Explicit Design of Uniformly Rough Pipe

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ABSTRACT : Pipe design normally requires pump power, discharge or pipe diameter for each condition given. Due to several investigators the pipe friction factor can now be estimated by explicit way for a wide range of flow condition. In various problems of pipe design, however, the flow condition cannot be pre-determined even for a uniformly rough pipe. In these cases a lot of iterations are often required to have an accurate solution with ordinary approach. This paper presents the direct computation method of discharge and pipe diameter without any iteration process. Introducing the power law of friction factor, various non-dimensional physical numbers are derived such as power-diameter number, power-discharge number, diameter-slope number and discharge-slope number. One of the physical numbers concerned with discharge or pipe diameter can be related to a combination of the others in an explicit way.

1. Introduction

In recent years computer techniques have been rapidly developed and PC's are world-widely in near-hand. This enables us to solve any complicated equations quite easily for engineering design, which often require iteration process for their solutions. When we have to choose the best one among various alternatives in a design process, however, the requirement of enormous number of iterations misguide us not to check all possible choices comprehensively. An explicit way of solution technique reduces the number of iterations significantly for each case so that the choice of the best alternative can be obtained in a very efficient way. It also aids to detect any error which may occur in the use of iteration precess. Furthermore, when an optimum combination of pipe size and pump power is due to be designed in a compicated pipe network, a lot of iterations may have to be required for each combination. In this case an explicit way of solution technique reduces the number of iterations tremendously and the best combination can be determined with a reasonable number of iterations.

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Even when a pipe is deployed on a sloping bed, the pump power required for a certain quantity of discharge through a pipe of pre-determined size can be estimated in an explicit way with an explicit equation of pipe friction factor. But the direct estimation of discharge or pipe diameter is not possible with the ordinary approach even when the friction factor can be determined by an explicit equation. The flow characteristics of pipe with uniform roughness has five flow regions such as laminar, transitional laminar, smooth turbulent, transitional turbulent and rough turbulent. When we encounter the problem of pipe design, the first step is to determine the flow condition among the five regions. In a certain flow condition the choice of flow region can be very difficult and sometimes the solution cannot be obtained because the choice of flow region is not decisive.

Various workers have tried to develop the explicit way of solving pipe design problems. Li (1974) has suggested three non-dimensional numbers by combining bed slope, pipe diameter, discharge, roughness height and viscosity in order to develop the explicit way of determining the pipe diameter. In his approach Colebrook-White equation of pipe friction factor was used, and the design graph of the three numbers was proposed for the estimation of proper pipe diameter. Simon (1986) mentioned that Asthana of Ethiopia has presented a design graph for the estimation of discharge. Asthana suggested four non-dimensional numbers such as diameter Reynolds number, roughness Reynolds number, relative roughness. On the other hand Hydraulic Research of U.K. has made a table of more than 150 pages for the estimation of pipe diameter and discharge. The explicit form of Colebrook-White equation developed by Barr (1975) was used for the product of the table, and the value of viscosity was given in advance for the water of 15℃.

Several problems are found in the previous approaches, which may have to be remedied or investigated by using totally different approaches. First, when the graphs they proposed are used for the pipe design, an interpolation process may have to be introduced and a slightly different answer is expected when it is used by different person. Second, an extra function may have to be developed for its use of a first approximate solution or optimum design. Third, as Yoo (1995), Yoo and Wun (1995), and Yoo and Han (1996) have explained and confirmed their findings using laboratory experimental results, the Colebrook-White equation of friction factor used for the commercial pipe is basically ill-logical. Colebrook (1938) assumed that 100 % of the pipe is covered by hydraulically-smooth wall and another 100 % by hydraulically-rough wall which means that 200 % exists in the same place. This is definately incorrect. Furthermore, any wall can be sometimes hydraulically-smooth or sometimes hydraulically-rough depending on a flow condition. The same value of roughness height for the same type of pipe was also reckoned to be improper and the roughness height of rough wall was found to be largely dependant on the pipe size because the pipe thickness is generally proportional to the pipe size. Fourth, the existing approaches did not consider the general situation of pipe condition, i.e., a pipe located on a sloping bed with a pumping power. For the proper development of explicit equations of pipe design, therefore, we have to use logically-correct friction factor equation of commercial pipe, and to consider the general case of pipe located on a sloping bed with a pumping power. The results might also have to be expressed by equations rather than by graphs for their general use. As a first step for this purpose we consider the uniformly-rough pipe but the general case of pipe located on a sloping bed with a pumping power.

As presented in Table 1, three types of pipe design can be classified for the case of single pipeline: Type A is for pump power, Type B for discharge, and Type C for pipe diameter. As for detailed classification, B-1 for discharge through a pumped pipe on a horizontal bed, B-2 for discharge through a pipe on a sloping bed with no pumping, C-1 for the diameter of pumped pipe on a horizontal bed, and C-2 for the diameter of pipe on a sloping bed with no pumping. Each condition of B-1 or B-2 is considered separately, and the related equation of Type B is formed by using the equation of B-1 as a basis. The related equation of Type C can be formed for the general case, and each case of C-1 or C-2 can be determined by using the general equation of Type C.

