Robust Stability and Transient Behavior of a Two - Degree - of - Freedom Servosystem

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2자유도 서보계의 강인한 안정성 및 과도특성

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Abstract

This paper considers robust stability and transient behavior of the Two - Degree - of - Freedom(2DOF) servosystem. A class of uncertainties allowed in the plant model is obtained, to which the servosystem is robustly stable for any gain of the integral compensator. This result implies that if the plant uncertainty is the allowable set defined by the condition, a high - gain compensation can be carried out preserving stability to achieve a high - speed tracking response. The transient behavior attainable by the limit of the high - gain compensation is calculated using the singular perturbation approach.

1. Introduction

One of the most basic problems occurring in control engineering is the design problem associated with finding realistic controllers to solve the robust servosystems problem. In this type of problem, it is well known to associate integral compensators in servosystems for constant reference signals in order to reject the steady - state tracking error[1].

However, if we have an exact mathematical model of the plant and there is no disturbance to the plant, the integral compensation is not necessary. From this point of view, a two - degree - of - freedom(2DOF) servosystem has been proposed in the recent literature[2,3]. The present paper considers robust stability and transient behavior of the 2DOF servosystem. A class of uncertainties allowed in the plant model is obtained, to which the servosystem is robustly stable for any gain of the integral compensator. This result implies that the high - gain compensation can be carried out preserving stability to achieve a high - speed tracking response.

In general, a high - gain compensation can not be carried out in the 1DOF servosystem because it can act to cause the system to be unstable. In this context, under the robust properties which we propose, the transient behavior attain able by the limit of the high - gain compensation is calculated using the singular perturbation approach.

2. Two - Degree - of - Freedom Servosystem

In the development to follow, the plant to be controlled is assumed to be described by the following linear time invariant model :

$$\dot{x}(t)=Ax(t)+Bu(t)$$
$$y(t)=Cx(t) \qquad (1)$$

where $x\in R^n$ is the state vector of the system, $u \in R^m$ are the inputs, $y\in R^m$ are the outputs to be regulated, and A, B and C represent real constant matrices of appropriate dimensions.

For this system, consider the step type reference signals as follows :

$$r(t)=\begin{cases} r_+ (t\geq 0) \\ r_- (t<0) \end{cases} \qquad (2)$$

and r_+ are given at $t=0$. So far, it is required that the plant outputs y track the reference signals r. For this purpose, assume that pair(A, B) is stabilizable and

$$rank\begin{bmatrix} A & B \\ C & 0 \end{bmatrix} = n+m \qquad (3)$$

Based on these assumptions, a 2DOF servosystem described in Fig. 1 has been proposed by Fujisaki and Ikeda[2]. In Fig. 1, it is considered that there exists F_0 such that $A+BF_0$ is stable, but, in Reference 2), an optimal regulator gain is used as F_0. And F_1, H_0 are

$$F_1=C(A+BF_0)^{-1}$$
$$H_0=[-C(A+BF_0)^{-1}B]^{-1} \qquad (4)$$

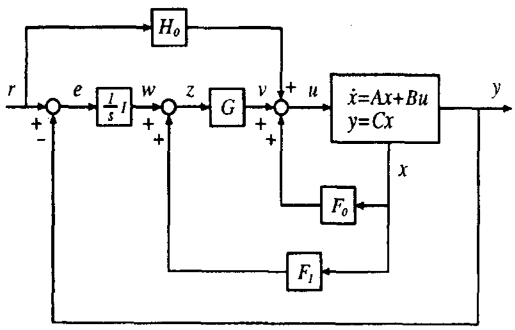

Fig.1. Two - Degree - of - Freedom Servosystem

In this context, for simplicity, assume step changes in the reference inputs take place only when the system is at steady state. Therefore, the servosystem shown in Fig. 1 is given by

$$\begin{bmatrix} \dot{x}(t) \\ \dot{w}(t) \end{bmatrix} = \begin{bmatrix} A+B(F_0+GF_1) & BG \\ -C & 0 \end{bmatrix}\begin{bmatrix} x(t) \\ w(t) \end{bmatrix} + \begin{bmatrix} BH_0 \\ I \end{bmatrix} r(t)$$
$$y(t)=\begin{bmatrix} C & 0 \end{bmatrix}\begin{bmatrix} x(t) \\ w(t) \end{bmatrix} \qquad (5)$$

