

## Increase of the Shock Thickness in Sea Water Due to the Molecular Relaxation Processes

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### Abstract

A relatively simple theoretical model for predicting the shock thickness is applied to the weak shock propagation through sea water, where the boric acid and the magnesium sulfate are the major relaxation processes. The relaxation effects increases the shock thickness by the factor of  $10^3$  compared with the thickness based on the classical absorption only. In seawater with the ambient pressure of 125 atm and 15°C temperature, the effects of the boric acid are dominant when the peak pressure is less than 0.3 atm and the effects of the magnesium sulfate are dominant when peak pressure is between 0.3 atm and 3 atm. For the shocks of peak pressure greater than 5 atm, the effects of the classical absorption theory is enough to describe the shock thickness. The effects of the ambient pressure and temperature on the shock thickness are also evaluated.

### I. Introduction

Many works have been done for predicting peak pressure and the decay time constant of underwater explosion waves; however, little efforts have been made for determining a detailed structure of the leading edges. For the structure design of MHC (Mine Hunter Crafts), a reference shock profile is defined for calculating the forcing function, in which a detailed profile of the rise phase affects the energy spectrum in the range of 0.1-5 kHz. The mine operation also requires a detailed information of a shock wave depending on the distance from a mine to a ship, depth, and temperature for predicting the level of the damage. Marsh, Mellen and Konrad investigated the effects of the finite amplitude and  $MgSO_4$  relaxation on the explosion waveform in sea water, and showed an uniform increase of the rise time within the range of their measurements[1]. Recently a molecular relaxation model for determining a rise phase of weak shocks propagating through the relaxing atmosphere was developed by Kang and Pierce[2], [3] where  $O_2$  and  $N_2$  are dominant relaxation processes. The model is based on the frozen shock profile assumption, in which the rise phase depends only on the peak pressure and local properties of a propagating medium. The rise phase can be assumed as a step pressure

jump, since the rise time is much shorter than the positive duration of a shock. For the rise time of sonic boom waveform, the model provides the lower bound to the rise time. It was found that the molecular relaxation processes are responsible to the basic mechanism of the increase of the rise time when the turbulence effects are negligible.

The theory is applied to predict the rise phase of weak shocks in sea water, in which, equivalently, two relaxation processes, boric acid and magnesium sulfate, are important[4]. The molecular relaxation processes in sea water depend on temperature, ambient pressure, salinity and pH. Boric acid,  $B(OH)_3$ , is responsible for the absorption in the lower frequency (100 Hz-1 kHz), and magnesium sulfate,  $MgSO_4$ , is responsible for the absorption in the higher frequency (1-100 kHz). The objectives of the present paper are to investigate how the molecular relaxation affects the shock profile and its shock thickness and to provide a lower bound to the shock thickness in sea water. The interested range of the peak pressure is from 0.05 atm to 5 atm where both the molecular relaxation and the nonlinear stiffening effects are important. The molecular relaxation model is briefly introduced in Section 2, and the temperature and pressure dependency of the physical coefficients of relaxation in sea water are explained in Section 3. Two important parameters (the effective diffusion parameter and the critical shock pressure) for characterizing a frozen shock profile are explained in Section 4, and some numerical results are discussed in Section 5.

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## II. Molecular relaxation model for a rise phase of weak shocks

In a homogeneous sea water, a traveling weak shock wave in a positive direction is governed by the steady state version of the augmented Burgers's equation and relaxation equations:

$$\delta \frac{dp}{d\xi} - \frac{\beta p}{2\rho c} (P_{sh} - p) + \sum_v (\Delta c)_v p_v = 0, \quad (1)$$

$$\frac{dp}{d\xi} - \frac{dp_v}{d\xi} - \frac{p_v}{\tau_v c} = 0, \quad v=1, 2. \quad (2)$$

where  $P_{sh}$  is the shock overpressure, and  $p$  and  $p_v$  are the shock pressure and the internal pressure variable related with  $v$ -type relaxation process, respectively. The coefficients,  $\delta$ ,  $\beta$ ,  $\rho$ , and  $c$  are the diffusion parameter for the classical absorption, nonlinearity coefficient, ambient density, and the equilibrium speed of the sound. The parameter  $(\Delta c)_v$  is the increment in the phase velocity at high frequency limit due to the freezing of the  $v$ -type relaxation process, and  $\tau_v$  the relaxation time. Here, 1 and 2 correspond to the boric acid and magnesium sulfate, respectively. The independent variable  $\xi$  is the coordinate system moving with the shock,

$$\xi = Vt - x, \quad (3)$$

$$V = c + \frac{\beta P_{sh}}{2\rho c}, \quad (4)$$

where  $V$  is the speed of shock propagation. In the  $\xi$  coordinate, the shock profile looks as if it were frozen, that is, a stationary profile.