Table 1. Three Types of Pipe Design for Single Pipeline

Types	Variables Required	Conditions Given	Remarks
A	P (Pumping Power)	Q, d, l, k_s, i	
B	Q (Discharge)	d, l, k_s, P, i	B-1 ;i=0 B-2 ;P=0
C	d (Pipe Diameter)	Q, l, k_s, P, i	C-1 ;i=0 C-2 ;P=0

<Note> P: pump power, Q: discharge, d: pipe diameter, l: pipe length, k_s: roughness height, i: bed slope

The basic equation of pipe design as presented in Table 1 is the equation of pump power required for the transport of fluid through a pipe on a sloping bed as followings:

$$P=\rho gQ\left(f\frac{l}{d}\frac{V^2}{2g}-i\,l\right) \tag{1}$$

where P, Q, d, l, k_s and i are noted as in Table 1, ρ is the density of fluid, g is the acceleration of gravity, V is the mean velocity of flow, and f is the pipe friction factor. The bed slope i has the positive sign when the downward slope is along the flow direction. When the pipe friction factor is estimated by any form of explicit equations, the pipe design of Type A can be determined by direct computation. The pipe design of Type B-1, B-2 or C can also be determined by direct computation, when the pipe friction factor is expressed by a power law like Blasius equation for a certain range of smooth turbulent flow or like Hagen-Poiseulle equation for laminar flow.

2. Friction Factor of Uniformly-Rough Pipe

Prandtl (1925) has proposed a very important concept for the description of turbulent flow, i.e., the concept of mixing length, and established a robust basis for the formation of friction factor equation. His theory was sufficiently verified by the laboratory experiments of Nikuradse (1933), and their results expressed by two major equations are world-widely approved for the uniformly-rough walled pipe without any dispute. One of the equations is used for the circular pipe of hydraulically-smooth wall, and the other for that of hydraulically-rough wall. The friction factor equation of Blasius is valid only at Reynolds number less than 150 000, while the equation of Prandtl and Nikuradse was found to be valid at Reynolds number up to 100 millian by Colebrook (1938). But the Prandtl-Nikuradse equation of smooth turbulent flow has to be solved by iteration process, because the friction factor f is found in both sides of the equation as followings:

$$\frac{1}{\sqrt{f}}=2\log R\sqrt{f}-0.8 \tag{2}$$

where R is Reynolds number. On the other hand the Prandtl-Nikuradse equation of rough turbulent flow can be solved directly, which is expressed by:

$$\frac{1}{\sqrt{f}}=2\ \log d_k+1.14 \tag{3}$$

if the relative roughness d_k were given. However, if the pipe diameter is the value we search for, the value of f cannot be determined immediately. This requires us to relate the friction factor to a power law related with relative roughness.

From the laboratory results of Nikuradse, five characteristic regions of pipe flow are detected; laminar, transitional laminar, smooth turbulent, transitional turbulent and rough turbulent flows. Smooth turbulent flow is often named as the flow in hydraulically smooth wall, and rough turbulent flow is often named as the flow in hydraulically rough wall. It has been widely agreed that the boundary condition can be determined by the value of non-dimensional number, 'friction-roughness Reynolds number' R_{*k}. That is, the boundary value of R_{*k} between smooth turbulent flow and transitional turbulent flow (ST point) has been widely approved to be approxiamtely 3, and that of R_{*k} between transitional turbulent flow and rough turbulent flow (TR point) to be about 100. However, Yoo (1993) found that the 'roughness Reynolds' number R_k is the right scale for the determination of characteristic flow region for the case of pipe flow. The value of R_k at ST point was found to be about 80, while the value of R_{*k} at ST point was found to vary between 2.1 and 4.5. On the other hand the value of R_k at TR point was found to be about 1140, while the value of R_{*k} at TR point was found to vary between 35 and 105. Therefore, from the laboratory results of Nikuradse, five characteristic regions of pipe flow are found to be classified as followings:

$R < 2000$	: Laminar flow
$2000 < R < 4000$	: Transitional Laminar flow
$4000 < R < 80d_k$	: Smooth Turbulent flow
$80d_k < R < 1140d_k$	: Transitional Turbulent flow

$1140d_k < R$: Rough Turbulent flow

Various workers have made the explicit form of Prandtl–Nikuradse equation such as Barr (1975), Chen (1985), Yoo (1993), Yoo and Kang (1995). The pumping power required with the given conditions of slope, pipe diameter, discharge, etc can be estimated immediately by employing one of the explicit forms of Prandtl–Nikuradse equation. On the other hand the discharge and pipe diameter have to be determined through iteration process when using the traditional method of solving related equations. If the friction factor equation were expressed by a power law like the equation of Blasius, however, any quantity required can be determined by an explicit form of equation. In the present study the friction factor equations are given by power laws in all characteristic regions including the regions of transitional laminar, transitional turbulent and rough turbulent as well as laminar and smooth turbulent flows as followings:

$$f = \alpha \, R^{\beta} \tag{4}$$

or

$$f = \alpha_k \, d_k^{\beta_k} \tag{5}$$

Using the power form of friction factor equation, the equations of discharge and pipe diameter can be represented by explicit forms. For the general case of pumped pipe on a sloping bed an implicit form of discharge equation is resulted, and further manipulation gives an approximate but explicit equation of Reynolds number which is directly related with the discharge.