The system derived above should be stable for working well as a 2DOF servosystem. To verify the stability of this system, we note the system matrix of (4) given by

$$\begin{bmatrix} A+B(F_0+GF_1) & BG \\ -C & 0 \end{bmatrix}$$
$$=\begin{bmatrix} I & 0 \\ -F_1 & I \end{bmatrix}\begin{bmatrix} A+BF_0 & BG \\ 0 & F_1BG \end{bmatrix}\begin{bmatrix} I & 0 \\ -F_1 & I \end{bmatrix}^{-1} \qquad (6)$$

and assume that there exists G such that F_1BG is a negative definite matrix. As an example, let a gain G be denoted as follows :

$$G=-R^{-1}(F_1B)^TW \qquad (7)$$

where it is considered that F_1B is nonsingular[2] and R, W are arbitrary positive definite matrices. In this case, a negative definite matrix F_1BG for the system (4) to be stable can be derived using the similar transformation of negative matrix as follows :

$$F_1BG = W^{-1/2}[-W^{1/2}F_1BR^{-1}(F_1B)^TW^{1/2}]W^{1/2} \quad (8)$$

From this fact, it is concluded that the system (4) is stable.

3. Robust stability Independent of Integral Compensation

In the previous chapter, it has been shown that if there are no modeling error of the system and disturbance to the plant, the 2DOF system given in Fig. 1 is stable for the arbitrary positive matrices R, W in which the gain G given in (7) is used.

But, if there are uncertainties as linear parameter variations of A, B and C, by selecting a gain G, the system may become unstable.

From this point of view, in this chater, a conditions for the system to be robustly stable independent of the gain G will be obtained.

Now, suppose there are uncertainties described as linear parameter variations of A, B and C, and disturbance inputs to the plant, then the system (1) is

$$\begin{aligned} \dot{x}(t) &= (A_0+\Delta A)x(t)+(B_0+\Delta B)u(t)+Dd(t) \\ y(t) &= (C_0+\Delta C)x(t) \end{aligned} \quad (9)$$

where A_0, B_0 and C_0 are nominal variables of A, B and C, and ΔA, ΔB and ΔC denote uncertainties.

In this case, the 2DOF servosystem described in (4) is represented as follows :

$$\begin{bmatrix} \dot{x}(t) \\ \dot{w}(t) \end{bmatrix} = \begin{bmatrix} A_0+\Delta A+(B_0+\Delta B)(F_0+GF_1) & (B_0+\Delta B)G \\ -(C_0+\Delta C) & 0 \end{bmatrix} \begin{bmatrix} x(t) \\ w(t) \end{bmatrix} + \begin{bmatrix} (B_0+\Delta B)H_0 \\ I \end{bmatrix} r(t) + \begin{bmatrix} D \\ 0 \end{bmatrix} d(t)$$

$$y(t) = \begin{bmatrix} C_0+\Delta C & 0 \end{bmatrix} \begin{bmatrix} x(t) \\ w(t) \end{bmatrix} \quad (10)$$

where F_0, F_1 and H_0 are calculated using the nominal variables A_0, B_0 and C_0. That is, there exists F_0 such that $A_0+B_0F_0$ is stable and

$$\begin{aligned} F_1 &= C_0(A_0+B_0F_0)^{-1} \\ H_0 &= [-C_0(A_0+B_0F_0)^{-1}B_0]^{-1} \end{aligned} \quad (11)$$

are obtained. Futhermore, assume the gain G given in (7) is obtained from the nominal value of B_0, and let R be fixed, such that

$$G = G_0W, \quad G_0 = -R^{-1}(F_1B_0)\mathrm{T} \quad (12)$$

here, consider the positive definite matrix W is adjustable.

