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LINEAR PROGRAMMING SOLUTIONS OF GENERALIZED LINEAR IMPULSIVE CORRECTION FOR GEOSTATIONARY STATIONKEEPING

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ABSTRACT

The generalized linear impulsive correction problem is applied to make a linear programming problem for optimizing trajectory of an orbiting spacecraft. Numerical application for the stationkeeping maneuver problem of geostationary satellite shows that this problem can efficiently find the optimal solution of the stationkeeping parameters, such as velocity changes, and the points of impulse by using the revised simplex method.

1. INTRODUCTION

Since the simplex method was introducd in 1947 by George B. Dantig, linear programming, which is an advanced mathematical field, have radically progressed in the field of the management science and operation research (OR). The simplex method, which is the first procedure of solving linear programming, has a simple algorithm in conception.

The simplex method of nonlinear optimization needs no initial estimate for the solution and can be made to converge to a global minimum although it requires a high computational price. Branham (1989) applied the simplex method to nonlinear optimization to determine the Pluto's mass and produce a satisfactory result.

Even though there are many advantages in linear programming, the optimal control theory has provided means by which the necessary conditions for optimal orbit control function can be derived. However, each method leads to subsidiary computational requirements that have proved troublesome in practice. Waespy (1970) described how the relatively efficient computational technics of linear programming can be used to obtain near-minimum fuel solutions and applied to the terminal guidance of an orbiting spacecraft. In his paper typical trajectories based on linearized equation of motions were calculated.

In 1939 L. V. Kantorovich, mathematician and economist of the U.S.S.R., introduced and solved the linear programming of the organization and planning of production (Gass 1994). A numerical algorithm of the generalized linear programming was first described by Lidov (1971). Bakhshiyan *et al.* (1980) developed this idea.

In this paper the generalized linear impulsive correction was applied to make a linear programming problem for an orbiting spacecraft. In order to show the effectiveness of the problem, for example, the stationkeeping maneuver problem of geostationary satellite was used. The optimal solution of the stationkeeping parameters, such as velocity changes, the points of impulses were obtained by using the revised simplex method.

2. GENERALIZED LINEAR IMPULSIVE CORRECTION

Let us consider an ideal correction for which errors of starting data are absent. In particular the execution errors of correction impulses are zero.

We regard each impulse vector $\vec{u_j}(j = 1, ...n; n$ - total number of impulses) as an element of Euclidean space $\mathbf{R}^{\mathbf{k_j}}$ of dimension k_j and the vector $\vec{l}$, which has to be controlled, as an element of Euclidean space $\mathbf{R}^{\mathbf{k}}$. Equation (1) shows the vector $\vec{u_j}$ may depend lineary on the vector (Bakhshiyan *et al.* 1980).

$$\vec{l} = f(\vec{l_0}, \vec{u_j}) = \vec{l_0} + \mathbf{B}_j \vec{u_j} \tag{1}$$

where $\vec{l_0}$ is starting vector, and $\mathbf{B_j}$ is a known $k_j \times k$ matrix which defines the linear mapping from $\mathbf{R}^{k_j}$ onto $\mathbf{R}^k$.
In special $\mathbf{B_j}$ may be expressed as in (2)

$$\mathbf{B_j} = \begin{pmatrix} \frac{\partial l_1}{\partial u_{jl}} & \cdots & \frac{\partial l_k}{\partial u_{jl}} \\ \cdots & \cdots & \cdots \\ \frac{\partial l_1}{\partial u_{jk_i}} & \cdots & \frac{\partial l_k}{\partial u_{jk_i}} \end{pmatrix} \tag{2}$$

When $\vec{l}, \vec{l_0}$ are given, to find optimal impulse vector $\vec{u_j}$, the generalized linear impulsive correction problem may be stated as following:

$$Minimize \quad \sum_{j=1}^{n} ||\vec{u_j}|| \tag{3}$$

$$subject \quad to \quad \sum_{j=1}^{n} \mathbf{B_j} \vec{u_j} = \vec{l} - \vec{l_0} = \vec{b}$$