Equations (1) and (2) are three nonlinear ordinary differential equations which can be solved by numerical integration, and the following boundary conditions are required:

$$p \rightarrow 0, \quad p_v \rightarrow 0; \quad \text{as } \xi \rightarrow -\infty, \quad (5)$$

$$p \rightarrow P_{sh}, \quad p_v \rightarrow 0; \quad \text{as } \xi \rightarrow +\infty. \quad (6)$$

However, Eqs. (1) and (2) are the nonlinear coupled equations, mathematical rigor is required to determine the initial values for numerical integration. Far ahead of the rise phase of a shock, since  $P_{sh}$  is much larger than  $p$ , Eq. (1) can be linearized. For this asymptotic region, the solution can be represented as  $p = Ae^{\lambda\xi}$  and  $p_v = A_v e^{\lambda\xi}$ . By substituting these type of solutions into the linearized equations, one can obtain a matrix equation of  $A$  and  $A_v$ . Then the determinant zero condition provides an unique eigenvalue,

say  $\lambda_a$ , which is a positive real root of a polynomial of the third degree. The details are given in Kang [3].

For numerical integration, one can choose any arbitrary value as an origin of  $\xi$ , since the governing equations are invariant under the time or spatial translations. If the linear asymptotic solution is valid at  $p \leq 0.05P_{sh}$ , then the shock profile can be written as

$$p = 0.05P_{sh} e^{\lambda_a \xi}, \quad \text{for } \xi \leq 0. \quad (7)$$

The relation between the two dependent variables are given as

$$\frac{p_v}{p} = \frac{\tau_v c \lambda_a}{\tau_v c \lambda_a + 1}, \quad (8)$$

from the asymptotic equation.

The shock profile for  $\xi > 0$  is determined by numerical integration starting with the initial values at  $\xi = 0$  given by Eqs. (7) and (8).

## III. Molecular relaxation processes and nonlinear parameter in sea water

As given in Section 2, the relaxation process is described by the two parameters:  $(\Delta c)_v$  and  $\tau_v$  (here, 1: boric acid, and 2: magnesium sulfate). In this paper, the formulations suggested by Fisher and Simmonds are used[4], in which the parameters are given as a function of temperature and ambient pressure. For a sake of convenience, they are given here as

$$(\Delta c)_1 = \frac{c^2}{2\pi} (1.03 + 0.0236T_c - 5.22 \times 10^{-4} T_c^2) \times 10^{-8}, \quad (9)$$

$$(\Delta c)_2 = \frac{c^2}{2\pi} (5.62 + 0.0752T_c)(1 - 10.3 \times 10^{-4} P + 3.7 \times 10^{-7} P^2) \times 10^{-8}, \quad (10)$$

$$\tau_1 = 2\pi(1315 T \exp[-1700/T])^{-1}, \quad (11)$$

$$\tau_2 = 2\pi(1.55 \times 10^7 T \exp[-3052/T])^{-1}, \quad (12)$$

and

$$\delta = \left( \frac{c}{2\pi} \right)^2 c(55.9 - 2.37T_c + 4.77 \times 10^{-7} T_c^2 - 3.48 \times 10^{-4} T_c^3) \times 10^{-15} f(P), \quad (13)$$

where

$$f(P) = 1 - 3.84 \times 10^{-4} P + 7.57 \times 10^{-8} P^2. \quad (14)$$

Here, the quantity  $T_c$  is the temperature in degree Celsius, and  $T$  is the absolute temperature,  $T = T_c + 273.1$ . The quantity  $P$  represents an ambient pressure in atmospheres (101325 Pa), and  $c$  is the speed of sound (m/s). These equations are valid for the sea water of Sa (salinity) = 35 ppm and  $pH = 8.0$ . The typical values in sea water of 15°C and 125 atm are  $(\Delta c)_1 = 4.708 \times 10^{-1}$  (m/s),  $(\Delta c)_2 = 2.20 \times 10^{-2}$  (m/s),  $\tau_1 = 1.532 \times 10^{-4}$  (sec),  $\tau_2 = 1.418 \times 10^{-6}$  (sec), and  $\delta = 2.578 \times 10^{-6}$  (m<sup>2</sup>/s). Comparing with the relaxation processes in air, one can show that boric acid plays the role of nitrogen, and magnesium sulfate does the role of oxygen.