The pipe friction factor equations of power form for all flow regions are presented in Table 2. When using the power form instead of the original logarithmic form for the pipe friction factor equations, the boundary values should be modified due to the errors caused by the approximation as follows:

$R < 2000$: Laminar flow

$2000 < R < 4000$: Transitional Laminar flow

$4000 < R < 80d_k^{1.1}$: Smooth Turbulent flow

$80d_k^{1.1} < R < 543d_k^{1.1}$: Transitional Turbulent flow

$543d_k^{1.1} < R$: Rough Turbulent flow

Nikuradse devised six conditions of relative roughness for his laboratory experiments, and among them when $d/k_s = 30$ the flow condition changes directly from laminar flow to rough turbulent flow without having the regions of smooth turbulent flow and transitional turbuent flow. We call this region 'transitional flow' region, and it is determined by:

Table 2. Values of α and β for the Pipe Friction Factor Equations of Power Form.

Flow Region	Boundary Condition	α	β
Laminar	$R < 2000$	64	-1
Transitional Laminar	$2000 < R < 4000$	0.0015	2/5
Smooth Turbulent I	$4000 < R < 1.5\ 10^5$	0.316	$-1/4$
Smooth Turbulent II	$1.5\ 10^5 < R < 4.2\ 10^6$	0.117	$-1/6$
Smooth Turbulent III	$4.2\ 10^6 < R < 9.3\ 10^8$	0.062	$-1/8$
Smooth Turbulent IV	$9.3\ 10^8 < R < 80d_k^{1.1}$	0.037	$-1/10$
Transitional Turbulent	$80d_k^{1.1} < R < 543d_k^{1.1}$	$0.075d_k^{-2/5}$	1/10
		α_k	β_k
Rough Turbulent I	$d_k < 212 < (R/543)^{0.91}$	0.175	$-1/3$
Rough Turbulent II	$212 < d_k < (R/543)^{0.91}$	0.112	$-1/4$

Note : $f = \alpha R^{\beta}$ or $f = \alpha_k d_k^{\beta_k}$ (rough turbulent flow)

$d_k < 49$ and $2000 < R < 543\ d_k^{1.1}$: Transitional flow

In the present study this flow region was not considered.

Fig. 1 shows the comparison between the laboratory results and the estimates computed by the power laws. As shown in the figure the computation results are shown to be in reasonable agreement with the laboratory measurements. Some detectable errors are found only in the region of transitional turbulent flow when $d_k < 61.2$ and $504 < d_k$. Except this region the agreement is found to be excellent, and the computation error is to be less than 2 % in all regions.

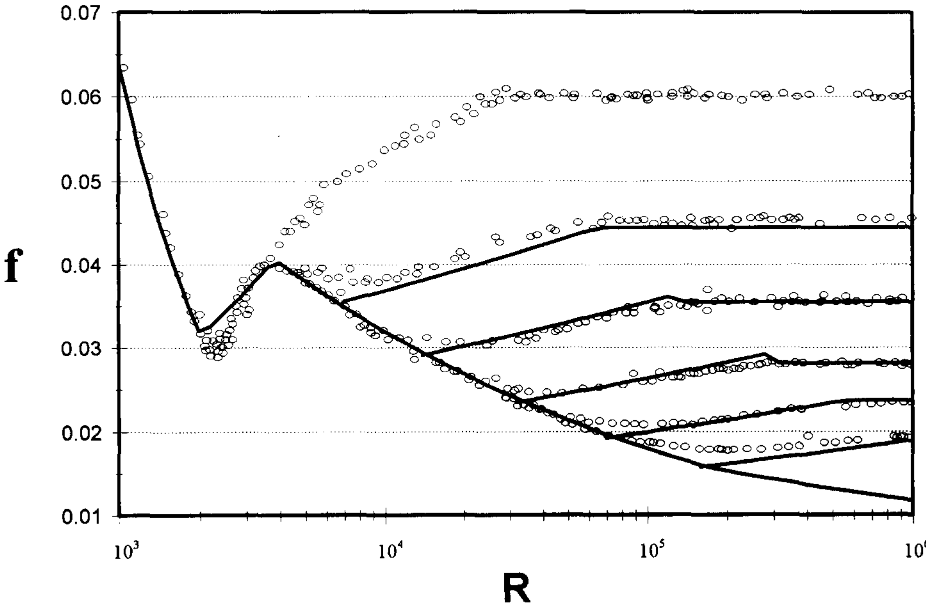

Fig. 1. Comparison between Laboratory Measurements and Power Law Computations

3. Discharge through Pumped Pipe (B–1)

For the computation of design type B–1, i.e., discharge through pumped pipe on a horizontal bed, we introduce the new non–dimensional number as followings:

$$B=\frac{1}{\nu}\left[\frac{Pd^2}{\rho l}\right]^{1/3} \tag{6}$$

which may be called 'power–diameter' number. Viscosity, density and pipe length are commonly introduced in the other physical numbers, and hence they are not included in the names. Introducing Eq. (4) into Eq. (1) with i=0, and using the 'power–diameter' number B, we have the relation of R with B as follows:

