Adaptive Precompensation of Wiener Systems

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Abstract

In this paper, an adaptive precompensator, which can reduce the distortion of a Wiener system effectively, is proposed. The previous techniques for adaptive precompensation, based on the Volterra series modeling to compensate the distortion of a nonlinear system, are not suitable for real-time implementation due to high computational burden and slow convergence rate. This paper presents an adaptive precompensation technique for the class of nonlinear systems, which can be represented by interconnection of a linear dynamic subsystem and a memoryless nonlinear subsystem, referred to as Wiener system. An adaptive algorithm for adjusting the parameters of a precompensator, structured by a Hammerstein model, is derived using the stochastic gradient method. Also, an adaptive precompensation technique which can effectively reduce nonlinear distortion in μ-law type of saturation characteristics is proposed. The validity of the proposed algorithm is confirmed through simulation by applying it to known Wiener systems and a typical loudspeaker model.

I. Introduction

Compensation of nonlinear distortion due to saturation in electronic devices or electromechanical components are becoming more important as the state of the art in electronic engineering continues to progress. Followings are some applications where small distortion produced by a nonlinear component dominates the overall performance: (a) Nonlinearity in amplifiers, especially in high data-rate communication channels or in satellite communication channels, produces intolerable distortion [1], [2]. (b) Loudspeakers consisting of major nonlinear sources, including the non-uniform magnetic field (nonlinear *Bl* product) and the nonlinear compliance of suspension and surround, produce the harmonic and intermodulation distortion, which are most significant in Hi-Fi audio systems [3].

Two potential adaptive compensation techniques which allow us to reduce the distortion of time-varying nonlinear channels are adaptive precompensation at the transmission side and adaptive equalization at the receiver side [2]. Although nonlinear equalization is effective means of compensating the distortion of a nonlinear channel, there exist situations where it is effective or necessary to place a compensator in front of the nonlinear system to be compensated. One example would be compensation of nonlinear distortion for high-power amplifiers in satellite communication channels [1] since it would appear logical to reduce nonlinear distortion at the transmitter where it occurs and where the transmitted bits are available. The other example would be distortion reduction of a loudspeaker [3], [5] or active noise cancellation [4], where the output of the nonlinear system is not an electric but an acoustic signal.

Recently, several adaptive precompensation techniques have been proposed to reduce nonlinear distortion of a time-varying system [1], [5], [6]. However, since these techniques are based on the Volterra series modeling, they are not practical for real-time implementation due to the high computational complexity as well as slow convergence [5], [6]. The block-oriented model is another approach for modeling nonlinear systems without requiring large filter coefficients. This approach is based on the assumption that a nonlinear system consists of relatively simple subsystems, and that structure of the system is known. Especially, the class of systems which can be represented by a linear dynamic subsystem followed by a zero-memory nonlinear subsystem are called the Wiener system. The Hammerstein system consists of the same subsystems connected in the reverse order. The systems of these forms have been employed to model nonlinear characteristics in many areas of signal processing. Also, numerous papers concerning on the identification of the block-oriented model without having access to signals

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interconnecting the subsystems have been published [7], [8], [9]. The subsystems were identified on the basis of both the input and output signals of the whole system. Whereas various solutions for identifying Wiener (or Hammestein) systems with different types of memoryless nonlinear subsystems were suggested, no attempt has yet been made to compensate the nonlinear distortion of the systems.

Therefore, in this paper, an adaptive precompensation technique for reducing the distortion of a Wiener system is proposed. In Section II, an appropriate precompensation structure (Hammerstein model) for compensating the distortion of a Wiener system is proposed, and an adaptive algorithm for adjusting the coefficients of the precompensator is derived using the stochastic gradient method [10], [11]. Here, the memoryless nonlinear subsystem part in a Wiener system is assumed to be well approximated by a polynomial form of finite order. Also, a special case of an adaptive precompensation technique for the situation where the nonlinear subsystem part is modeled by the μ-law function is discussed. In Section III, the validity of the proposed algorithm is demonstrated via computer simulation by applying it to known Wiener systems and a typical loudspeaker model. Conclusion is made in Section IV.

II. Adaptive Precompensation of Wiener Systems

Let $u(n)$ and $y(n)$ represent the input and output signals, respectively, of a discrete-time causal nonlinear system. Then, the Volterra series expansion for a nonlinear system is given by [12], [13], [14]

$$y(n)=f_0+\sum_{k=0}^{\infty}\left[\sum f_k(m_1, m_2,\cdots,m_k)\,u(n-m_1)\cdots u(n-m_k)\right] \quad (1)$$

where $f_k\,(m_1, m_2,\cdots,m_k)$ is known as the k-th order Volterra kernel. Since the Volterra modeling of a nonlinear system requires a great deal of computation [15], only nonlinear systems incorporating up to second or third order terms have been actually realized [16], [17]. Another approach is based on the block-oriented model where a nonlinear system consists of relatively simple subsystems, and the structure of the system is known. The signals interconnecting the subsystems are usually inaccessible to measurements. If a linear dynamic system is followed by a memoryless nonlinear system as shown in Fig. 1(a), the block-oriented model is called the Wiener model. In

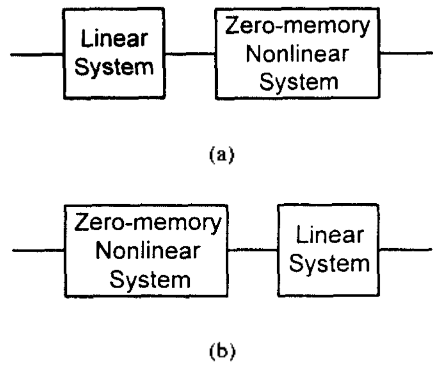

Fig. 1. Wiener model and Hammerstein model
(a) Wiener model
(b) Hammerstein model