Now, let the nominal part of the system matrix(10) be denoted by $\tilde{A}_0(W)$ and the uncertain part be denoted by $\Delta\tilde{A}(\mathrm{W})$, then they are given as follow :

$$\tilde{A}(W) = \begin{bmatrix} A_0+B_0F_0+B_0G_0WF_1 & B_0G_0W \\ -C_0 & 0 \end{bmatrix}$$

$$\Delta\tilde{A}(W) = \begin{bmatrix} \Delta A_0+\Delta B_0F_0+\Delta B_0G_0WF_1 & \Delta B_0G_0W \\ -\Delta C & 0 \end{bmatrix} \quad (13)$$

Based on these assumptions, to show the stability of the system, we consider the following positive definite matrix :

$$\tilde{P}(W) = \begin{bmatrix} I & F^T \\ 0 & I \end{bmatrix} \begin{bmatrix} P & 0 \\ 0 & W \end{bmatrix} \begin{bmatrix} I & 0 \\ F_1 & I \end{bmatrix} \quad (14)$$

where P is a solution to the following Lyapunov inequality :

$$P(A_0+B_0F_0)+(A_0+B_0F_0)^TP<0 \quad (15)$$

The existence of $p>0$ is guaranteed by the stability of $A_0+B_0F_0$. From these results, the following Theorem is obtained.

Theorem 1 Assume that

$$\tilde{P}(I)\tilde{A}_0(I)+\tilde{A}^T_0(I)\tilde{P}_0(I)<0 \quad (16)$$

holds, and

$$\tilde{P}_0(I)[\tilde{A}_0(I)+\Delta A(I)]+[\tilde{A}_0(I)+\Delta\tilde{A}(I)]^T\tilde{P}(I)<0 \tag{17}$$

is satisfied for the arbitrary ΔA, ΔB and ΔC, then the system denoted in (10) is stable for the arbitrarily adjustable gain W.

Proof : From (13) and (14),

$$\tilde{A}_0(W)+\Delta\tilde{A}(W)=$$

$$\left[\tilde{A}_0(I)+\Delta\tilde{A}(I)\right]\begin{bmatrix} I & 0 \\ -F_1+WF_1 & W \end{bmatrix} \tag{18}$$

$$P(W)=\begin{bmatrix} I & -F_1^T+F_1^TW \\ 0 & W \end{bmatrix}\tilde{P}(I) \tag{19}$$

hold. Using (18), (19) and the condition (17),

$$\tilde{P}(W)[\tilde{A}_0(W)+\Delta\tilde{A}(W)]+[\tilde{A}_0(W)+\Delta\tilde{A}(W)]^T\tilde{P}(W)$$
$$=\begin{bmatrix} I & -F_1^T+F_1^TW \\ 0 & W \end{bmatrix}$$
$$\cdot\{\tilde{P}(I)[\tilde{A}_0(I)+\Delta\tilde{A}(I)]+[\tilde{A}_0(I)+\Delta\tilde{A}(I)]^T\tilde{P}(I)\}$$
$$\cdot\begin{bmatrix} I & 0 \\ -F_1+WF_1 & W \end{bmatrix}<0 \tag{20}$$

is obtained for the arbitrary $W>0$, such that the system (10) is robustly stable. Thus, it is concluded that Theorem 1 holds.