$$R=\gamma\ B^{\delta} \tag{7}$$

where

$$\gamma=\left[\frac{8}{\alpha\pi}\right]^{\frac{1}{3+\beta}} \tag{8}$$

$$\delta=\frac{3}{3+\beta} \tag{9}$$

For the region of rough turbulent flow, it is not necessary to use the friction factor equation of power form, Eq. (5), which has a limited validity in a certain region. In the design type B–1, we can use the logarithmic form Eq. (3) which is accurate in a wide condition. Introducing Eq. (3) instead of Eq. (5) into Eq. (1) with i=0, and using the 'power–diameter' number B, we have the relation of R with B for the region of rough turbulent flow as follows:

$$R=1.36\ (2\log d_k+1.14)^{2/3}\ B \tag{10}$$

Eq. (7) can be used for all flow regions except the region of rough turbulent flow. In each flow region only the parameters of γ and δ vary depending on the values of α and β appropriate to each region as presented in Table 3. The flow region can also be determined explicitly by checking the value of B. The boundary values of B can be pre–determined by applying Eq. (7) with the proper values of γ and δ to the boundary values of R. That is, the boundary value of B at ST point is estimated by applying Eq. (7) with $\gamma=3.12d_k^{4/31}$ and $\delta=0.968$ to $R_{ST}=80d_k^{1.1}$. Then B_{ST} is estimated to be $28.6d_k$. All values of B_{LT}, B_{TS}, B_{ST} and B_{TR} can be pre-determined by the same way, and the boundary values of B are also presented in Table 3.

Table 3. Parameters of γ and δ for the Computation of Discharge for Type B-1

Flow Region	Boundary Condition	γ	δ
Laminar	$B < 465$	0.200	3/2 (1.500)
Transitional Laminar	$465 < B < 1017$	8.904	15/17 (0.882)
Smooth Turbulent I	$1017 < B < 3.3\ 10^4$	2.126	12/11 (1.091)
Smooth Turbulent II	$3.3\ 10^4 < B < 6.4\ 10^5$	2.984	18/17 (1.059)
Smooth Turbulent III	$6.4\ 10^5 < B < 1.1\ 10^8$	3.641	24/23 (1.043)
Smooth Turbulent IV	$1.1\ 10^8 < B < 28.6d_k$	4.305	30/29 (1.034)
Transitional Turbulent	$28.6d_k < B < 206.7d_k$	$3.12\ d_k^{0.129}$	30/31 (0.968)
Rough Turbulent	$206.7d_k < B$	$1.36(2\log d_k + 1.14)^{0.667}$	1.000

Note: $R = \gamma\ B^{\delta}$

As discussed above, for the case of pumped pipe on a horizontal bed, the discharge or mean velocity or Reynolds number of pipe flow is wholely dependent on B and the flow condition can also be pre-determined by the value of B with d_k. Fig. 2 shows the distribution of R against B with various values of d_k. In the region of laminar flow with $B < 465$, Reynolds number rapidly increases with the value of B. In the region of transitional laminar flow with $465 < B < 1017$, the increase of R becomes relatively mild. Depending on the relative roughness, the region of transitional turbulent flow starts from different positions and the slope of the curve has also different degree of steepness. Large relative roughness or small value of d_k means early start of transitional turbulent flow and rough turbulent flow, and mild slope.

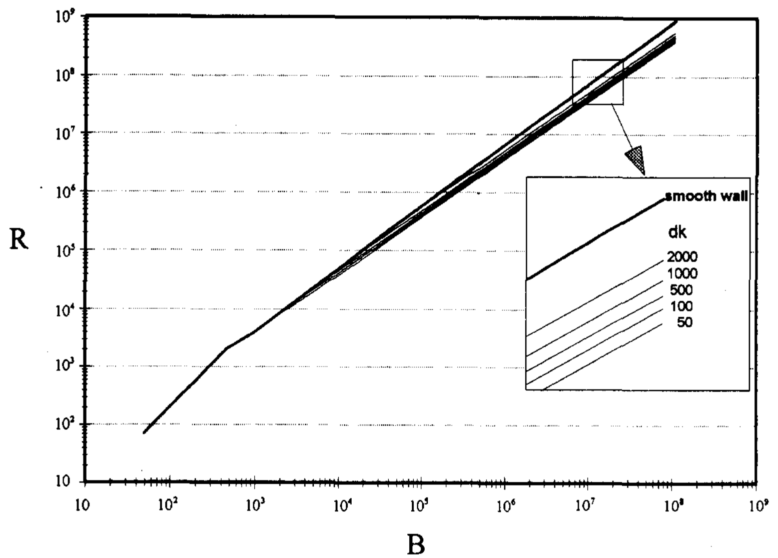

Fig. 2. R against B with Various Values of d_k

4. Discharge on a Sloping Bed (B-2)

For the computation of design type B-2, i.e, discharge through a pipe on a sloping bed with no pumping power, we introduce another two non-dimensional numbers as followings:

$$F=\frac{V}{\sqrt{gdi}} \tag{11}$$

$$N=\frac{R}{F}=\frac{\sqrt{gd^3i}}{\nu} \tag{12}$$

where F is called as 'slope Froude number' and N is called as 'R-F number' or 'diameter-slope number'. Here d is not the water depth, which is often used for determining the characteristics of open channel flow. The introduction of N makes the explicit formulation possible, because it does not include the velocity which is the quantity we search for.