Hammerstein model, the same subsystems are connected in the reverse order as shown in Fig. 1(b). Since the Wiener and Hammerstein models can be considered as special cases of Volterra series expansion, the Volterra kernels for Wiener and Hammerstein models must satisfy the relationship given by (2) and (3), respectively [7].

$$f_k(m_1, m_2,\cdots,m_k)=o_k\, f_k(m_1)f_k(m_2)\cdots f_k(m_k) \quad (2)$$

$$f_k(m_1, m_2,\cdots,m_k)=\begin{cases} o_k\, f_k(m, m,\cdots m) & \text{for } m_1=m_2=m_k=m \\ 0 & \textit{otherwise} \end{cases} \quad (3)$$

where o_k denotes scaling constant.

Fig. 2 shows the block diagram of a proposed adaptive precompensator which can reduce the distortion in a Wiener system. The proposed scheme is composed of a system estimator, which estimates the parameters of a Wiener system using an adaptive algorithm, and an adaptive precompensator with which the total system becomes linearized. If the memoryless nonlinear part of the Wiener system can be approximated by a polynomial form of finite order, the input and output relationship of the system estimator is given by

$$\hat{y}(n)=\sum_{l=1}^{N_a} a_l\left(\sum_{k=1}^{N_h} h_k\, u(n-k)\right)^l \quad (4)$$

where N_h and N_a denote the memory length of a linear filter, h_k, and the order of a nonlinear filter, a_l, respectively. The coefficients of the system estimator, h_k and a_l, are adjusted to minimize the mean square error, $E\{e_1^2(n)\}$, between $y(n)$ and $\hat{y}(n)$ using only the input and output signals of the system [7].

Assuming that correct parameters of the Wiener system

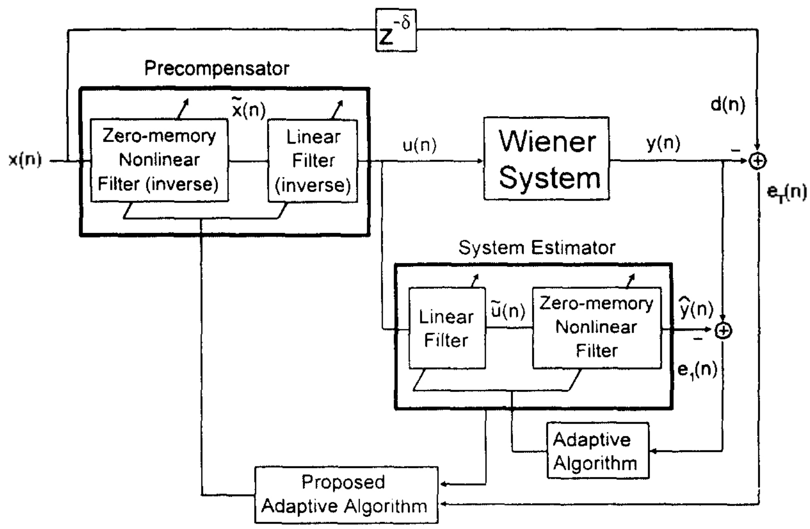

Fig. 2. A block diagram of an adaptive precompensator for Wiener system

are estimated, an adaptive precompensator, which is ideally the inverse of the Wiener system, can be designed to reduce distortion of the Wiener system. Since an appropriate structure of the precompensator for Wiener system can be easily shown to be the Hammerstein model, the precompensator in Fig. 2 is constructed by a memoryless nonlinear inverse filter followed by a linear inverse filter. By using a polynomial form of finite order as a memoryless nonlinear inverse filter, the precompensator can be expressed as

$$u(n)=\sum_{i=1}^{N_p} p_i \sum_{j=1}^{N_s} s_j\, x^j(n-i) \tag{5}$$

where N_p and N_s denote the memory length of a linear inverse filter, p_i, and the order of a nonlinear inverse filter, s_j, respectively. The error of the total system is defined by

$$e_T(n)=d(n)-y(n) \tag{6}$$

where the desired signal, $d(n)$, is the delayed version of input signal, $x(n)$, by δ samples to account for causality of the precompensator. The coefficients of the precompensator is obtained by minimizing the mean square error, $E\{e_T^2(n)\}$, of the total system. An adaptive algorithm for updating the coefficients of the precompensator is given by applying the stochastic gradient method as follows [10], [11]:

$$p_m(n+1)=p_m(n)-\frac{\alpha_p}{2}\,\hat{\nabla}_{p_m}(n),\ m=1,2,\cdots,N_p \tag{7}$$

$$s_m(n+1)=s_m(n)-\frac{\alpha_s}{2}\,\hat{\nabla}_{s_m}(n),\ m=1,2,\cdots,N_s \tag{8}$$

where α_p and α_s represent step-size constants of a linear inverse filter, p_m, and a nonlinear inverse filter, s_m, respectively. The step-size constant controls stability and convergence rate of the algorithm. The term, $\hat{\nabla}_{p_m}(n)$, in (7) represents an instantaneous estimate of gradient of $E\{e_T^2(n)\}$ with respect to the coefficients of linear inverse filter, p_m, defined by

$$\hat{\nabla}_{p_m}(n)\equiv\frac{\partial e_T^2(n)}{\partial p_m(n)} \tag{9}$$

$$=-2\,e_T(n)\,\frac{\partial y(n)}{\partial p_m(n)}\,. \tag{10}$$

Assuming that the coefficients of the system estimator have converged to correct values of the Wiener system, the output of the system estimator, $\hat{y}(n)$, can replace the output of the Wiener system, $y(n)$, in (10). Since $\hat{y}(n)$ is a function of input signals, $u(n-1)$, $u(n-2)$,$\cdots$,(10) can be rewritten by

$$\hat{\nabla}_{p_m}(n)=2\,e_T(n)\sum_{r=1}^{N_s}\frac{\partial\,\hat{y}(n)}{\partial u(n-r)}\,\frac{\partial u(n-r)}{\partial p_m(n)}\,. \tag{11}$$