We can express $\vec{u_j}$ in magnitude x_j and it's direction $\vec{\gamma_j}$.

$$\vec{u_j} = x_j \vec{\gamma_j} \tag{4}$$

Where $||\gamma_j|| = 1$. Therefore $||u_j|| = x_j||\gamma_j|| = x_j$ and problem (3) is reduced to the following generalized linear programming problem,

$$\text{to find } x_j, \ \vec{\gamma_j}$$

$$Minimize \quad \sum_{j=1}^{n} x_j \tag{5}$$

$$subject \quad to \quad \sum_{j=1}^{n} \mathbf{B_j} x_j \vec{\gamma}_j = \vec{b}$$

$$x_j \geq 0$$

The simplex method can be applied to problem (5) and optimality condition is as follows:

$$\Delta_j = \overset{\min}{||\gamma_j||} = 1 \; (1 - \pi^T \mathbf{B_j} \gamma_j) \tag{6}$$

where π^T is Lagrangian multiplier. If $\Delta_j \geq 0$ then the current point is considered as optimal. In the opposite case the new basis for which the minimum in (6) is attained is introduced and until optimality condition is specified, the procedure is repeated.

3. STATEMENT OF THE PROBLEM

Consider the problem of optimal correction of the satellite orbital parameters, in special determining the configuration of an orbit. The corrected orbital parameters are as follows:

γ_a : apogee altitude
γ_p : perigee altitude
ω : argument of perigee

Also, the impulsive correction is performed by thrusters in the following directions (Figure 1) :

V_x : velocity of tangential direction
V_y : velocity of radial direction

However the geostationary station keeping maneuver is usually performed in the tangential direction. Hence radial direction is not considered, namely $V = V_x$ and $k_j = 1 \quad \forall j = 1, ..., n$.

The time when the correction should be made is determined by the eccentric anomaly E. At the correction moment E_j the spacecraft receives impulses ΔV_j where j is the number of correction ($j = 1, ..., n$) and n is total number of impulses.

The required variations of orbital parameters $\Delta r_a, \Delta r_p, \Delta\omega$ are small. Hence the correction impulses are small too. Consequently it is possible to write with a sufficient accuracy the following linear dependence.

$$\Delta l = \sum_{j=1}^{n} \mathbf{B}(E_j) \Delta V_j \tag{7}$$

where $\Delta l = (\Delta\gamma_a, \Delta\gamma_p, \Delta\omega)$,

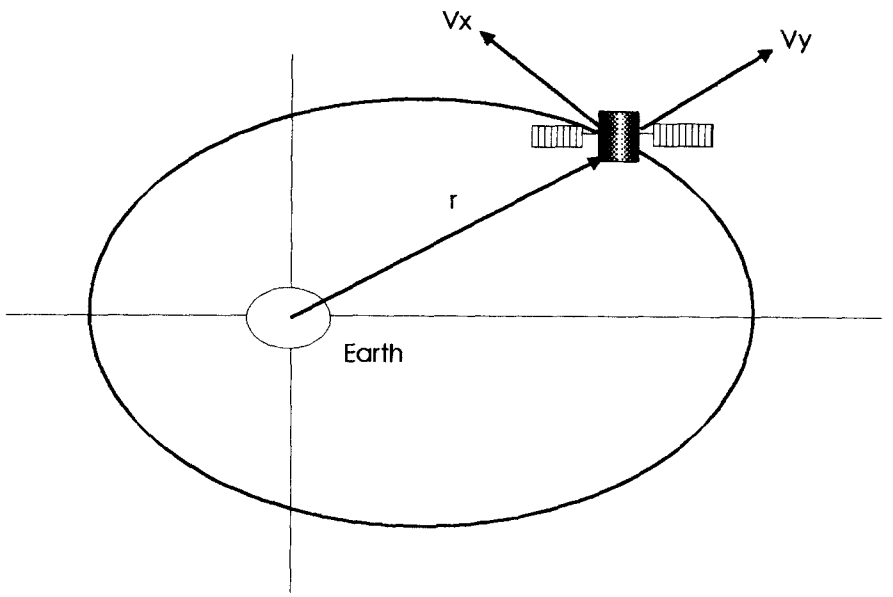

Figure 1. Typical geostationary orbit.