Now, it will be shown that there always exits a positive matrix P for which the assumption (16) in Theorem 1 holds. For this we rewrite

$$\tilde{Q}=-[\tilde{P}(I)\tilde{A}_0(I)+\tilde{P}^T{}_0(I)\tilde{P}(I)] \tag{21}$$

where

$$\tilde{P}(I)=\begin{bmatrix} P+F_1^TF_1 & F_1^T \\ F_1 & I \end{bmatrix} \tag{22}$$

$$\tilde{A}_0(I)=\begin{bmatrix} A_0+B_0F_0+B_0G_0F_1 & B_0G_0 \\ -C_0 & 0 \end{bmatrix} \tag{23}$$

and decmpose $\tilde{Q}$ into four blocks as

$$\tilde{Q}=\begin{bmatrix} Q_{11} & Q_{12} \\ Q_{21} & Q_{22} \end{bmatrix} \tag{24}$$

$$\begin{aligned} Q_{11}= & -P(A_0+B_0F_0)-(A_0+B_0F_0)^TP-PB_0G_0F_1 \\ & -(PB_0G_0F_1)^T-F_1^TF_1B_0G_0F_1 \\ & -(F_1^TF_1B_0G_0F_1)^T \\ Q_{12}= & -PB_0G_0-F_1^TF_1B_0G_0-(F_1B_0G_0F_1)^T \\ Q_{21}= & -(PB_0G_0)^T-(F_1^TF1B_0G_0)^T-F_1B_0G_0F_1 \\ Q_{22}= & -F_1B_0G_0-(F_1B_0G_0)^T \end{aligned} \tag{25}$$

In this case, in order to show that $\tilde{Q}$ is positive definite, consider G_0 given in (12). Since F_1B_0 is nonsingular, we can see that Q_{22} is positive definite. Therefore, $\tilde{Q}>0$ is equivalent to that

$$\begin{aligned} Q_{11}-Q_{12}Q_{22}^{-1}Q_{21}= & -P(A_0+B_0F_0) \\ & -(A_0+B_0F_0)^TP-(1/2)PB_0R^{-1}B_0^TP \end{aligned} \tag{26}$$

is positive definite.

Here, the sum of the first and second term in the right side is negative definite as seen from (15). The third term is positive semidefinite, but square in P. This shows that, by choosing a appropriately small P satisfying (15) which always exists, (26) is negative definite such that $\tilde{Q}$ of (21) is negative definite.

Above, it has been shown that there exists a feedback gain F_0 such that the pair (A_0, B_0) is stable. Here, consider the optimal regulator gain as a F_0, given as follows :

$$F_0=-R^{-1}B_0^TP \tag{27}$$

where P is a positive semidefinite solution of the following Riccati equation,

$$PA_0+A_0^TP-PB_0R^{-1}B_0^TP+Q=0 \tag{28}$$

where R and Q are positive definite matrices. In this case, since the right - side of (26) becomes $Q+(1/2)PB_0R^{-1}B_0^TP$, we can see that (21) is positive definite so that (16) holds.

4. High Gain Integral Compensation and Transient Behavior

In the previous chapter, it has been shown that if there are uncertainties which satisfy the condition (17) then, the 2DOF servosystem (10) is robustly stable for any arbitrary positive definite matrix W. Thus, by the high gain integral compensation, the steady state error can be rejected and the high - speed tracking response is expected.

In this chapter, it will be calculated the transient behavior attainable by the limit of high - gain compensation using the singular perturbation approach under the assumption (17). For the system (10), deduce the coordinate transformation

$$\begin{bmatrix} x(t) \\ z(t) \end{bmatrix} = \begin{bmatrix} I & 0 \\ F_1 & 1 \end{bmatrix} \begin{bmatrix} x(t) \\ w(t) \end{bmatrix} \tag{29}$$

then the system (10) is

$$\begin{bmatrix} \dot{x}(t) \\ \dot{z}(t) \end{bmatrix} = \begin{bmatrix} A_0+\Delta A+(B_0+\Delta B)F_0 & (B_0+\Delta B)G_0 W \\ F_1(\Delta A+\Delta BF_0)-\Delta C & F_1(B_0+\Delta B)G_0 W \end{bmatrix} \cdot \begin{bmatrix} x(t) \\ z(t) \end{bmatrix} + \begin{bmatrix} (B_0+\Delta B)H_0 \\ F_1\Delta BH_0 \end{bmatrix} r(t) + \begin{bmatrix} D \\ F_1 D \end{bmatrix} d(t)$$

$$y(t) = [C_0+\Delta C \quad 0] \begin{bmatrix} x(t) \\ z(t) \end{bmatrix} \tag{30}$$