For simplicity of notation, we define the derivative of Wiener system as

$$g(r;n)\equiv\frac{\partial\,\hat{y}(n)}{\partial u(n-r)}\,. \tag{12}$$

The above equation can be rewritten by substituting (4) into (12) as

$$g(r;n)=\sum_{l=1}^{N_a} l\, a_l\, h_r \Big(\sum_{k=1}^{N_h} h_k\, u(n-k)\Big)^{l-1}. \tag{13}$$

Also, the derivative of a precompensator output with respect to p_i, the second part of the summation term in (11), can be defined as

$$b_m(r;n) \equiv \frac{\partial u(n-r)}{\partial p_m(n)} . \tag{14}$$

Then, (14) can be rewritten by substituting (5) into (14) as

$$b_m(r;n)=\sum_{j=1}^{N_s} s_j\, x^j\,(n-r-m). \tag{15}$$

Finally, an adaptive algorithm for the linear inverse part of the precompensator can be obtained by substituting (11), (13), and (15) into (7) as follows:

$$p_m(n+1)=p_m(n)+\alpha_p\, e_T(n)\sum_{r=1}^{N_h}\sum_{l=1}^{N_a} l\, a_l\, h_r \Big(\sum_{k=1}^{N_h} h_k\, u(n-k)\Big)^{l-1}\sum_{j=1}^{N_s} s_j x^j\,(n-r-m). \tag{16}$$

An adaptive algorithm for the nonlinear inverse part of the precompensator can be derived in a similar way. The term, $\hat{\nabla}_{s_m}(n)$, given in (8), is defined by an instantaneous estimate of the gradient of $E\{e_T^2(n)\}$ with respect to the coefficient of the nonlinear inverse filter, s_m, as

$$\hat{\nabla}_{s_m}(n) \equiv \frac{\partial e_T^2(n)}{\partial s_m(n)} \tag{17}$$

$$= -2\, e_T(n)\,\frac{\partial \hat{y}(n)}{\partial s_m(n)}. \tag{18}$$

Also, (18) can be expressed as

$$\hat{\nabla}_{s_m}(n) = -2\, e_T(n)\sum_{r=1}^{N_h}\frac{\partial \hat{y}(n)}{\partial u(n-r)}\;\frac{\partial u(n-r)}{\partial s_m(n)} \tag{19}$$

Since the first part of summation term in (19) has already defined in (12), we define, here, the second part of summation term in (19) as

$$c_m(r;n) \equiv \frac{\partial u(n-r)}{\partial s_m(n)} . \tag{20}$$

Then, (20) can be expressed by substituting (5) into (20) as

$$c_m(r;n)=\sum_{i=1}^{N_p} p_i\, x^m\,(n-r-i). \tag{21}$$

Finally, an adaptive algorithm for the nonlinear part of the precompensator is obtained by substituting (13), (19), and (21) into (8) as follows:

$$s_m(n+1)=s_m(n)+\alpha_s\, e_T(n)\sum_{r=1}^{N_h}\sum_{l=1}^{N_a} l\, a_l\, h_r \Big(\sum_{k=1}^{N_h} h_k\, u(n-k)\Big)^{l-1}\sum_{i=1}^{N_p} p_i\, x^m\,(n-r-i). \tag{22}$$

Note that, in order to obtain the coefficients of the nonlinear inverse filter, s_m, of the precompensator, both the coefficients of the linear inverse filter, p_m, of the precompensator and the estimates, h_k and a_l, of the system estimator are required. Consequently, (16) and (22) constitute an adaptive algorithm for the precompensator which can reduce the distortion of a Wiener system.

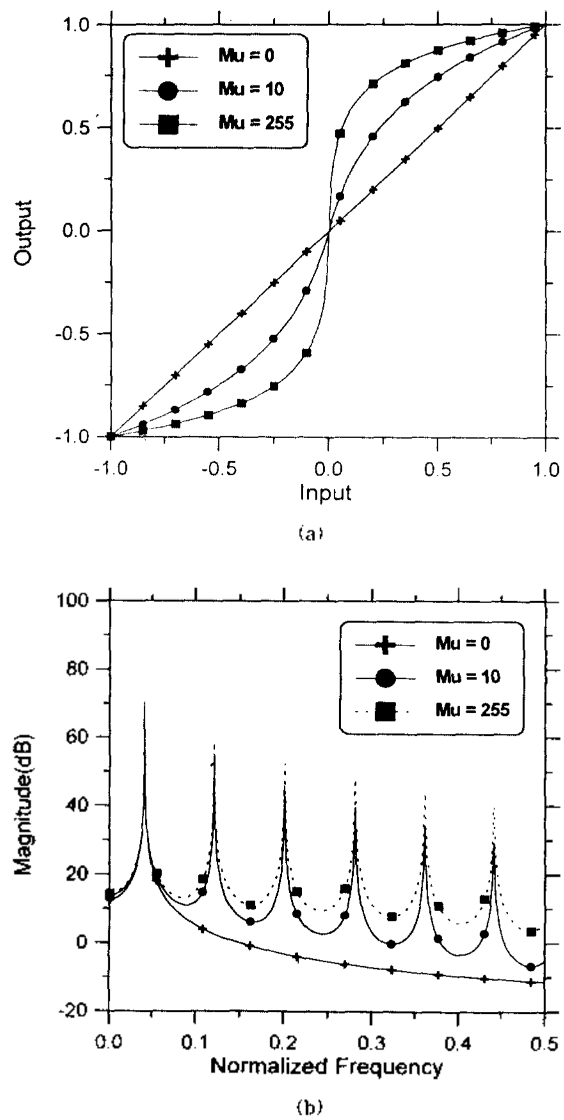

Fig 3. Characteristics of the μ-law function
(a) input-output relationship for the μ-law function
(b) frequency responses of the μ-law function

