Theoretical Study of Coherent Acoustic Inverse Method for Bubble Sizing in Bubbly Water

*Bok-Kyoung Choi, and *Suk-Wang Yoon

Abstract

The bubble size distribution is critical information to understand sound propagation and ambient noise in the ocean. To estimate the bubble size distribution in a bubbly water, the sound attenuation has been only used in the conventional acoustic bubble sizing method without considering the sound speed variation. However, the effect of the sound speed variation in bubbly water cannot be neglected because of its compressibility variation. The sound attenuation is also affected by the sound speed variation. In this paper, a coherent acoustic bubble sizing inverse technique is introduced as a new bubble sizing technique with considering sound speed variation as well as the sound attenuation. This coherent sizing method is theoretically verified with the bubble distribution functions of single-size, Gaussian, and power-law functions. Its numerical test results with the coherent acoustic bubble sizing method show good agreement with the given bubble distributions.

I. Introduction

Bubbles are very efficient sound sources and scatterers in water. The bubbles in the ocean can resonate radiate and reradiate sounds at their individual or collective resonance frequencies [1, 2]. One size bubble has one resonance frequency. The bubble size distribution and void fraction of bubbly water are important parameters to understand sound propagation and acoustic roles of bubbles in the ocean.

The acoustic bubble sizing techniques with sound attenuation in bubbly water were investigated by Clay and Medwin [3, 4] and many other researchers [5-13] based on a bubble resonance theory. They used the attenuation information of sound through bubbly water in order to get bubble size distribution because sound is strongly attenuated at individual resonance frequencies of bubbles. In their works the sound attenuation was determined from the extinction cross section of bubbles without considering the sound speed variation in the bubbly water. However, the bubbly water is generally very dispersive for sound waves because of its compressibility variation and sound attenuation. The compressibility variation implies the sound speed variation. The sound attenuation depends not only on the damping mechanisms but also on the sound speed variation of the bubbly water. Commander *et al.* [12, 13] theoretically calculated the off-resonance contributions to acoustic bubble sizing with sound attenuation, but coherency between phase speed and attenuation of sound waves was not considered. The off-resonance contribution is to consider the attenuation effect at near resonance frequency. Such conventional acoustic inverse method is referred as the "incoherent acoustic bubble sizing method".

To represent the effect of sound speed variation due to bubbles, it is more feasible to use more complete expression of the wave number including sound speed information of the bubbly water. Choi *et al.* [14, 15] recently reported an inverse method for acoustic bubble sizing considering the sound speed variation. They used direct inversion of the wave number in bubbly water for bubble sizing estimation.

In this paper, we investigate the contribution of sound speed variation affected on extinction cross section of bubbles and used the physically induced different formula. Using this extinction cross section of bubbles including the sound speed variation, the bubble size distribution can be more accurately estimated in the acoustic inverse method as well as the direct inversion of wave number. We will refer to this method as the "coherent acoustic bubble sizing method".

*Physical Oceanography Division, Korea Ocean Research & Development Institute
** Acoustics Research Laboratory, Department of Physics, Sung Kyun Kwan University
Manuscript Received 96. 7. 30

II. Theoretical approach

1. Induction of coherent attenuation

For various-size bubbles in a bubbly water, the conventional sound attenuation through the bubbly water can be written in the following integral form:

$$\alpha(w, a) = \int_{a_{min}}^{a_{max}} \sigma(w, a) n(a)\, da, \tag{1}$$

where a_{max} and a_{min} are the maximum and minimum radii of the bubbles, respectively, and n is the bubble number density of radius a and ada. From reference 3 the extinction cross section σ of a bubble is given by

$$\sigma(w, a) = \frac{4\pi a \delta(w, a)}{(w_o^2/w^2 - 1)^2 + |\delta(w, a)|^2} \frac{w}{c}, \tag{2}$$

where a is the bubble radius, c the sound speed in a pure water, w_o the angular resonance frequency of the bubble, w the angular frequency of propagating sound, and δ the thermal, viscous and radiation damping constant of the bubble [3].

To estimate the bubble size distribution using the conventional acoustic bubble sizing method, Eq. (1) is inversely solved. In this estimation, the sound speed variation in a bubbly water is not considered. The sound attenuation α in Eq. (1) is referred as the "incoherent attenuation". For sound propagation in bubbly water, the pure water sound speed c in Eq. (2) is not appropriate since the surrounding medium is the bubbly water. Therefore the sound speed c of pure water in the right hand side of Eq. (2) should be replaced by $Re(c_m)$ of the bubbly water.

