac{k}{n} (P_u + P_c) \tag{13}$$

(Note that N is defined to be T/T_c in Section 2.) Thus to get a higher throughput, we should use a higher code rate and allow more users to communicate. Since P_E and the throughput are merely increasing functions of the number of effective users and code rate the code rate which satisfies the P_E bound maximizes the channel throughput.

When the effective number of users is given and the fading environment is known, we calculate P_E for all code rates ($k=1,2,\cdots,n$) by making use of (6), (9), (10), and (12). We then select the highest code rate which results in less P_E than the given bound. Figure 3 shows the code rate for maximum throughput when the number of effective users and P_E bound are given under time selective fading. In this figure, $n=15$, N=1023, the signal power $\varepsilon = PT$, and the pure AWGN power is N_0. As expected, the code rate becomes high as the effective number of users decreases, and the effect of the pure AWGN on P_E is more severe when γ is smaller.

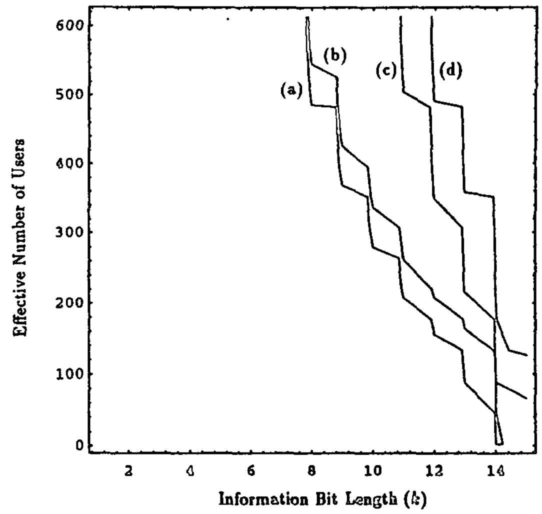

Fig. 3. Code Rate Bound for Time Selective Fading and Triangular Channel Covariance Function ($P_E = 10^{-1}$).

(a) γ=1.0, λ=1.5, $\frac{\varepsilon}{N_0}$ =10dB
$\Rightarrow C$=8.689 × 10^{-4}, D=5.011 × 10^{-2}
(b) γ=1.0, λ=1.5, $\frac{\varepsilon}{N_0}$ =0
$\Rightarrow C$=8.689 × 10^{-4}, D=1.081 × 10^{-4}
(c) γ=0.05, λ=1.5, $\frac{\varepsilon}{N_0}$ =10dB
$\Rightarrow C$=3.530 × 10^{-4}, D=4.970 × 10^{-2}
(c) γ=0.05, λ=1.5, $\frac{\varepsilon}{N_0}$ =0
$\Rightarrow C$=3.530 × 10^{-4}, D=-3.041 × 10^{-4}.

2. Estimation of Effective Number of Users

In FH-SSMA systems, the bit error probability is characterized by the hit probability [10]. If a hitting occurs, the bit information is lost entirely. It is thus expected that the error performance will be degraded fast as the number of users becomes larger than the optimum number of users. In a DS-SSMA system, however, increase of users is felt by all users as the gradual degradation of bit error probability performance. Consequently, any number of users can communicate simultaneously without much loss of information if the error correcting code rate is controlled properly.

In mobile communication systems, since the base station should assign a random signature sequence to each user, the base station normally knows the number of users. The effective number of users, however, may be different from the number of real users due to the interference from near cells as explained at the beginning of Section 3. Therefore we need to estimate the effective number of users to adequately change the code rate.

Figure 4 shows our estimation scheme, where we assume slotted communication: that is, each user transmits simultaneously one packet per slot. In Figure 4, T_s is the slot interval, $T_M = LT_s$ is the channel monitoring interval (CMI), L is the channel monitoring slot number, R_i is the number of backloged packets in the ith slot, and K'_i is the estimate of K' in the ith transmission slot. We assume that the code rate is constant during a CMI and can be changed at the start of every CMI. We assume K is constant over a CMI, which can be achieved by not accepting new users until the end of a CMI and allowing them to communicate at the beginning of the next CMI. Once the estimation of the effective number of users is accomplished, we can optimize the code rate based on the estimated effective number of users. The effect of the interference from near cells, the number of effective users minus the number of real users, can be used for several CMIs since the near cell interference is assumed to be slowly-varying.

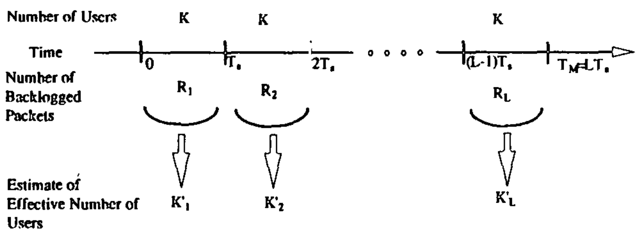

Fig. 4. Estimation of Effective Number of Users.