So far, the memoryless nonlinear part of the Wiener system and that of the precompensator are modeled by a polynomial form of finite and known order, which can provide good approximation for systems with smooth nonlinear characteristics. However, in order to accurately model saturation characteristics, typical phenomena of memoryless nonlinear systems, a polynomial function with infinite order is generally required. Thus, in this section, an adaptive precompensation technique, which enables us to avoid this difficulty if the Wiener system possesses the μ-law type of saturation characteristic, is proposed. Fig. 3 shows input-output relationship for the μ-law characteristic and corresponding frequency responses for different values of μ. Here, the value of μ controls the degree of nonlinearity in Wiener systems. For instance, a Wiener system with the value of $\mu=0$ reduces to a linear system. Note that a polynomial function with infinite order is generally required to model the μ-law characteristic in Fig. 3 whereas only one parameter, μ, is sufficient to model the nonlinear part of the Wiener system. A system estimator with the μ-law function as a memoryless nonlinear part is given by [18]

$$\tilde{u}(n)=\sum_{k=1}^{N_h} h_k\, u(n-k) \tag{23}$$

$$\hat{y}(n)=\tilde{u}_{\max}\,\frac{\log(1+\mu|\tilde{u}(n)|/\tilde{u}_{\max})}{\log(1+\mu)}\cdot sign(\tilde{u}(n)),$$

$$-1 < \tilde{u}(n)/\tilde{u}_{\max} < 1 \tag{24}$$

where $\tilde{u}(n)$ and $\hat{y}(n)$ denote an output signal passing through only the linear part, h_k, and an output signal after the nonlinear part, modeled by the μ-law function, of a system estimator, respectively. Also, $\tilde{u}_{\max}$ represents the maximum value of the input signal, $\tilde{u}(n)$, for the μ-law function. Also, log and $sign(\cdot)$ represent the natural logarithm and the function which takes only sign of the argument, respectively.

An updating algorithm for a parameter, μ, of the μ-law function and coefficients, h_k, of the linear filter in the system estimator can be obtained by minimizing the mean square error, $E\{e_1^2(n)\}$, between the output signal of the Wiener system, $y(n)$, and the output signal of the system estimator, $\hat{y}(n)$, as follows [10], [11]:

$$h_l(n+1)=h_l(n)-\alpha_h\,\hat{\nabla}_{h_l},\ l=1, 2,\cdots,N_h \tag{25}$$

$$\mu(n+1)=\mu(n)-\alpha_\mu\,\hat{\nabla}_\mu \tag{26}$$

where α_h and α_μ denote step-size parameters governing convergence rates of the linear filter and the μ-law function, respectively. Also, $\hat{\nabla}_{h_l}$ and $\hat{\nabla}_\mu$ represent instantaneous estimates of gradients of $E\{e_1^2(n)\}$ with respect to linear filter coefficients and μ, respectively. If we define the error signal as

$$e_1(n)=y(n)-\hat{y}(n). \tag{27}$$

The instantaneous estimates of gradients, $\hat{\nabla}_{h_l}$ and $\hat{\nabla}_\mu$, can be obtained by using (23) and (24) as follows:

$$\hat{\nabla}_{h_l}=-2e_1(n)\cdot\frac{\partial\hat{y}(n)}{\partial h_l} \tag{28}$$

$$=-2e_1(n)\cdot\frac{\mu|u(n-l)|}{\log(1+\mu)\cdot\log(1+\mu|\tilde{u}(n)|/\tilde{u}_{\max})}\cdot sign(u(n-l))$$

$$\hat{\nabla}_\mu=-2e_1(n)\,\frac{\partial\hat{y}(n)}{\partial\mu}$$

$$=-2e_1(n)\,\tilde{u}_{\max}$$

$$\frac{\dfrac{|\tilde{u}(n)|}{\tilde{u}_{\max}+\mu|\tilde{u}(n)|}\log(1+\mu)-\dfrac{\log(1+\mu|\tilde{u}(n)|/\tilde{u}_{\max})}{1+\mu}}{(\log(1+\mu))^2}$$

$$\cdot\, sign(\tilde{u}(n)). \tag{29}$$

Thus, by substituting (28) and (29) into (25) and (26), the parameters of the system estimator can be obtained.