The new extinction cross section σ_m with the correction due to the sound speed variation is

$$\sigma_m(w, a) = \frac{4\pi a \delta(w, a)}{(w_o^2/w^2 - 1)^2 + |\delta(w, a)|^2} \frac{w}{Re(c_m)}, \tag{3}$$

where the sound speed in bubbly water can be obtained by real part of the complex sound speed cm induced by dispersion relation of sound in bubbly water from appendix 6 of reference 3.

Thus, the coherent attenuation m can be written as

$$\alpha_m(w, a) = \int_{a_{min}}^{a_{max}} \sigma_m(w, a) n(a)\, da, \tag{4}$$

In the following section, we will use the coherent attenuation to estimate the bubble size distribution by solving inversely Eq. (4).

The incoherent and the coherent attenuations calculated by Eq. (1) and (4), respectively, and the sound speed in bubbly mixture are shown in Fig. 1 for single-size bubble distribution of bubble radius 0.5 mm for the void fraction of 0.001%, where the void fraction β is defined as the fraction of the total gas volume in the bubbly water volume. In Fig. 1(a), the dotted and solid lines denote the incoherent and coherent attenuations, respectively. Both cases give similar results. The sound speed is shown in Fig 1(b).

Figure 2 shows the case of the void fraction of 0.01%. Note that the coherent attenuation is very different from the incoherent one, and is affected by the sound speed variation up to 11 kHz. This coherent attenuation should be corresponded to the experimentally observed sound attenuation in the bubbly water.

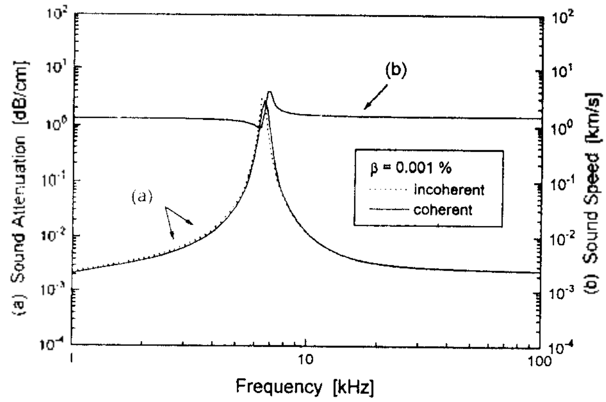


Figure 1. Sound attenuation and sound speed for single-size bubble distribution of bubble radius 0.5 mm with the void fraction β = 0.001%. (a) sound attenuations, (b) sound speed.

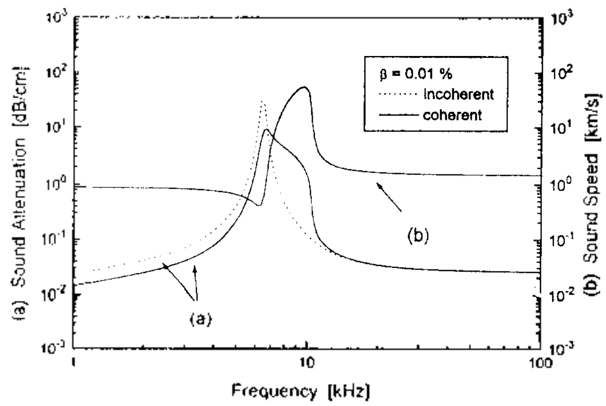


Figure 2. Sound attenuation and sound speed for single-size bubble distribution of bubble radius 0.5 mm with the void fraction β = 0.01%. (a) sound attenuations, (b) sound speed.