Here, let the arbitrary matrix $\tilde{W}$ be the positive definite and μ be positive number, and denote W as follows :

$$W=(1/\mu)\tilde{W} \tag{31}$$

For a fixed $\tilde{W}$,, the decrease of μ represents the increase of W. Introducing

$$\tilde{z}(t)=(1/\mu)z(t) \tag{32}$$

we rewrite (30)

$$\begin{bmatrix} \dot{x}(t) \\ \mu\dot{\tilde{z}}(t) \end{bmatrix} = \begin{bmatrix} A_0+\Delta A+(B_0+\Delta B)F_0 & (B_0+\Delta B)G_0 \tilde{W} \\ F_1(\Delta A+\Delta BF_0)-\Delta C & F_1(B_0+\Delta B)G_0 \tilde{W} \end{bmatrix} \begin{bmatrix} x(t) \\ \tilde{z}(t) \end{bmatrix} + \begin{bmatrix} (B_0+\Delta B)H_0 \\ F_1\Delta BH_0 \end{bmatrix} r(t) + \begin{bmatrix} D \\ F_1 D \end{bmatrix} d(t)$$

$$y(t) = [C_0+\Delta C \quad 0] \begin{bmatrix} x(t) \\ \tilde{z}(t) \end{bmatrix} \tag{33}$$

To apply the singular perturbation approach to the system (33), it is needed to confirm stability of the fast system, that is, the stability of the (2, 2) block $F_1(B_0+\Delta B)G_0\tilde{W}$ of the system matrix. Now, note that the (2,2) block of the condition (17)

$$F_1(B_0+\Delta B)G_0+[F_1(B_0+\Delta B)G_0]^T<0 \tag{34}$$

Multiplying this inequality by $\tilde{W}$ from the right and left, a Lyapunov inequality is obtained as follows :

$$\tilde{W}[F_1(B_0+\Delta B)G_0\tilde{W}]+[F_1(B_0+\Delta B)G_0\tilde{W}]^T\tilde{W}<0 \tag{35}$$

(35) can be considered as a Lyapunov inequality, where $F_1(B_0+\Delta B)G_0\tilde{W}$ is the system matrix and $\tilde{W}$ is the solution of the Lyapunov inequality abovementioned. From this, it is concluded that $F_1(B_0+\Delta B)G_0\tilde{W}$ is a stable matrix under the assumption (17). Since the fast subsystem of (33) is stable, in the limit of $\mu \to 0$, the behavior $z(t)$ is represented as

$$\tilde{z}(t)=-[F_1(B_0+\Delta B)G_0\tilde{W}]^{-1} \cdot \{[F_1(\Delta A+\Delta BF_0)-\Delta C]x(t) + F_1\Delta BH_0 r(t)+F_1 Dd(t)\} \tag{36}$$

Then the behavior of the slow subsystem of (33) is given as follows :

$$\dot{x}(t)=[A_0+\Delta A+(B_0+\Delta B)F_0]x(t) + (B_0+\Delta B)G_0\tilde{W}\tilde{z}(t)+(B_0+\Delta B)H_0 r(t) + Dd(t)=\{A_0+\Delta A+(B_0+\Delta B)F_0$$

$$-(B_0+\Delta B)G_0\tilde{W}[F_1(B_0+DB)G_0\tilde{W}]^{-1}$$
$$\cdot\ [F_1(\Delta A+DBF_0)-\Delta C]\}x(t)$$
$$+\{(B_0+\Delta B)H_0-(B_0+\Delta B)G_0\tilde{W}$$
$$\cdot\ [F_1(B_0+\Delta B)G_0\tilde{W}]^{-1}F_1\Delta BH_0\}r(t)$$
$$+\{D-(B_0+\Delta B)G_0\tilde{W}$$
$$\cdot\ [F_1(B_0+\Delta B)G_0\tilde{W}]^{-1}F_1D\}d(t)$$
$$y(t)=(C_0+\Delta C)x(t) \quad (37)$$

From this, weprove the following Theorem.