Since there exists an exact inverse function of the μ-law function, we need an adaptive algorithm only for the linear inverse filter, p_i, of the precompensator. If we define $\tilde{x}(n)$ as a signal passing through the inverse of the μ-law function, then the output signal of the precompensator can be expressed as

$$u(n)=\sum_{i=1}^{N_p} p_i\,\tilde{x}(n-i) \tag{30}$$

where

$$\tilde{x}(n)=\frac{\tilde{u}_{\max}}{\mu}\left\{(1+\mu)^{\frac{|x|}{\tilde{u}_{\max}}}-1\right\}\cdot sign(x(n)) \tag{31}$$

Assuming that the output signal of the system estimator, $y(n)$, well approximates the output signal of the Wiener system, $\hat{y}(n)$, the total error can be approximated by

$$e_T(n)\approx d(n)-\hat{y}(n). \tag{32}$$

Finally, an adaptive algorithm for the linear inverse filter of the precompensator is obtained by minimizing $E\{e_T^2(n)\}$ with respect to p_i as follows:

$$p_m(n+1) = p_m(n) - \alpha_p \hat{\nabla}_{p_m},\ m = 1, 2, \cdots, N_p \tag{33}$$

where

$$\hat{\nabla}_{p_m} = -2e_T(n)\frac{\partial \hat{y}(n)}{\partial p_m}$$

$$= \frac{-2e_T(n)\,\mu \left| \sum_{k=1}^{N_h} h_k \tilde{x}(n-k-q) \right|}{\log(1+\mu)(1+\mu|\tilde{u}(n)|/\tilde{u}_{max}} \tag{34}$$

$$\cdot\, sign\left(\sum_{k=1}^{N_h} h_k \tilde{x}(n-k-q)\right).$$

In summary, for Wiener systems possessing the μ-law type of saturation characteristic, the precompensator can be designed by following the steps:

(i) estimate the parameter, μ, and filter coefficients, h_k, of the system estimator by iterating (25)-(26) and (28)-(29)

(ii) estimate the coefficients of the linear inverse filter, p_i, by inserting the estimated values from (i) into (34) and by iterating (33)-(34)

(iii) construct the precompensator by inserting the estimated values, μ and p_i, into (30)-(31)

(iv) repeat (i)-(iii) if the system is time-varying

Ⅲ. Simulation

In this section, the feasibility of applying the algorithms, proposed in section II, to reduction of distortion in nonlinear systems is demonstrated by computer simulation. The first two examples are concerned with the compensation of distortion in nonlinear systems using Wiener model with a polynomial form of finite order. In the third example, distortion of a nonlinear system is compensated by Wiener model with the μ-law function.

A. Precompensation of a known Wiener system

The nonlinear system used for this simulation is a known Wiener system represented by interconnection of a linear subsystem of 10 memory length, given by (35), and a memoryless nonlinear subsystem formed by the third-order polynomial, given by (36).

$$\begin{aligned}\tilde{u}(n) &= 0.0\cdot u(n-1) + 0.4\cdot u(n-2) + 0.08\cdot u(n-3)\\ &+ 0.15\cdot u(n-4)\\ &+ 0.05\cdot u(n-5) + 0.01\cdot u(n-6) + 0.005\cdot u(n-7)\\ &+ 0.002\cdot u(n-8) + 0.001\cdot u(n-9) + 0.0005\cdot u(n-10)\end{aligned} \tag{35}$$

$$y(n) = 1.5\cdot\tilde{u}(n) + 0.08\cdot\tilde{u}^2(n) + 0.02\cdot\tilde{u}^3(n) \tag{36}$$

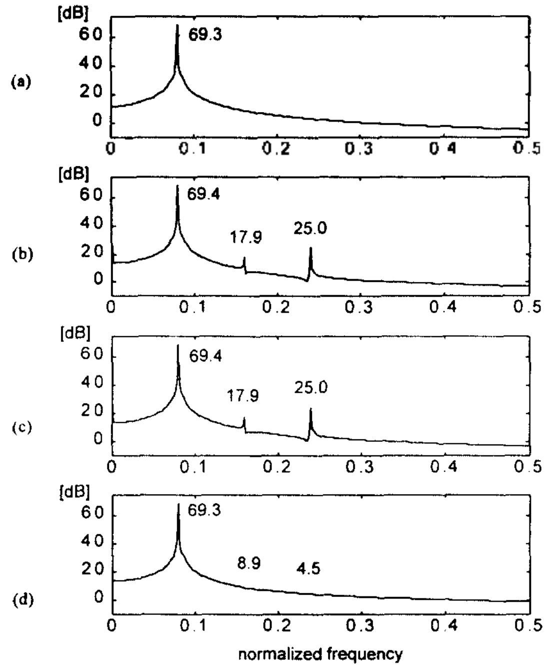

Fig. 4. Input and output spectrum of the known Wiener system (input: a sine wave with normalized frequency 0.08)
(a) input spectrum
(b) output spectrum of the known Wiener system
(c) output spectrum of the system estimator
(d) output spectrum when the proposed precompensator is used

In order to compensate the distortion present in this system, the parameters of the Wiener system, given by (35)-(36), were first estimated by the system estimator, The memory length of the linear filter, N_h, and the order of the nonlinear filter, N_a, in the system estimator were assigned to the same values as the ones of the Wiener system, 10 and 3, respectively. The input signal used for simulation was a white random signal with uniform distribution over [-1, 1]. After having checked the convergence of mean square errors in system estimation, we used the adaptive algorithm, derived in section II, to obtain the coefficients of the precompensator. Here, the order of the nonlinear inverse filter, N_s, and the memory length of the linear inverse filter, N_p, in the precompensator were set to 5 and 20, respectively. The parameters of the precompensator were estimated by applying a white random signal with uniform distribution. Next, in order to investigate the performances of the system estimator and the precompensator more clearly, we applied a sinusoidal signal to the precompensated system and compared its result to the case where no precompensation was performed. Fig. 4(a) shows the spectrum of an input signal with a normalized frequency 0.08 and amplitude 2. Fig. 4(b) shows the corresponding output spectrum when no precompensation was performed. Note that the linear distortion occurs at the same frequency as the frequency content of the input signal while nonlinear distortions occur at the second and third harmonics due to polynomial form of nonlinearity up to third order. By comparing it with Fig. 4(c), which shows the output spectrum of the system estimator, one can see that the system estimator works perfectly. Fig. 4(d) shows the output spectrum when the proposed precompensator was applied to the Wiener system. Note that the second and third harmonic components are considerably reduced by about 9 dB and 21.5 dB, respectively. The number of multiplications required for the precompensation part in the proposed approach is 25 while 285 multiplications, even with taking into account the symmetric property of Volterra kernels, is needed to implement the precompensator part in the previous approaches utilizing the Volterra series expansion.