2. Inverse technique

The coherent attenuation of Eq. (4) can be written in a matrix form:

$$\alpha_{m_i} = \sigma_{m_{i,j}} n_j \, \Delta a, \tag{5}$$

where Δa is the radius increment between bubble radii and the subscript i and j represent the elements of the sound frequency and bubble radius, respectively. The bubble number density n can be estimated from Eq. (5) by an inverse method with the coherent attenuation α_m and the extinction cross section σ_m [16-18]:

$$n = (\sigma_m^T \sigma_m)^{-1} \sigma_m^T \frac{\alpha_m}{\Delta a}. \tag{6}$$

In this paper, the singular value decomposition (SVD) method [16, 17] is used. The SVD method has an advantage to deal properly with errors in an ill-conditioned problem [18]. The SVD method can decompose the matrix σ_m in Eq. (5) into

$$\sigma_{m_{i,j}} = U_{i,j} \Lambda_{j,j} V_{j,j}^T. \tag{7}$$

where U and V^T are the orthonormal matrices and Λ is the diagonal matrix consisting of the singular values of σ_m. Since U and V^T are orthonormal, they satisfy the following relations:

$$U^T U = U U^T = J, \quad V^T V = V V^T = J, \tag{8}$$

where J is the identity matrix.

The bubble number density matrix n can be given by

$$n_j = V_{j,j} \Lambda_{j,j}^{-1} U_{i,j}^T \frac{\alpha_{m_i}}{\Delta a}. \tag{9}$$

Since the coherent attenuation α_m includes the effect of sound speed variation, the estimated bubble size distribution from Eq. (9) also includes its effect. Therefore, the bubble number density estimated from Eq. (9) will be more realistic correct solution.

Ⅲ. Numerical estimation of bubble size distribution using coherent acoustic bubble sizing method

To test the conventional incoherent and the present coherent acoustic bubble sizing methods, three cases of bubble distribution were chosen: single-size, Gaussian and power-law functions. In each distribution, both the incoherent and the coherent methods are applied to the void fractions of 0.001% and 0.01%, respectively.

1. Single-size bubble distribution

The bubble size distributions estimated from the incoherent and the coherent methods for a single-size case of the bubble radius of 0.5 mm with the void fraction of 0.001% are shown in Fig. 3. The bubble radius is in the range from 25 m to 1 mm with the incremental radius step of $\Delta a = 25$ μm. its corresponding resonance frequency is in the range from 130 kHz to 3 kHz. The histogram is the given bubble distribution, the symbol × denotes the incoherent acoustic bubble sizing case, and the symbol • indicates the coherent one. Even for a very low void fraction with 0.001% as shown in Fig. 3, the coherent

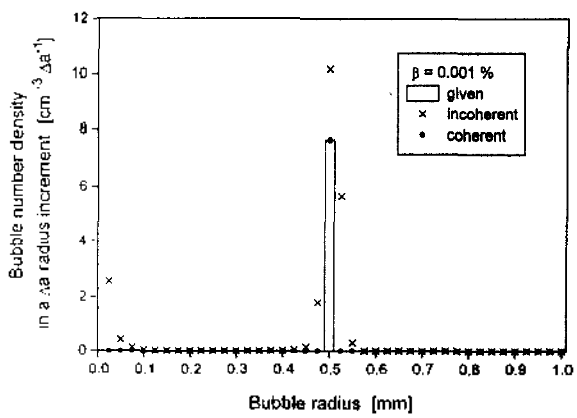


Figure 3. Bubble size distributions estimated by the incoherent and the coherent methods for a single-size distribution of the bubble radius 0.5 mm with the void fraction of 0.001%.

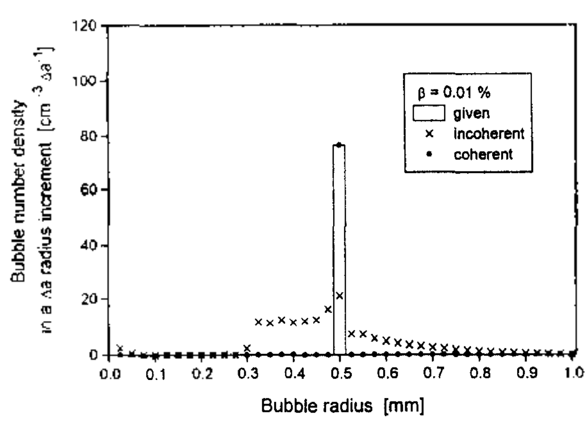


Figure 4. Bubble size distributions estimated by the incoherent and the coherent methods for a single-size distribution of the bubble radius 0.5 mm with the void fraction of 0.01%.

method shows good agreement with the given bubble distribution, but the incoherent method does not represent it because the incoherent estimation does not consider the sound speed variation as shown in Fig. (1b).