When no power is applied to a pipe line, i.e., P=0, with $h_f/l=i$, Eq. (1) becomes Chezy equation as follows:

$$V=\sqrt{\frac{2gdi}{f}} \tag{13}$$

Introducing Eq. (4) into Eq. (13), and using the 'R-F number' N, we have the relation of F or R with N as follows:

$$F=\left(\frac{2}{\alpha}\right)^{\frac{1}{2+\beta}} N^{\frac{-\beta}{2+\beta}} \tag{14}$$

$$R=\left(\frac{2}{\alpha}\right)^{\frac{1}{2+\beta}} N^{\frac{2}{2+\beta}} \tag{15}$$

Both equations of (14) and (15) can be used for a wide range of flow condition from laminar flow up to transitional flow. In the region of smooth turbulent flow, it can be used by changing the values of α and β as presented in Table 2. In the case of Type B-2, however, we can apply Eq. (2) for smooth turbulent flow, which is valid for a wide range of flow condition. For a uniform flow, if we apply Chezy equation (13) to Eq. (2), the friction factor is automatically eliminated and we obtain the widely-valid equation of F in the region of smooth turbulent flow as follows:

$$F=\sqrt{2}[2\log N-0.5] \tag{16}$$

Similarly for the region of rough turbulent flow, if we apply Chezy Eq. (13) to Eq. (3), the friction factor is automatically eliminated and we obtain the widely-valid equation of F in the region of rough turbulent flow as follows:

$$F=\sqrt{2}[2\log d_k+1.14] \tag{17}$$

Reynolds number can be determined by multiplying N to F estimated by the appropriate equation for the flow region. In the regions of laminar, transitional laminar and transitional turbulent flows, Eq. (14) or (15) have to be used for the estimation of F or R. In the region of transitional turbulent flow, by applying appropriate values of α and β Eq. (15) becomes as follows:

$$R=4.77\, d_k^{\frac{4}{21}}\, N^{\frac{20}{21}} \tag{18}$$

Substituting Eq. (18) into the boundary values of R at ST point and TR point gives that $N_{ST}=19.3$

Table 4. R against N for the Computation of Discharge for Type B-2

Flow Region	Boundary Conditions	R
Laminar	$N < 253$	$0.031\ N^2$
Transitional Laminar	$253 < N < 577$	$20.05\ N^{0.833}$
Smooth Turbulent	$577 < N < 19.3d_k^{0.96}$	$N(2.83\log N - 0.71)$
Transitional Turbulent	$19.3d_k^{0.96} < N < 144.1d_k^{0.96}$	$4.77d_k^{0.19}N^{0.952}$
Rough Turbulent	$144.1d_k^{0.96} < N$	$N(2.83\log d_k + 1.61)$

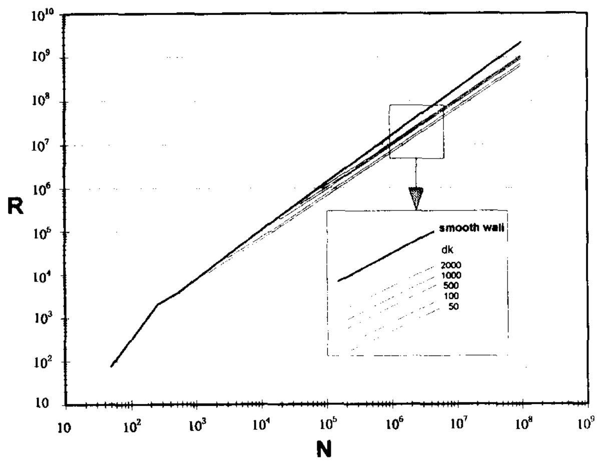

Fig. 3 R against N with Various Values of d_k

$d_k^{0.96}$ and $N_{TR}=144.1d_k^{0.96}$. The values of N_{LT} and N_{TS} can also be pre-determined by the same way, and Table 4 presents the boundary values of N and the appropriate equations of R for each flow region for the design type B-2. Fig. 3 shows the variation of R against N with various values of d_k.

5. Discharge on General Case (Type B)

The design type B requires the discharge in the general case of pumped pipe on a sloping bed with the given values of pipe diameter, pipe length, roughness height, bed slope and pump power. For all regions of flow except the rough turbulent flow region, we can apply Eq. (4) to Eq. (1) and then we have

$$R=R_0\left(1+\frac{\pi}{4}\frac{N^2R}{B^3}\right)^{\frac{1}{3+\beta}} \tag{19}$$

where R_0 is the Reynolds number for the case of pumped pipe on a horizontal bed, that is, Type B-1, which can be immediately estimated by Eq. (7) or Eq. (10) and/or one of the equations presented in Table 3.

For further simplification, we introduce the enhancement ratio $\eta = R/R_0$ and the combined non-dimensional number N_B, which is given by:

$$N_B=\frac{R_0N^2}{B^3}=\frac{R_0\nu\rho g d l i}{P} \tag{20}$$

Then Eq. (19) is simplified as follows:

$$\eta=\left(1+\frac{\pi}{4}\eta N_B\right)^{\frac{1}{3+\beta}} \tag{21}$$

We now have the problem to estimate η from Eq. (21) which includes it in both sides and hence Eq. (21) is possible to be solved only by iteration process.