The nonlinear coefficient  $\beta$  is defined as

$$\beta = 1 + \frac{1}{2} \frac{B}{A}, \quad (15)$$

which is also a function of temperature, ambient pressure and salinity. The typical value of  $\beta$  in sea water is 3.6 at 20°C. The pressure and temperature dependency of  $B/A$  in water was given by Hagelberg *et. al.* [6] and the temperature and salinity dependency in sea water was given by Coppens *et. al.* [7] as a form of tabulated data. For the numerical calculation, the values of  $B/A$  are calculated by using the linear interpolation, and it is assumed that the pressure dependency in water is maintained in sea water.

#### IV. Shock thickness in low frequency limit and critical shock pressure

In low frequency limit, the augmented Burgers's equation reduces to Burgers's equation, and the shock thickness (defined by the slope at the point of a middle shock overpressure) is given by

$$l = \frac{8\rho c}{\beta P_{sh}} \delta_{eff}, \quad (16)$$

where

$$\delta_{eff} = \delta + c(\Delta c)_1 \tau_1 + c(\Delta c)_2 \tau_2. \quad (17)$$

Here, the quantity  $\delta_{eff}$  is an effective diffusion parameter in the relaxing sea water. For a shock with  $P_{sh} = 0.3$  atm in sea water  $T = 15^\circ\text{C}$ ,  $P = 125$  atm,  $Sa = 35$  ppm the shock thickness are as follows: when only the classical absorption are considered,  $l = 0.0287$  cm; when the classical absorption and magnesium sulfate are included,  $l = 1.730$  cm; when the classical absorption and boric acid are included,  $l = 12.30$  cm; and, if all processes are included,

then  $l = 12.83$  cm. For comparison, if the classical absorption is included,  $l P_{sh} = 8.61 \times 10^{-3}$  (cm-atm), which is very close to the value in Beyers[8]; on the other hand,  $l P_{sh} = 3.8$  (cm-atm), when the both relaxation are considered. Thus, it is easy seen that the relaxation processes increase the shock thickness by the factor of  $10^3$ .

A critical shock pressure in relaxing sea water is defined as

$$P_{cr} = \frac{2\rho c}{\beta} \sum_v (\Delta c)_v \quad (18)$$

The physical meaning of  $P_{cr}$  is explained as follows: when the shock overpressure is less than the critical pressure, molecular relaxation effects are dominant, and the shock has a long precursor. On the contrary, when the shock overpressure is larger than the critical pressure, the shock begins with an abrupt jump,  $P_{jump} = P_{sh} - P_{cr}$ . Since  $(\Delta c)_2 > (\Delta c)_1$ , initial pressure rise is more related with  $\text{MgSO}_4$ , and the late climb to  $P_{sh}$  is more related with  $\text{B(OH)}_3$  because of the longer relaxation time of  $\text{B(OH)}_3$ . The typical value of  $P_{cr}$  is equal to 0.22 atm for  $T = 15^\circ\text{C}$ ,  $P = 125$  atm in sea water.