Theorem 2 Assume the condition given in (17) holds for the uncertainties ΔA, ΔB and ΔC. In this case, let $W\rightarrow\infty$, then the behavior of the 2DOF servosystem (10) approaches

$$\dot{x}(t)=(A_0+B_0F_0)x(_t)+B_0H_0r(t)$$
$$+\{I-(B_0+\Delta B)[F_1(B_0+\Delta B)]^{-1}F_1\}\}$$
$$\cdot\ [(\Delta A+\Delta BF_0)x(t)+\Delta BH_0r(t)+Dd(t)]$$
$$+(B_0+\Delta B)[F_1(B_0+\Delta B)]^{-1}\Delta Cx(t)$$
$$y(t)=(C_0+\Delta C)x(t) \quad (38)$$

This theorem means that the limit of the behavior depends generally on the plant uncertainties and disturbance inputs. As the special case, assume that the uncertainties ΔA, ΔB and the matrix D which describes a type of disturbance satisfy the following matching conditions[5]

$$\text{Range}\Delta A\subset\text{Range } B_0$$
$$\text{Range}\Delta B\subset\text{Range } B_0$$
$$\text{Range}D\subset\text{Range } B_0 \quad (39)$$

and $\Delta C=0$. Then it is clear that (38) is reduced to

$$\dot{x}(t)=(A_0+B_0F_0)x(t)+B_0H_0r(t)$$
$$y(t)=C_0x(t) \quad (40)$$

From this, it is verified that the behavior of the system (10) coincides with that of the the servosystem for the nominal system without the integral compensator. Also, it is behavior of the 2DOF servosystem in the abscence of the uncertainties and disturbance imputs to the plant.

5. A Numerical Example

In previous, it has been shown that if the condition (17) holds for the modelling error of the system, by high gain compensation, the transient behavior of the servosystem approaches that of (38). Now, these results are illustrated by numerical simulation. Consider the following A, B, and C as the system matrices :

$$A=\begin{bmatrix}-0.04 & 0.02 & 0.020 & -0.50\\ 0.05 & -1.01 & 0.002 & -4.02\\ 0.10 & 0.29+a_1 & -1.710 & 1.3+a_2\\ 0 & 0 & 1 & 0\end{bmatrix}$$
$$B=\begin{bmatrix}0.44 & 0.18\\ 3.05+b_1 & -7.6\\ -5.52 & 4.9+b_2\\ b_3 & 0\end{bmatrix}$$
$$C=\begin{bmatrix}1+c_1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\end{bmatrix} \quad (41)$$

where a_1, a_2, b_1, b_2, b_3, c_1 are uncertainties which satisfy the condition (17) suppose that these parameters are

$$a_1=0.22,\quad a_2=1.2,$$
$$b_1=-1.1,\quad b_2=-1.5$$
$$b_3=-0.5,\quad c_1=0.5 \quad (42)$$

In this case, we show the effective of the gain W. Assume there is no disturbance input to the plant, for simplicity. F_0 as the feedback gain obtained from the optimal regulator theory is used. Using the weighting matrices

$$R=\text{diag}\{10,\ 15\},\ Q=\text{diag}\{10,\ 15,\ 10,\ 5\} \quad (43)$$

the positive definite solution to the Riccati equation (28) is obtained as follows :

$$P = \begin{bmatrix} 19.088 & 0.316 & 0.034 & -7.488 \\ 0.316 & 2.380 & 1.083 & -0.184 \\ 0.034 & 1.083 & 1.994 & 1.459 \\ -7.488 & -0.184 & 1.459 & 14.417 \end{bmatrix} \quad (44)$$