Approximate but explicit expression of η has been devised by numerical computation. As expressed by Eq. (21) η is a function of N_B and β. The most probable occurance in a real field situation might be found in the range of R between 4 000 and 500 000. Therefore only the regions of smooth turbulent flow I and II were considered in the present study with $\beta = -1/4$ and $\beta = -1/6$ respectively. By applying wide range of N_B values the distribution of η has been characterized for the development of explicit equation of η. Four regions were classified: $-1 < N_B < 0$, $0 < N_B < 1$, $1 < N_B < 4$, and $4 < N_B$. The region of $N_B < -1$ may hardly occur, and hence it was not considered in the present study. The regions of smooth turbulent flow Ⅲ and Ⅳ were not investgated, either, because the regions are also regarded to hardly occur. The results are presented in Table 5. For checking the boundary conditions, approximate values of R have to be computed and they are presented in the table using the relevant equations of η and R_0.

Table 5. Enhancement Ratio η in Each Region of N_B

(a) $-1 < N_B \leq 0$

Flow Region		Boundary Condition	η
Laminar		$R < 2000$	$\exp(0.385\ N_B)$
Tr. Laminar		$2000 < R < 4000$	$0.236\ N_B + 1$
S. Turbulent	I	$4000 < R < 1.5\times10^5 < R_{ST}$	$0.269\ N_B + 1$
	II	$1.5\times10^5 < R < R_{ST}$	$0.264\ N_B + 1$
T. Turbulent		$R_{ST} < R < R_{TR}$	$0.247\ N_B + 1$
R. Turbulent		$R_{TR} < R$	$0.258\ N_B + 1$

<Note> Reynolds Number used for checking boundary condition

Laminar and Tr. Laminar Flow: $R=18.71\ N^2\ B^{-1.235}+8.904\ B^{0.882}$

Smooth Turbulent Flow: $R=2.351\ N^2\ B^{-0.882}+2.984\ B^{1.059}$

Tr. Turbulent and R. Turbulent Flow: $R=2.404d_k^{0.258}\ N^2\ B^{-1.065}+3.12d_k^{0.129}\ B^{0.968}$

(b) $0 < N_B \leq 1$

Flow Region		Boundary Condition	η
Laminar		$R < 2000$	$\exp(0.385\ N_B)$
Tr. Laminar		$2000 < R < 4000$	$0.220\ N_B + 1$
S. Turbulent	I	$4000 < R < 1.5\times10^5 < R_{ST}$	$0.290\ N_B + 1$
	II	$1.5\times10^5 < R < R_{ST}$	$0.278\ N_B + 1$
T. Turbulent		$R_{ST} < R < R_{TR}$	$0.247\ N_B + 1$
R. Turbulent		$R_{TR} < R$	$0.258\ N_B + 1$

<Note> Reynolds Number used for checking boundary condition

Laminar and Tr. Laminar Flow: $R=17.442\ N^2\ B^{-1.235}+8.904\ B^{0.882}$

Smooth Turbulent Flow: $R=2.475\ N^2\ B^{-0.882}+2.984\ B^{1.059}$

Tr. Turbulent and R. Turbulent Flow: $R=2.404d_k^{0.258}\ N^2\ B^{-1.065}+3.12d_k^{0.129}\ B^{0.968}$

(c) $1 < N_B < 4$

Flow Region		Boundary Condition	η
Laminar		$R < 2000$	$\exp(0.866\sqrt{N_B}-0.499)$
Tr. Laminar		$2000 < R < 4000$	$\exp(0.353\sqrt{N_B}-0.156)$
S. Turbulent	I	$4000 < R < 1.5\times10^5 < R_{ST}$	$\exp(0.486\sqrt{N_B}-0.235)$
	II	$1.5\times10^5 < R < R_{ST}$	$\exp(0.464\sqrt{N_B}-0.221)$
T. Turbulent		$R_{ST} < R < R_{TR}$	$\exp(0.404\sqrt{N_B}-0.185)$
R. Turbulent		$R_{TR} < R$	$\exp(0.424\sqrt{N_B}-0.197)$

<Note> Reynolds Number used for checking boundary condition

Laminar and Tr. Laminar Flow: $R=7.618\ B^{0.882}\exp(1.053\ N\ B^{-1.059})$

Smooth Turbulent Flow: $R=2.392\ B^{1.059}\exp(0.802\ N\ B^{-0.971})$

Tr. Turbulent and R. Turbulent Flow: $R=2.593d_k^{0.129}\ B^{0.968}\exp(0.714d_k^{0.065}\ N\ B^{-1.02})$

(d) $4 < N_B$

Flow Region		Boundary Condition	η
Laminar		$R < 2000$	$0.768\,N_B + 0.345$
Tr. Laminar		$2000 < R < 4000$	$0.6\sqrt{N_B} + 0.52$
S. Turbulent	I	$4000 < R < 1.5\times10^5 < R_{ST}$	$(0.323\sqrt{N_B} + 0.8)^2$
	II	$1.5\times10^5 < R < R_{ST}$	$0.97\sqrt{N_B} + 0.07$
T. Turbulent		$R_{ST} < R < R_{TR}$	$0.75\sqrt{N_B} + 0.36$
R. Turbulent		$R_{TR} < R$	$0.82\sqrt{N_B} + 0.27$