Figure 4 shows the bubble size distributions estimated by the incoherent and the coherent methods for the given single-size bubble distribution with the void fraction of 0.01%. The coherent sizing method shows almost perfect agreement with the given bubble distribution. For the void fraction 0.01%, the incoherent method brings much more errors from the given bubble distribution for the low void fraction 0.001%. There are more discrepancies at small size bubbles than at large ones in the incoherent method. The incoherent estimation errors are in the range from 0.3 mm to 0.5 mm of bubble radius. They correspond to the bubble resonance frequencies from 11 kHz to 6 kHz, which appear the frequencies as the difference between the incoherent and the coherent attenuations due to the sound speed variation shown in Fig. 2. The estimation errors in the incoherent method are mainly due to the sound speed variation. Such sound speed variation strongly depends on the void fraction.

2. Gaussian bubble distribution

For a Gaussian bubble distribution the bubble number density function is given by

$$n(a) = \frac{1}{\sqrt{2\pi}\,\eta} \exp\left[- \frac{(a - a_o)^2}{2\eta^2} \right], \tag{10}$$

where a is the bubble radius, a_o is mean bubble radius at the maximum peak in the given Gaussian distribution and η is arbitrary constant.

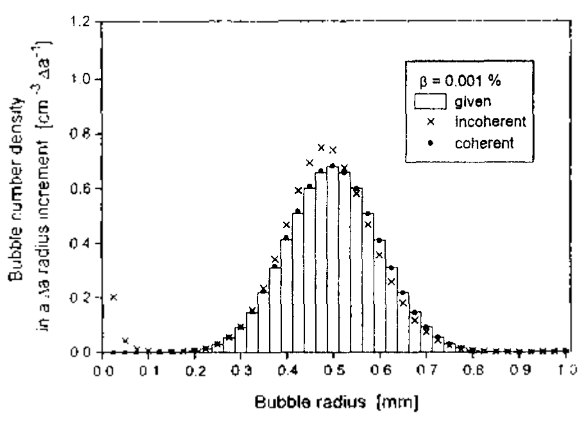


Figure 5. Bubble size distributions estimated by the incoherent and the coherent methods for a Gaussian bubble distribution with the void fraction of 0.001%.

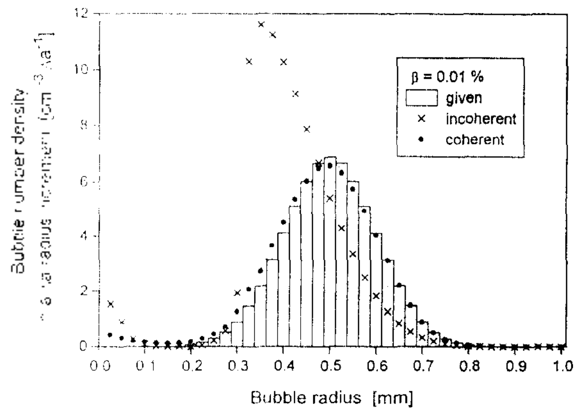


Figure 6. Bubble size distributions estimated by the incoherent and the coherent methods for a Gaussian bubble distribution with the void fraction of 0.01%.

For a Gaussian bubble distribution of mean bubble radius at 0.5 mm with the void fraction of 0.001%, the bubble size distributions estimated by the incoherent and the coherent methods are shown in Fig. 5. For a very low void fraction with 0.001%, the incoherent method shows reasonable agreement to the given bubble distribution while the coherent method almost perfectly estimates the given distribution. For the void fraction of 0.01%, the sound speed variation is large, and the coherent attenuation is affected by the sound speed variation. As shown in Fig. 6, the coherent sizing estimations (•) show much better agreement with the given bubble distribution than the incoherent ones (×). The peak of the incoherent bubble sizing estimation shifts to the smaller size bubble region rather than that of the given bubble distribution (bar). Such errors are also due to the ignorance of sound speed variation effect.

3. Power-law bubble distribution

For a power law bubble distribution, the bubble number density function is given by

$$n(a) = a^{-b}, \tag{11}$$

where the constant value of b is experimentally given as 3.7 for ocean surface bubble distribution [19].