$$\mathbf{B}(E_j) = \frac{\partial L}{\partial V}(E_j) = \left\{ \frac{\partial \gamma_a}{\partial V}(E_j), \frac{\partial \gamma_p}{\partial V}(E_j), \frac{\partial \omega}{\partial V}(E_j) \right\}^T .$$

The partial derivatives of each orbital parameters are as follows (Nazirov *et al.* 1992).

$$\frac{\partial \gamma_a}{\partial V}(E) = 2a\sqrt{\frac{p}{\mu}}\sqrt{\frac{1+e}{1-e}}\frac{1+\cos E}{\sqrt{1-e^2\cos^2 E}}$$

$$\frac{\partial \gamma_p}{\partial V}(E) = 2a\sqrt{\frac{p}{\mu}}\sqrt{\frac{1+e}{1-e}}\frac{1-\cos E}{\sqrt{1-e^2\cos^2 E}}$$

$$\frac{\partial \omega}{\partial V}(E) = \frac{2}{e}\sqrt{\frac{p}{\mu}}\frac{\sin E}{\sqrt{1-e^2\cos^2 E}}$$

By using (5), (6) and (7), we can formulate the linear programming in order to find optimal ΔV_j.

$$Minimize \quad \sum_{j=1}^{n} |\Delta V_j|$$

$$subject \quad to \quad \Delta l = \sum_{j=1}^{n} \mathbf{B}(E_j)\Delta V_j \tag{8}$$
$$\Delta V_j \geq 0, E_j \geq 0$$

4. NUMERICAL EXAMPLE

We will apply the problem (8) to geostationary station keeping maneuver and verify the effectiveness of the optimal solutions, which are obtained by using simplex method. Usually in EWSK (East-West Station Keeping) the drift rate change ($\Delta\dot{\lambda}$) is constantly maintained.

The required velocity changes (ΔV) are calculated from equation (9) as drift rate change (Agrawal 1986).

$$\Delta V = 2.83\Delta\dot{\lambda} \ \ m/s \tag{9}$$

The variation of velocity changes as drift rate change are showed in Table 1. On the other hand, the velocity change achived from the optimal solutions of linear programming problem (8), are described in Table 2.

Table 1. Typical longitude station keeping.

Longitude Station Keeping		
Longitude Deadband(deg)	$\Delta V(m/s)$	$\Delta\dot{\lambda}(deg/day)$
$\pm$ 0.1	0.15	0.053
$\pm$ 0.2	0.21	0.074
$\pm$ 0.5	0.33	0.117
$\pm$ 1.0	0.46	0.163
$\pm$ 2.0	0.66	0.233
$\pm$ 3.0	0.80	0.283

Table 2. Velocity change calculated by this paper.

$\Delta\lambda(deg/day)$	$\Delta\gamma_a^*(km)$	$\Delta\gamma_p^*(km)$	$\Delta\omega^*(deg)$	$\Delta V(m/s)$
0.053	8.09	0.131	5.08	0.148
0.074	11.3	0.164	6.37	0.210
0.117	18.0	0.214	8.32	0.332
0.163	25.2	0.250	9.76	0.463
0.233	36.1	0.288	11.2	0.662
0.283	43.9	0.307	12.0	0.805

$*$ The orbital parameters used in calculating $\Delta\gamma_a$, $\Delta\gamma_p$, $\Delta\omega$ are as follows:
$a = 42.163km, e = 0.00025, \omega = 80°$