<Note> Reynolds Number used for checking boundary condition

Laminar and Tr. Laminar Flow: $R = 15.941\,B^{-0.176}N + 4.630\,B^{0.882}$

Smooth Turbulent Flow: $R = 5.0\,B^{-0.068}N + 0.209\,B^{1.059}$

Tr. Turbulent and R. Turbulent Flow: $R = 4.133 d_k^{0.194} B^{-0.05} N + 1.123 d_k^{0.129} B^{0.968}$

For the region of rough turbulent flow, Eq. (5) is introduced into Eq. (1). Then we have the enhancement ratio as follows:

$$\eta = \left(1 + \frac{\pi}{4}\eta\,N_B\right)^{\frac{1}{3}} \tag{22}$$

Eq. (22) has to be solved by iteration process, too. By similar process, approximate but explicit expression of η has been developed. The results are also presented in Table 5.

Fig. 4 shows the comparison between the explicit approximations from Table 5 and the accurate solutions by numerical computation from Eq. (21) or (22). As shown in the figure, the comparison is found to be in excellent agreement in most regions. In the regions of laminar, smooth turbulent, transitional turbulent, and rough turbulent flows, the agreements are found to be almost perfect. In the region of transitional laminar flow, some computation errors are observed, but even in this region the error is detected to be less than 1 %.

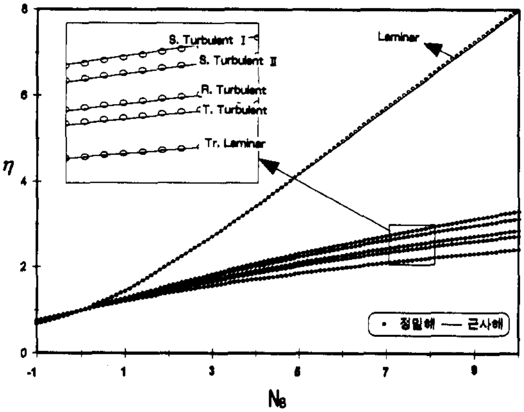

Fig. 4. Comparison of η between Explicit Approximations and Accruate Solutions

6. Pipe Diameter (Type C)

The design type C requires the pipe diameter in the general case of pumped pipe on a sloping bed with the given values of discharge, pipe length, roughness height, bed slope and pump power. For this computation we introduce five non-dimensional numbers as follows:

$$S=\frac{\nu d}{Q} \tag{23}$$

$$E=\frac{1}{\nu}\left(\frac{PQ^2}{\rho l}\right)^{\frac{1}{5}} \tag{24}$$

$$G=\frac{1}{\nu}\left(Q^3 gi\right)^{\frac{1}{5}} \tag{25}$$

$$K=\frac{\nu k_s}{Q} \tag{26}$$

$$T=(E^5+G^5)^{\frac{1}{5}}=\frac{1}{\nu}\left(\frac{PQ^2}{\rho l}+Q^3\ gi\right)^{\frac{1}{5}} \tag{27}$$

S is reversely proportional to Reynolds number, and hence it may be called 'reversed Reynolds number'. Although S is reversely proportional to Reynolds number R, R has two unknown values, V and d, while S has only one unknown value, d. E is called 'power-discharge number', G 'discharge-slope number', K 'reversed roughness Reynolds number', and T 'combined E-G number' or 'power-discharge-slope number'.

For all regions of flow except the rough turbulent flow region, we can apply Eq. (4) to Eq. (1) and then we have the relation of S with the combined number T as follows:

$$S=\varepsilon\ T^{\zeta} \tag{28}$$

where

$$\varepsilon=\left[\frac{\alpha}{2}\left(\frac{4}{\pi}\right)^{2+\beta}\right]^{\frac{1}{5+\beta}} \tag{29}$$

$$\zeta = -\frac{5}{5+\beta} \tag{30}$$

For the region of rough turbulent flow, substituting Eq. (5) into Eq. (1) gives the same form of the Eq. (28), but with the different parameters of ε and ζ as follows:

$$\varepsilon = \left[\frac{\alpha}{2}\left(\frac{4}{\pi}\right)^2\right]^{-\frac{\zeta}{5}} K^{1+\zeta} \tag{31}$$

$$\zeta = -\frac{5}{5-\beta} \tag{32}$$

Eq. (28) can be used for all flow regions with Eqs. (31) and (32) for the region of rough turbulent flow and Eqs. (29) and (30) for the other flow regions. In each flow region only the parameters of ε and ζ vary depending on the values of α and β appropriate to each region as presented in Table 6. The flow region can also be determined explicitly by checking the value of T. The boundary values of T can be pre-determined by applying Eq. (28) with the proper values of ε and ζ to the boundary values of R. That is, the boundary value of T at ST point is estimated by applying Eq. (28)