From this, F_0 is

$$F_0 = \begin{bmatrix} -0.917 & -0.142 & 0.767 & 1.191 \\ -0.080 & 0.842 & -0.115 & -0.489 \end{bmatrix} \quad (45)$$

F_1 and H_0 are given as follow, respectively :

$$F_1 = \begin{bmatrix} -2.006 & -0.024 & 0.007 & 0.795 \\ 0.030 & -0.179 & -0.078 & 0.061 \end{bmatrix}$$

$$H_0 = \begin{bmatrix} 0.994 & 0.152 \\ 0.107 & -1.003 \end{bmatrix} \quad (46)$$

and gain G is

$$G = G_0 W = \begin{bmatrix} 0.099 & 0.011 \\ 0.010 & -0.065 \end{bmatrix} W \quad (47)$$

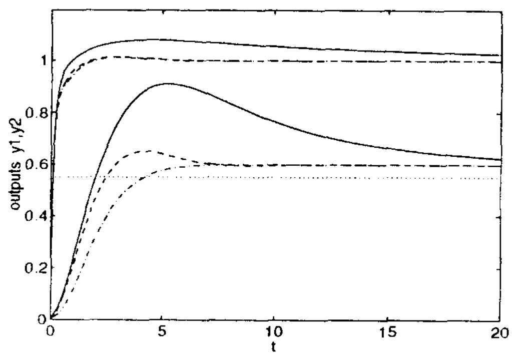

Fig. 2. Step Response(α=1)

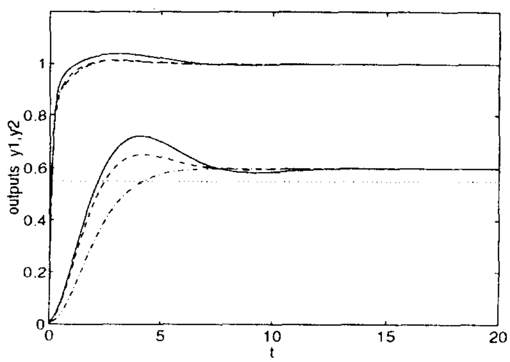

Fig. 3. Step Response(α=10)

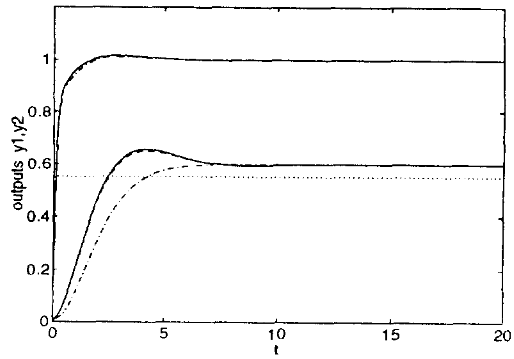

Fig. 4. Step Response(α=100)

Where $r_+ = [0.6\ 1]^T$ as the reference signals is considered, and let the initial state x_0 and the initial value of integral compensator w_0 be 0. And, for simplicity, let the adjustable gain $W = \alpha I$.

Fig. 2, Fig. 3 and Fig. 4 are simulation results corresponding to the case of α=1, 10 and 100, respectively. The solid lines show the behaviors of the controlled outputs, While the dashdot lines indicate the nominal behaviors and the dashed lines are the limit(eg.(38)) attainable by a sufficiently large α, respectively.

From these results, it is concluded that the controlled outputs (solid) approach those (dashed) of (38), not those (dashdot) of the nominal system.

6. Concluding Remarks

In this paper, a robust stability condition has been presented for a 2DOF servosystem, which is independent of any size of the gain of integral compensation. And, based on the robust stability condition, the transient behaviors are obtained by high gain compensation. Using high gain integral compensation, it has been demonstrated that the high - speed tracking responses are archived, as expected.

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