#### V. Numerical results and discussion

For numerical integration of Eqs. (1) and (2), the 4th order Runge-Kutta integration technique is used. The formulae for the speed of sound and the ambient density are

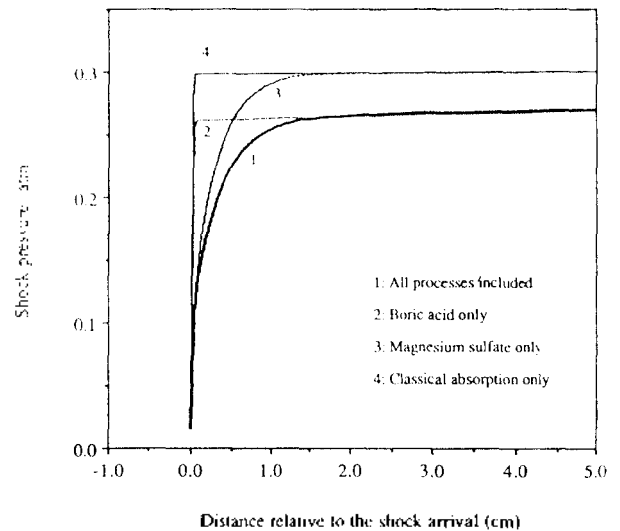


Figure 1. Roles of the relaxation processes in sea water,  $P_{sh} = 0.3$  atm,  $T = 15^\circ\text{C}$ ,  $P = 125$  atm,  $B/A = 5.44$ , and  $Sa = 35$  ppm. The initial rise follows the curve by magnesium sulfate (curve 3), and the late climb follow the curve 2 given by boric acid.

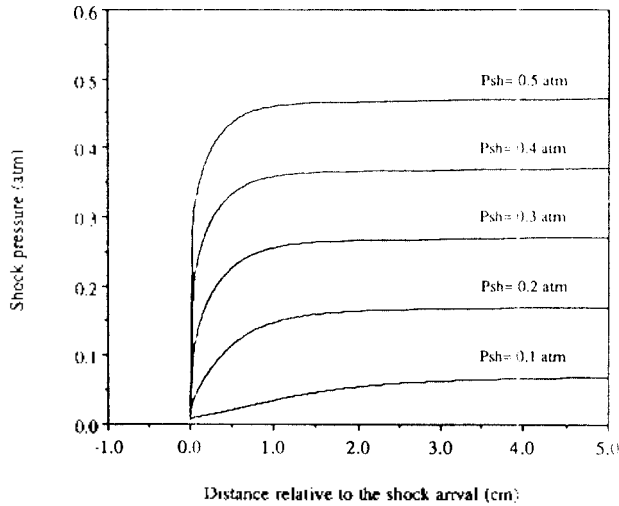


Figure 2. Effects of peak pressure on the shock profile,  $P = 125$  atm,  $T = 15^\circ\text{C}$ ,  $B/A = 5.44$  and  $Sa = 35$  ppm. The critical pressure is 0.22 atm. There are initial jump in shock pressure when  $P_{sh}$  is larger than the critical pressure.

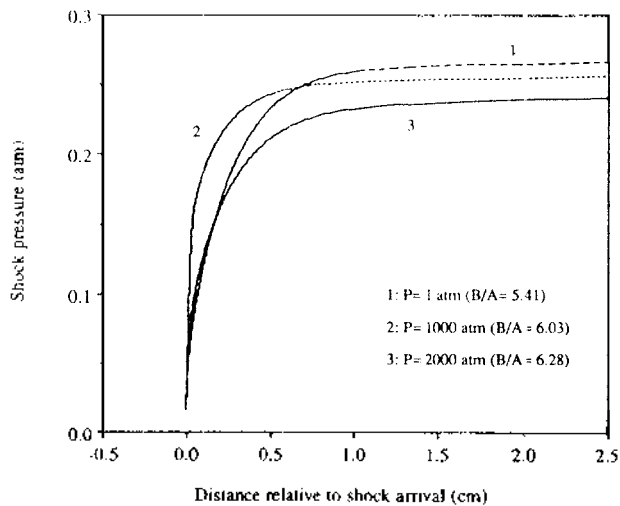


Figure 3. Effects of the ambient pressure on the shock profile,  $P_{sh} = 0.3$  atm,  $T = 30^\circ\text{C}$ , and  $Sa = 35$  ppm. The shock in higher ambient pressure (in deep underwater) has a slow climb to the peak pressure.

also available in the textbook by Pierce as a function of temperature and ambient pressure[5]. In all figures of shock profiles, the precursors ahead of 0.05% of  $P_{sh}$  are omitted.