Figure 7 shows the bubble size distributions estimated by the incoherent and the coherent methods for a power-law bubble distribution with the void fraction of 0.001%. In this very low void fraction, the incoherent and the coherent methods show similar results because of the small variation of sound speed. However, for the void fraction of 0.01% as shown in Fig. 8, the coherent sizing estimations

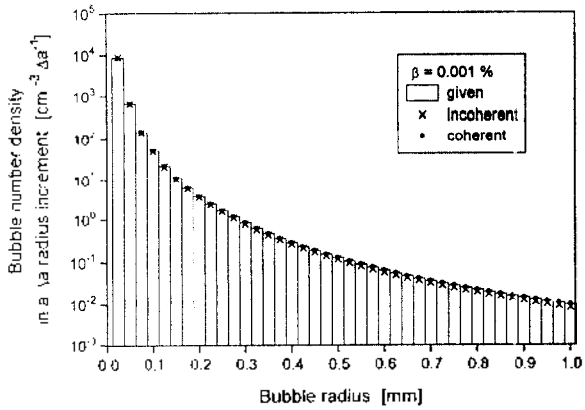


Figure 7. Bubble size distributions estimated by the incoherent and the coherent methods for a power-law bubble distribution with the void fraction of 0.001%.

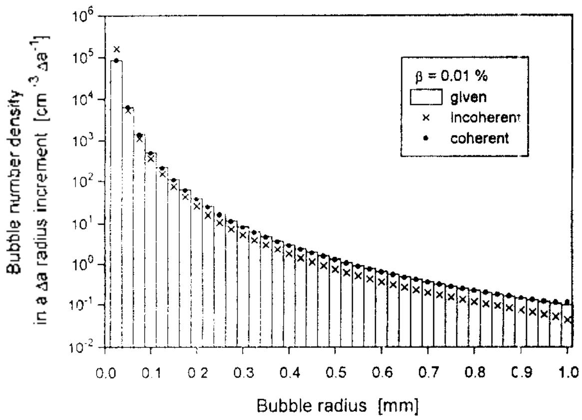


Figure 8. Bubble size distributions estimated by the incoherent and the coherent methods for a power-law bubble distribution with the void fraction of 0.01%.

(•) give much better agreement with the given bubble distribution than the incoherent ones (×). Note that the bubble number density in the ordinate in Fig. 7 and 8 are logarithmic scales. In Fig. 8, the incoherent estimation of the bubble number density in a 25 m radius increment is 1.6×10^5 and the coherent one is 8.4×10^4, while the given density is 8.3×10^4 [cm^{-3} Δa^{-1}]. In the incoherent estimation there are many errors in the smaller bubbles rather than the larger ones.

Ⅳ. Conclusions

To estimate more accurately the bubble number density in water, a coherent acoustic bubble sizing technique was developed. The effect of sound speed variation cannot be neglected in acoustic bubble sizing techniques for bubbly water. The sound attenuation measurements are also influenced by the sound speed variation in bubbly water. The sound speed variation as well as the sound attenuation were considered in the coherent acoustic bubble sizing method while the incoherent acoustic bubble sizing method is only based on the sound attenuation information. In the coherent method, the coherent attenuation was adopted, where the coherent attenuation can be given from the extinction cross section considering the sound speed in the bubbly water rather than that in a pure water. The numerical test results for the given three bubble distributions of single-size, Gaussian, and power-law functions show that the coherent method gave much accurate estimations than the incoherent ones, and can be a very powerful tool to estimate the bubble size distribution in water.

In the ocean it might be difficult to measure simultaneously the sound speed and sound attenuation in a bubbly water for many different frequencies. Such restrictions make the underdetermined system in the SVD method [18]. Application of the coherent inverse method to an underdetermined system with fewer data of sound speed and sound attenuation demands further study in future.