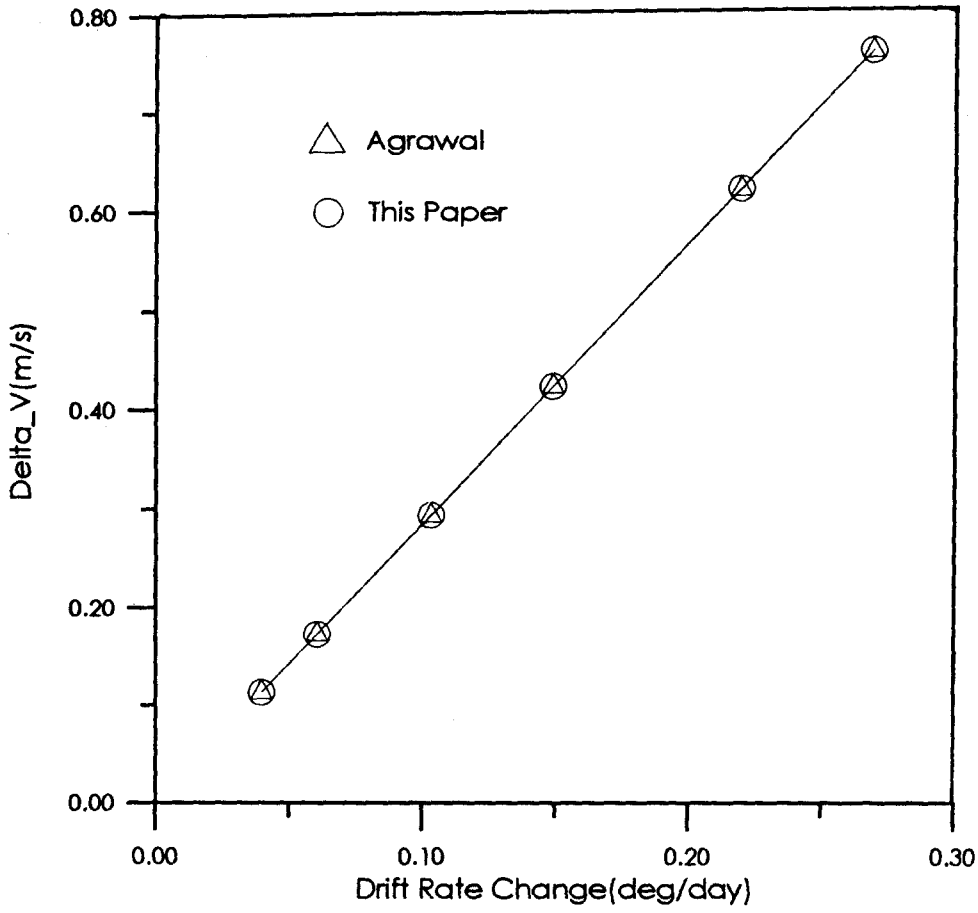

Figure 2. Comparison the optimal solution with the analytical solution.

Figure 2 shows the satisfactory coincidence between the analytical results and the linear programming solutions.

5. CONCLUSION

In spite of the simplicity in conception of algorithm and the powerful convergence of linear programming, its application to the orbit control program is comparatively fewer. In order to apply these advantages to the orbit control problem, the problem of generalized linear impulsive correction was presented for the numerical optimization of an optimal flight trajectory, which is essentially non-linear problem.

Table 3. Velocity change at each impulsive point.

$\Delta V_{apogee}(m/s)$	$\Delta V_{perigee}(m/s)$
0.147	0.00239
0.207	0.00299
0.328	0.00389
0.459	0.00456
0.657	0.00525
0.799	0.00559

To find the optimal solutions, the revised simplex method was applied and the computer program, using FORTRAN 77, was developed. The geostationary station keeping maneuver as a numerical example for the generalized linear impulsive correction illustrated that an optimal impulsive correction vector could be calculated by the present method. Furthermore correction points and number of impulses could be determined as Table 3.

As the results of comparison with the existing analytical solutions, they coincided. Hence we showed that the generalized impulsive correction problem is an efficient tool for achieving the optimal solution of the orbit control problem, which has depended on the existing optimal control theory.

Finally, the velocity change may have an upper bound as fuel state of the satellite so that the generalized impulsive correction problem should be modified to the upper bound problem.

Although in this paper one velocity direction was considered, three dimensional velocity direction should be incorporated into the present optimization process to complete the generalized impulsive correction problem.

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