B. Precompensation of a loudspeaker model

In this section, the proposed precompensation technique is applied to reduction of distortion in a loudspeaker model. The principal causes for nonlinear distortion of a loudspeaker at lower frequencies are nonuniform Bl product versus the voice coil, and nonlinearities in the compliance of the suspension and surround [3]. The state-space equation for a typical loudspeaker model taking into account these nonlinear effects is given by [5]

$$X(n+1)=\begin{bmatrix} -0.1 & 0 & -0.2 \\ 0 & 1 & 1 \\ 0.6 & -0.5 & -0.15 \end{bmatrix} X(n)+\begin{bmatrix} 0.4 \\ 0 \\ 0 \end{bmatrix} u(n) \qquad (37)$$

$$+\begin{bmatrix} -0.04x_2(n)x_3(n)-0.05x_2^2(n) \\ -0.08x_2^3(n)+0.01x_1(n)x_2(n)+0.02x_1(n)x_2^2(n) \end{bmatrix}$$

$$y(n)=(0\ 1\ 0)^T X(n)$$

where $u(n)$, $y(n)$, and $X(n)$ denote an input signal, an output signal, and a state vector, respectively. The third term on the RHS of (37) represents the main source of nonlinear distortion caused by the nonlinear Bl factor and the nonlinear suspension of a loudspeaker. Fig. 5 shows the second-and third-order Volterra kernels of the loudspeaker model, which were estimated by approximating the state-space equation in (37) with the Volterra series expansion. The memory lengths of the first-order Volterra kernel, second-order Volterra kernel, and third-order Volterra kernel were set to 17, 10, 10, respectively. Fig. 5(a), (b), and (c) show the first-order Volterra kernel, second-order Volterra kernel, and third-order Volterra kernel with the first argument set to 5, respectively. From Fig. 5, one can see that an approximate second-order Volterra kernel and third-order Volterra kernel can be obtained by multiplying the first-order Volterra kernel as given by (2), implying that Wiener modeling of a loudspeaker is adequate.

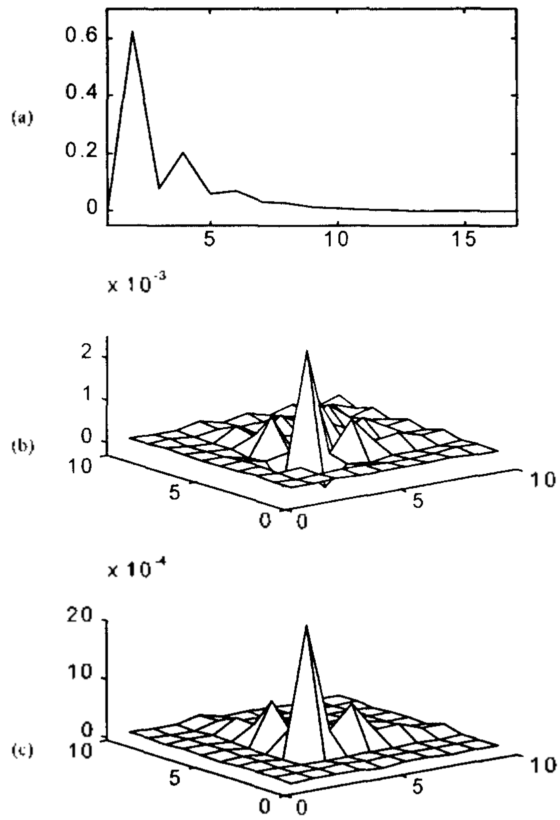

Fig. 5. Volterra modeling of the typical loudspeaker model
(a) first-order Volterra kernel ($f_1(i)$)
(b) second-order Volterra kernel ($f_2(i)$)
(c) third-order Volterra kernel ($f_3(i)$)

As in the previous simulation, the loudspeaker model was first approximated by the system estimator (Wiener model with $N_h=15$ and $N_a=3$) and then linearized by the precompensator (Hammerstein model with $N_p=20$ and $N_s=5$), all with a random input. Next, an sinusoidal signal was applied to the precompensated system to see how well the system estimator and the precompensator perform. Fig. 6(a) and (b) show the spectrum of an input signal with a normalized frequency 0.08 and amplitude 2, and corresponding output spectrum, respectively. Shown in Fig. 6(c) is the output spectrum of the system estimator (Wiener model) which approximates the output spectrum