References

1. S. W. Yoon and B. K. Choi, "Active and Passive Acoustic Roles of Bubbles in the Ocean," *Bubble Dynamics and Interface Phenomena* edited by J. R. Blake, Kluwer Academic, Dordrecht, pp. 151-160, 1994.
2. S. W. Yoon, L. A. Crum, A. Prosperetti and N. Q. Lu, "An investigation of the collective oscillations of a bubble cloud, *J. Acoust. Soc. Am.* **89**, pp. 700-706, 1991.
3. C. S. Clay, and H. Medwin, in *Acoustical Oceanography: principle and applications*, John Wiley & Sons, New York, pp. 178-215, 1977.
4. H. Medwin, "Acoustical determinations of bubble-size spectra," *J. Acoust. Soc. Am.* **62**, pp. 1041-1044, 1977.
5. H. Medwin, J. Fitzgerald, and G. Rautmann, "Acoustical Miniprobing for Ocean Microstructure and Bubbles," *J. Geophy. Res.* **80**, pp. 405-413, 1975.
6. G. B. Crawford and D. M. Farmer, "On The Spatial Distribution of Ocean Bubbles," *J. Geophy. Res.* **92**, pp. 8231-8243, 1987.
7. S. Vagle and D. M. Farmer, "The Measurement of Bubble-Size Distributions by Acoustical Backscatter," *J. Atmos. & Ocean. Tech.* **9**, pp. 630-644, 1992.
8. S. A. Thorpe, "Measurements with an Automatically Recording Inverted Echo Sounder; ARIES and the Bubble Clouds," *J. Phy. Ocean.* **16**, pp. 1462-1478, 1986.
9. S. A. Thorpe, P. Bowyer, and D. K. Woolf, "Some Factors

Affecting the Size Distributions of Oceanic Bubbles," *J. Phy. Ocean.* **22**, pp. 382-389, 1992.

10. J. W. Cartmill, and M. Y. Su, "Bubble size distribution under saltwater and freshwater breaking waves," *Dynam. Atmos. & Oceans* **20**, pp. 25-31, 1993.
11. M. Y. Su, D. Todoroff, and J. Cartmill, "Laboratory Comparisons of Acoustic and Optical Sensors for Microbubble Measurement," *J. Atmos. & Ocean. Tech.* **11**, pp. 170-181, 1994.
12. K. W. Commander, and E. Moritz, "Off-resonance contributions to acoustical bubble spectra," *J. Acoust. Soc. Am.* **85**, pp. 2665-2669, 1989.
13. K. W. Commander, and R. J. McDonald, "Finite-element solution of the inverse problem in bubble swarm acoustics," *J. Acoust. Soc. Am.* **89**, pp. 592-597, 1991.
14. B. K. Choi, H. R. Lee, and S. W. Yoon, "Coherent acoustic bubble sizing in the ocean," *J. Acoust. Soc. Am.*, **96**, pp. 3235, 1994.
15. B. K. Choi, and S. W. Yoon, "Acoustic bubble sizing considering the sound speed variation in bubbly water," *Proceedings of 10th Underwater Acoustics Symposium*, pp. 25-32, 1995.
16. G. Strang, *Linear Algebra and its Application*, Academin Press, Inc., Orland, pp. 137-152, 1980.
17. W. Press, B. P. Flannery, S. A. Teukolsky, and W. T. Vetterling, *Numerical Recipes*, Cambridge Univ. Press, New York, pp. 52-64, 1986.
18. J. H. Lee, "A review of tracer inverse problems in oceanography," *Ocean Res.*, **15**, pp. 53-69, 1993.
19. J. Wu, "Bubbles in the Near-Surface Ocean : A General Description," *J. Geophy. Res.* **93**, pp. 587-590, 1988.

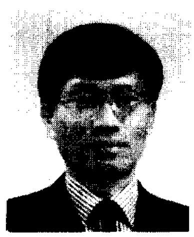
▲Bok-Kyoung Choi

Bok Kyoung Choi received the M.S. and Ph.D. degrees in Department of Physics from Sung Kyun Kwan University, in 1991 and 1996, respectively.

Since 1996, he has been with the Physical Oceanography Division, Korea Ocean Research and Development Institute.

His research interests include the underwater acoustics and physical acoustics. He is a member of the Korean Physical Society, The Acoustical Society of Korea, the Acoustical Society of America and the Korean Society of Oceanography.

▲Suk-Wang Yoon



Suk Wang Yoon received the B.S. and M.S. degrees in Physics from Sogang University in 1975 and 1978, respectively, and the Ph.D. degree in Physical Acoustics from The University of Texas at Austion in 1983.

From 1984 to 1987 he was a faculty member of Physics at the U. S. Naval Postgraduate School. From 1989 to 1994 he worked as Visiting Professor and a Consulting Research Scientist at Physics Department and the U. S. National Center for Physical Acoustics, University of Mississippi. Since 1985 he is a Professor of Physics at Sung Kyun Kwan University. Since 1996 he has been working as a Visiting Professor of Bioengineering and Applied Physics Laboratory at University of Washington. He has been a member of the International Commission on Acoustics since 1995.