At first, Figure 1 shows the individual role of the relaxation process to the rise phase of shock. As explained in the previous section, the initial rise of the curve 1 follows the curve of magnesium sulfate only (curve 3), which is responsible for the higher frequency absorption; then it

follows the curve by boric acid (curve 2), which is responsible for the lower frequency absorption. Note the smoothing effect of the relaxation processes compared with the curve 4 by the classical absorption only. Secondly, the effects of peak pressure are shown in Figure 2. When the peak pressure is less than a critical pressure (here,  $P_{sh} = 0.22$  Pa), there is a precursor in the earlier stage of the rise phase. A larger peak pressure means a thinner shock. In underwater shock propagation, the rise time increases with the propagation distance because of the range dependency of the peak pressure and the decrease of nonlinear effects. Next, the effects of local properties of sea water, mainly ambient pressure and temperature on the shock profiles are shown in Figures 3 and 4. A shock in deep underwater takes longer time to reach its peak pressure. However, the effects of temperature are not easy to identify in Figure 4. For a shock in cold water, the initial rise is slow and the late climb is fast compared to the shock in warm water.

In the present paper, the shock thickness is defined as the thickness from 10% of the peak pressure to 90% of the peak pressure. Figure 5 demonstrates that when the peak shock pressure is less than the critical pressure, molecular relaxation is very important (especially, boric acid). In the boric acid dominant region, the shock thickness can be well estimated by considering the boric acid only as shown by the straight line 1. When the peak shock pressure is larger than 3 atm, the molecular relaxation effects are not important since the nonlinear stiffening effects are dominant. In the intermediate region, the magnesium sulfate plays the major role given by the straight line 2.

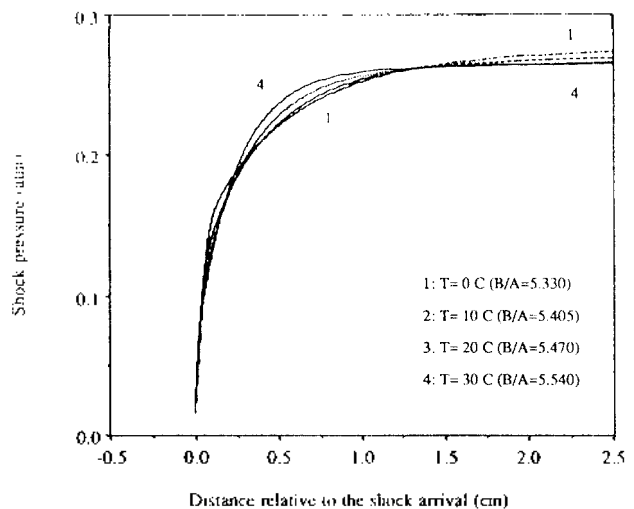


Figure 4. Effects of the temperature on the shock profile,  $P_{sh} = 0.3$  atm,  $P = 125$  atm, and  $Sa = 35$  ppm.

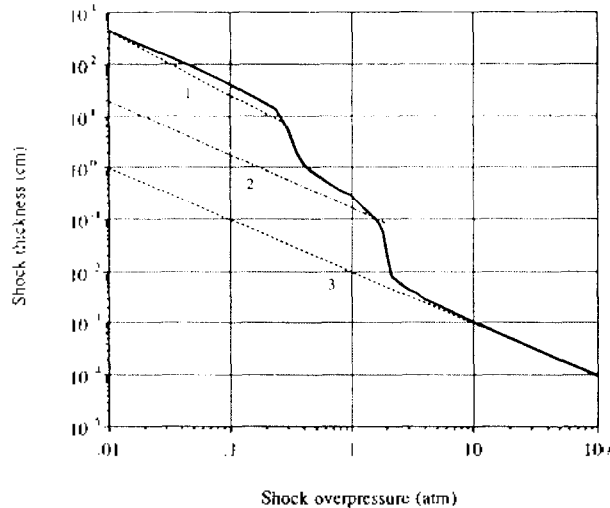


Figure 5. Effect of the peak pressure on the shock thickness,  $T = 15^\circ\text{C}$ ,  $P = 125\text{atm}$ ,  $B/A = 5.44$ , and  $Sa = 35\text{ ppm}$ . The straight dotted lines 1, 2, and 3 correspond to the shock thickness by considering  $\text{B(OH)}_3$ ,  $\text{MgSO}_4$ , and classical absorption, respectively. Molecular relaxation is important when  $P_{sh}$  is less than 3 atm. For lower peak pressure region ( $P_{sh} \leq 0.3\text{ atm}$ ), boric acid effects are dominant and magnesium sulfate effects are dominant for the region  $0.3\text{ atm} < P_{sh} < 3\text{ atm}$ .