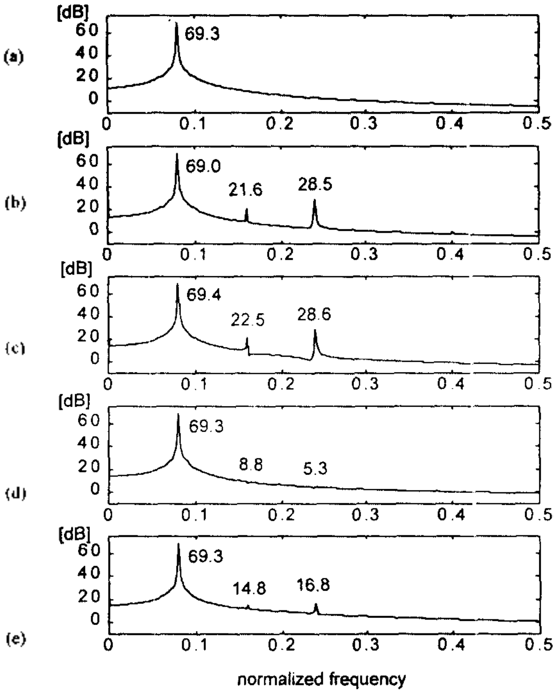

Fig. 6. Input and output spectrum of the typical loudspeaker model (input : a sine wave with normalized frequency 0.08)
(a) input spectrum
(b) output spectrum of the typical loudspeaker model
(c) output spectrum of the system estimator
(d) output spectrum when the proposed precompensator is used
(e) output spectrum when the previous approach is used

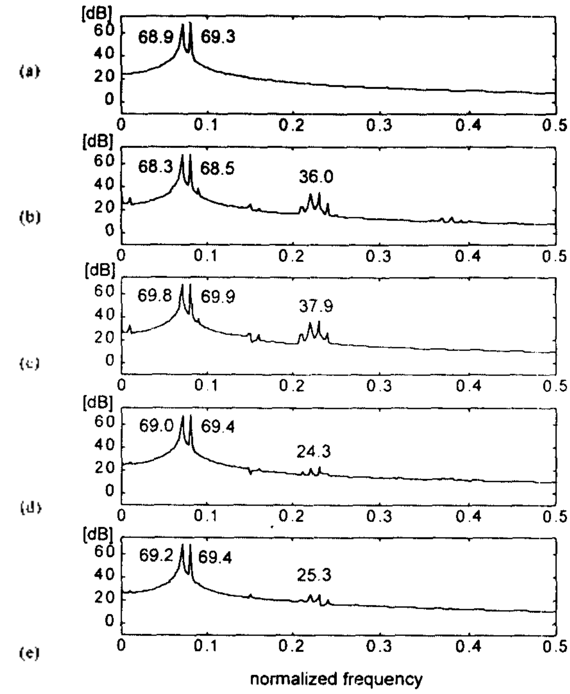

Fig. 7. Input and output spectrum of the typical loudspeaker model (input : a sine wave with normalized frequencies 07 and 0.08)
(a) input spectrum
(b) output spectrum of the typical loudspeaker model
(c) output spectrum of the system estimator
(d) output spectrum when the proposed precompensator is used
(e) output spectrum when the previous approach is used

of the loudspeaker model. From Fig. 6(d), one can see that nonlinear distortions at the second and third harmonic frequencies are considerably reduced by about 13.7 dB and 23.3 dB, respectively, when the proposed precompensator is used. By comparing this result with Fig. 6 (e), obtained by the previous approach [5], it can be said that the precompensator proposed in this paper can reduce the distortion of a loudspeaker model effectively, even with a small number of filter coefficients. Note that the number of multiplications required for the precompensation part in the proposed approach (previous approach) is 25 (292 when symmetric property is taken into account). In general, computational burden for the proposed approach increases linearly as the order and memory length of the model increase, while it increases exponentially for the previous approach using the Volterra series expansion. On the other hand, Fig. 7 shows the results of precompensation when intermodulation distortion occurs. In terms of Wiener modeling and precompensation of the loudspeaker, all results are similar to the ones in Fig. 6. Note that about 13.6 dB of intermodulation distortion is reduced by the proposed approach whereas about 12.6 dB is reduced by the previous approach.

C. Precompensation Wiener model with the μ-law function

The nonlinear system used for this simulation is a known Wiener system represented by cascade of a linear subsystem of 3 memory length, given by (38), and a memoryless nonlinear subsystem formed by the μ-law function ($\mu = 10$), given by (24).

$$\tilde{u}(n) = 0.2\,u(n-1) - 0.5\,u(n-2) + 0.2\,u(n-3) \qquad (38)$$

As in the previous simulation, we, first, estimated the parameters of the Wiener system, h_k and μ, by step (i) and, then, adjusted the parameters of the linear inverse filter ($N_p = 40$) in the precompensator by step (ii), all with random inputs. Fig. 8(b) shows the output spectrum when an input signal with a normalized frequency 0.04

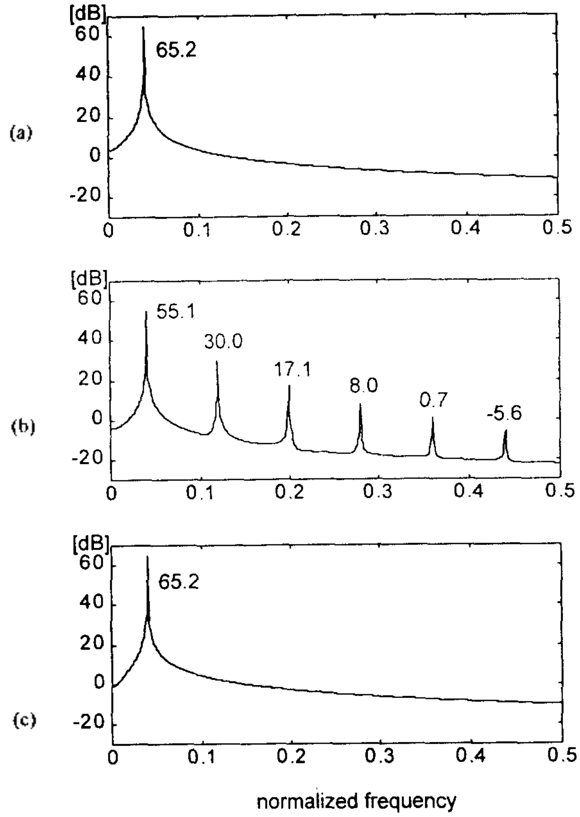

Fig. 8. Input and output spectrum of Wiener system with μ-law function (input: a sine wave with normalized frequency 0.04)
(a) input spectrum
(b) output spectrum of the Wiener system
(c) output spectrum when the proposed precompensator is used