Table 6. Parameters ε and ζ in the Equation of S Related with T

Flow Region	Boundary Condition	ε	ζ
Laminar	$T < 754.1$	2.526	−5/4 (−1.25)
Tr. Laminar	$754.1 < T < 1594.9$	0.294	−25/27 (−0.926)
S. Turbulent I	$1594.9 < T < 5.0\times104$	0.741	−20/19 (−1.053)
S. Turbulent II	$5.0\times10^4 < T < 1.3\ K^{-0.5}$	0.609	−30/29 (−1.034)
Tr. Turbulent	$1.3\ K^{-0.5} < T < 13.6\ K^{-0.5}$	$0.6\ K^{0.073}$	−10/11 (−0.909)
R. Turbulent I	$13.6\ K^{-0.5} < T < 0.002\ K^{-1}$	$0.694\ K^{0.063}$	−15/16 (−0.938)
R. Turbulent II	$13.6\ K^{-0.5} < 0.002\ K^{-1} < T$	$0.633\ K^{0.048}$	−20/21 (−0.952)

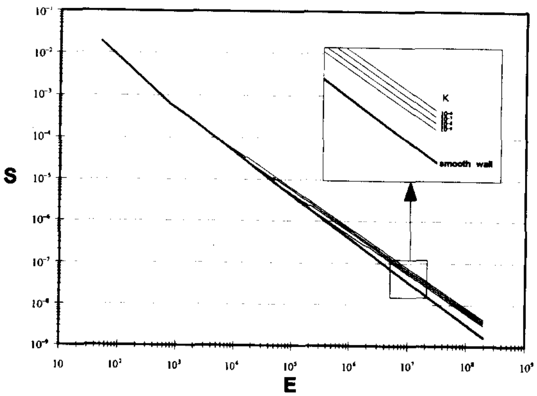

Fig. 5. S against T with Various Values of K

with $\varepsilon=0.6K^{0.073}$ and $\zeta=-0.909$ to $R_{ST}=80d_k^{1.1}$. Then T_{ST} is estimated to be $1.3K^{-0.5}$. All values of T_{LT}, T_{TS}, T_{ST} and T_{TR} can be pre-determined by the same way, and the boundary values of T are also presented in Table 6.

As discussed above, for the general case of pumped pipe on a sloping bed the reversed Reynolds number S is wholely dependent on the power-discharge-slope number T. Contrary to the design type B, Type C does not require an iteration process for its solution. Type C-1 can be considered by assuming $i=0$ with the same equation (28), and Type C-2 by $P=0$ also with the same equation. Fig. 5 shows the distribution of S against T with various values of K. With the increase of T the rate of increase of S is found to decrease, and S increases with K.

7. Conclusions

Three types of pipe design have been considered for the direct estimation of quantities concerned. Type A is to determine pump power, Type B for discharge, and Type C for pipe diameter. Pump power can be determined by ordinary approach if the friction factor were expressed by any type of explicit form, but Types B and C needs special treatment for direct computation. It was found that the power law of friction factor equation allows us to develop the explicit representation of equations for the direct estimation of the quantities designer wants to determine. By re-structuring the relevant equations with the power law of friction factor equation, several non-dimensional numbers were introduced such as power-diameter number B, slope Froude number F, Reynolds-Froude number N, reversed Reynolds number S, power-discharge number E, discharge-slope number G, power-discharge-slope number T, and reversed roughness Reynolds number K.

For the general case of Type B to compute the discharge through a pumped pipe on a sloping bed, the relevant quantity cannot be determined by direct estimation, but the explicit formulation is possible for each special case; Type B-1 for a pumped pipe on a horizontal bed and Type B-2 for a sloping pipe without pumping power. For the pipe design of Type B-1, R is found to be fully dependent on B with relative roughness d_k, and for the pipe design of Type B-2, F or R is found to be fully dependent on N with relative roughness d_k. For the general case of Type B the enhancement ratio of R against R_o of Type B-1 was determined by numerical computation, and its explicit expression was developed by trial and error after checking the distribution of the enhancement ratio in various conditions. On the other hand the pipe diameter of Type C was found to be fully detemined by direct estimation. For the pipe design of Type C, S is found to be fully dependent on T with K. The special case of Type C-1 or C-2 can be designed by using the same equation for the general case of Type C.

The results of the present study are based on the laboratory results of Nikuradse experiments for the friction factor of circular pipe covered with uniform roughness. If the inner wall of pipes were not in the same condition as that of Nikuradse pipe, different approaches might have to be introduced. Particularly the friction factor of commercial pipes has different characteristics in its distribution, and it may require a new method. Several equations have been suggested for the estimation of friction factor of commercial pipe. The equations of Hazen–Williams and Weston are purely based on laboratory results, and the equation of Colebrook–White is obtained from the ill-logical combination of smooth–turbulent friction factor equation and rough–turbulent friction factor equation, both of which are developed by Prandtl and Nikuradse. Colebrook (1938) estimated the roughness heights of various commercial pipes suitable for the use of Colebrook–White equation by testing the equation against a considerable number of data collected from several sources. Yoo (1995) has, however, found that Colebrook–White equation is logically poor, and suggested that the friction factor can be properly estimated by considering the probability of rough wall which does not cover 100 % of a commercial pipe. The method of explicit design for a commercial pipe might also be developed by using the results of the present study.

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