More specifically, at the ith transmission slot, we can obtain K'_i from

$$KP_R(K'_i|k) = R_i, \tag{14}$$

where k is the information bit length trimmed to satisfy P_E limit for the previously estimated effective number of users and $P_R(K'_i|k)$ denotes the retransmission probability for K'_i effective users when the code rate is $r = k/n$. This probability can be calculated using (9) and (10), or

$$P_R(A|k) = P_d|_{K'=A} = 1 - P_u|_{K'=A} - P_c|_{K'=A} \tag{15}$$

for Reed-Solomon error correcting codes.

At the end of the CMI, we take

$$\overline{K_L'} = \max\left\{\frac{1}{L}\sum_{i=1}^{L} K_i', K\right\} \tag{16}$$

as the estimate of the effective number of users. If the number of new calls minus that of dismissed calls during the CMI is K_n, we assume that $\overline{K_L'} + K_n$ effective users are on the line in the next CMI and change the code rate using the results in Section 3.1 based on this number.

The duration of the CMI can be adjusted. If the estimate of the effective number of users is decided to have converged within certain accuracy, we may stop the estimation procedure in a CMI. Convergence of the estimated value can be determined by, for example,

$$\frac{Var[K_J']}{\overline{K_J'}} \le \alpha \tag{17}$$

where

$$Var[K_J'] = \frac{1}{J}\sum_{i=1}^{J}(K_i')^2 - \overline{K_J'}^2 \tag{18}$$

and α is a threshold set to satisfy the estimation accuracy. (That is, if (17) is satisfied, we stop the estimation procedure at the Jth slot.)

Figure 5 shows simulation results of the estimation where k_{opt} is the information bit length which results in the maximum throughput under the P_E bound 10^{-4}. In Figure 5, L=20 and the fading is time selective with $\lambda = 1.5$, $\gamma = 1.0$, and $\frac{\varepsilon}{N_0}$ = 10dB. We can see that even when the code rate is incorrectly selected in a CMI, we can estimate the effective number of users to some degree of accuracy.

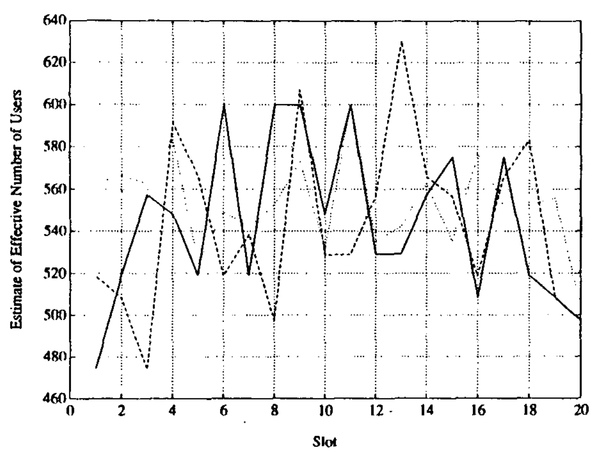

Fig. 5. Simulation Results for K=500 and K'=550.
(a) When k_{opt} is used $\Rightarrow$ $\overline{k_{20}'}$ = 544 ——
(b) When $k_{opt}-1$ is used $\Rightarrow$ $\overline{k_{20}'}$ = 543 : - - -
(c) When $k_{opt}+1$ is used $\Rightarrow$ $\overline{k_{20}'}$ = 553 : ……

The estimate of the effective number of users can then be used to change the code rate adaptively based on the results of Section 3.1. A simulation result for L=20 is shown in Figure 6. In Figure 6, it is assumed that the number of users is constant over 5 CMIs and the estimation procedure is accomplished every 5 CMIs. Note that the estimation results in incorrect code rate when the number of users is small. This seems to stem from the AWGN approximation of MAI, which is based on the assumption of large enough number of users.

IV. Summary

In this paper, we proposed an adaptive code rate change scheme in DS-SSMA systems. In the proposed scheme, the error correcting code rate was changed according to an estimate of the effective number of users, which has significant effects on the bit error probability.

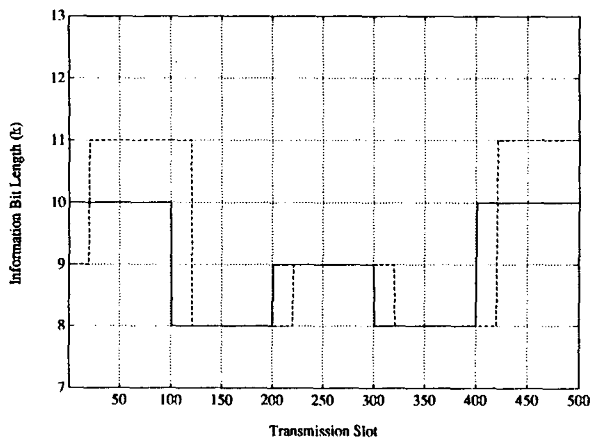

Fig. 6. Simulation for a Time Selective Fading with Triangular Channel Covariance Function: $\gamma=1.0$, $\lambda=1.5$, $\frac{\epsilon}{N_0}=10\text{dB}$, $P_E=10^{-1}$(real line - theory, dashed line - simulation).

We estimated the effective number of users by making use of the retransmission requests.

The criterion for the change of the code rate based on the estimated channel states was to maximize the throughput under given error bound.

In Part 2, we will consider a scheme in which the error correcting code rate will be changed according to the estimated fading environment, which also has significant effects on the bit error probability.

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