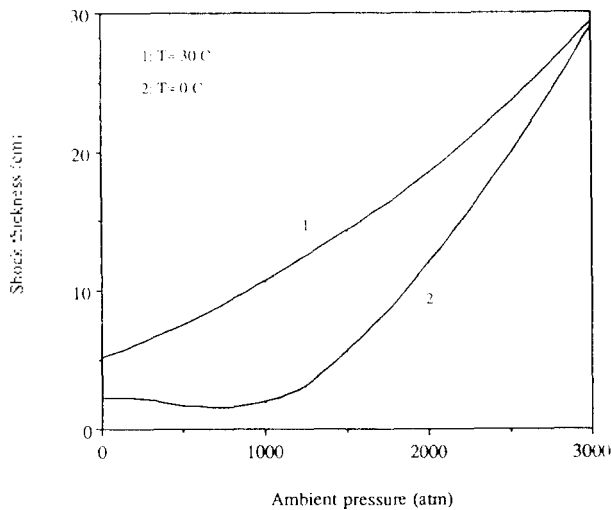


Figure 6. Effects of the ambient pressure on the shock thickness up to 3000 atm,  $P_{sh} = 0.3\text{ atm}$  and  $Sa = 35\text{ ppm}$ . The shock thickness gradually increase as the ambient pressure increases.  $B/A$  has a maximum value around 3000 atm.

The molecular relaxation increases the shock thickness by the factor of  $10^3$ . Both effects of ambient pressure and temperature on the shock thickness are shown in Figures 6 and 7. As the ambient pressure increases, the thickness gradually increases. The shock thickness increases as the

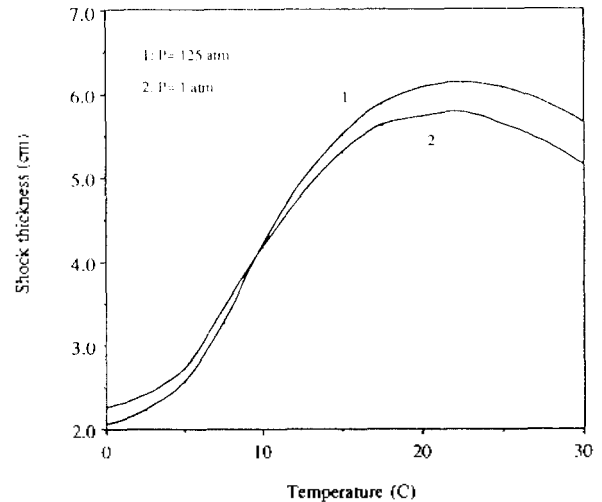


Figure 7. Effect of temperature on the shock thickness,  $P_{sh} = 0.3\text{ atm}$  and  $Sa = 35\text{ ppm}$ . The shock thickness has the peak value at  $T = 20\text{-}25^\circ\text{C}$ .

temperature increases, but it decreases after the peak around  $20\text{-}25^\circ\text{C}$ .

## VI. Concluding remark

A frozen shock profile and its shock thickness in sea water are determined based on the molecular relaxation model. The predicted shock thickness may be the lower bound to the shock thickness, since the inhomogeneity of the medium is neglected. In sea water,  $\text{MgSO}_4$  is responsible for the initial rise of a shock, and  $\text{B(OH)}_3$  is responsible for the late climb to the peak pressure. For lower shock overpressure region ( $P_{sh} \leq 0.3\text{ atm}$ ), the effects of  $\text{B(OH)}_3$  are dominant, and for higher shock overpressure region ( $0.3\text{ atm} < P_{sh} \leq 3\text{ atm}$ ), the effects of  $\text{MgSO}_4$  are dominant. When  $P_{sh} \geq 5\text{ atm}$  the molecular relaxation is not important because the nonlinear stiffening effects are dominant. The shock thickness increases as the temperature and the ambient pressure increase (up to 3000 atm and  $25^\circ\text{C}$ ). In the future, detailed comparisons of the theory with experimental data in sea water are expected.

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