and amplitude 1, shown in Fig. 8(a), was applied to the Wiener system. From Fig. 8(c), one can see that the proposed precompensation technique with the inverse μ-law function is very effective in reducing the distortion of a saturation component if its input-output characteristic can be approximated by the μ-law function. Note that the number of parameters to be estimated for the precompensation part is minimal, i.e., N_p plus one.

Ⅳ. Conclusion

Precompensation techniques for Wiener systems, which can be represented by cascade of linear dynamic and nonlinear elements, are proposed in this paper. Compared to the previous approaches using Volterra series modeling for compensation of distortions in general class of nonlinear systems, the proposed approach can reduce the distortion of a nonlinear system most effectively, i.e., with minimum computational complexity and fast convergence rate, if the nonlinear system can be approximated by the Wiener model. Futhermore, if the saturation characteristic of a memoryless nonlinear component in the Wiener system can be approximated by the μ-law function, the precompensator proposed in this paper can reduce nonlinear distortion with only one additional parameter, μ. Although the structures and algorithms for precompensator in this paper are derived under the assumption that the nonlinear system can be approximated by the Wiener model, the precompensator for Hammerstein system can be easily designed in the same manner.

References

1. S. Benedetto and E. Biglieri, "Nonlinear equalization of digital satellite channel," *IEEE Journal on Selected Areas in Communications*, Vol. SAC-1, No. 1, pp. 57-62, 1980.
2. S. Benedetto, E. Biglieri, and V. Castellani, *Digital Transmission Theory*, Prentice-Halt, Englewood Cliffs, NJ, 1987.
3. A. J. M. Kaiser, "Modeling of the nonlinear response of an electrodynamic loudspeaker by a Volterra series expansion," *J. Audio Eng. Soc.*, Vol. 35, No. 6, pp. 421-432, Jun. 1987.
4. S. J. Elliott and P. A. Nelson, "Active Noise Control." *IEEE Signal Processing Magazine*, pp. 12-35, Oct. 1993.
5. X. Y. Gao and W. M. Snelgrove, "Adaptive Linearization of a Loudspeaker," *Proc. Int. Conf. Acoust., Speech, Signal Processing*, Toronto, pp. 3589-3592, May 1991.
6. Y. H. Lim, H. W. Kang, Y. S. Cho and D. H. Youn, "An Adaptive Nonlinear Prefilter for the Compensation of Distortion in Nonlinear System," *IEEE Signal Processing ATHOS Workshop on Higher-Order Statistics*, Spain, pp. 357-360, Jun. 1995.
7. S. Y. Mao and P. X. Lin, "A Test of Nonlinear Autoregressive Models," *Proc. Int. Conf. Acoust., Speech, Signal Processing*, New York, pp. 2276-2279, 1988.
8. W. Greblicki and M. Pawlak, "Nonparametric Identification of Hammerstein Systems," *IEEE Tran. on Information Theory*, Vol. 35, No. 2, pp. 409-418, Mar. 1989.
9. W. Greblicki, "Nonparametric Identification of Wiener Systems," *IEEE Tran. on Information Theory*, Vol. 38, No. 5, pp. 1487-1493, Sep. 1992.
10. B. Widrow and S. D. Sterns, *Adaptive Signal Processing*, Englewood Cliffs, NJ: Prentice-Hall,, 1985.
11. S. Haykin, *Adaptive Filter Theory*, 2nd Eds., Englewood Cliffs, NJ: Prentice-Hall, 1991.
12. M. Schetzen, *The Volterra and Wiener Theories of Nonlinear Systems*, New York: Wiley, 1980.
13. M. Schetzen, "Nonlinear System Modeling Based on the Wiener Theory," *Proc. IEEE*, Vol. 69, No. 12, pp. 1557-1573, Dec. 1981.
14. S. A. Billings, "Identification of a Nonlinear Systems-A

Survey," *IEE Proc.*, Vol. 127, No. 6, pp. 272-285, Nov. 1980.

15. V. J. Mathews, "Adaptive Polynomial Filters," *IEEE Signal Processing Magazine*, pp. 10-26, Jul. 1991.
16. Y. S. Cho, S. B. Kim, E. L. Hixson and E. J. Powers, "A Digital Technique to Estimate Second-Order Distortion Using Higher Order Coherence Spectra," *IEEE Tran. on Signal Processing*, Vol. 40, No. 5, pp. 1029-1040, May. 1992.
17. T. H. Koh and E. J. Powers, "Second-Order Volterra Filtering and Its Application to Nonlinear System Identification," *IEEE Tran. on ASSP*, Vol. 33, No. 6, pp. 1445-1455, Dec. 1985.
18. L. R. Rabiner and R. W. Schafer, *Digital Processing of Speech Signals*, Englewood Cliffs, NJ: Prentice-Hall, 1978.

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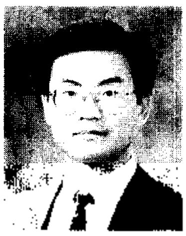

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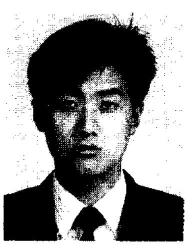

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