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지리정보시스템을 이용한 유역에서의 지형지수 산정
Calculation of Watershed Topographic Index with Geographic Information System

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Abstract

The multiple flow direction algorithm to calculate the spatial variation of the saturation tendency, i.e. topographic index, is integrated into the Geographic Information System, GRASS. A procedure is suggested to consider the effect of a tile system on calculating the topographic index. A small agricultural subwatershed (3.4 km^2) is used for this study. The impact of a tile system on the groundwater table can be effectively considered by the Laplace's equation to the DEM. The analysis shows that a tile system has a high degree of saturation compared to the case without tile drainage, and the predicted riparian area is well fitted to the actual watershed condition. A procedure is suggested to consider the effect of tile system on calculating the topographic index.

요 지

포화도의 공간적 변화인 지형지수를 계산하기 위한 다중흐름 알고리듬이 지형정보시스템인 GRASS와 연계되어 개발되었다. 본 연구는 3.4 km^{2}의 소농경지 배수구역에 대하여 적용하였다. 지하수위에 대한 타일 시스템의 영향이 DEM 모형과 Laplace식에 의해서 효과적으로 고려 될 수 있었다. 본 연구결과, 타일 시스템을 고려한 유역은 타일 시스템을 고려하지 않은 경우와 비교하여 높은 포화도를 가지고 있는 것으로 나타났으며, 예측된 riparian 유역은 실제의 유역조건과 잘 일치되고 있었다. 본 연구에서는 지형지수를 산정함에 있어 타일 시스템의 효과를 고려해야 함을 제안하였다.

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1. Introduction

Runoff can be described as the result of the various patterns of water movement in a basin after a rainfall event. These patterns of flux are strongly affected by the local saturation state, which can be estimated by application of TOPMODEL (Beven and Kirkby, 1979) to the watershed. The hydrological state of a watershed varies spatially because of the spatial variation of the rainfall event, topography and hydraulic conductivity. The existence of macropores and preferential flows along high conductivity layers tends to dominate the flow mechanism and to reduce the effect of spatial variation of rainfall and of hydraulic conductivity in a small subwatershed. Many researchers (Kirkby and Chorley, 1967; Dunne et al., 1975; Beven and Wood, 1983) have found that the topography is the main factor governing the runoff from watersheds in humid, temperate climates. Troch et al. (1993) found a large discrepancy between the hydraulic conductivities at the catchment scale and in laboratory measurements on soil samples. Hence, the hydraulic conductivities obtained from the soil classification cannot be used to improve the hydrologic simulation of the model. Therefore, the spatial information of saturation tendency (i.e. the topographic index) can be effectively obtained by considering only the topographical features in the watershed.

Since the highly saturated portion in a watershed generally is the riparian portion at the hillslope hydrologic scale, the topographic index (wetness index) (Beven and Kirkby, 1979) is a useful tool for predicting the riparian area in upland watersheds.

Many agricultural watersheds in the U.S. Midwest are equipped with tile drains. The introduction of a tile drainage system in an upland watershed makes the runoff response more complicated. Tile drainage is used in agricultural watersheds to lower the moisture content of the upper soil layer and improve the production of crops. The tile system accelerates the soil drainage, increases the average saturation deficit and permits more penetration of water into the soil layer. Since tile systems strongly affect the elevation of the water table, the runoff generation patterns are different from those of a watershed without tile drainage.

As the preliminary step for the runoff simulation at hillslope hydrologic scale, the calculation of the topographic index in an agricultural watershed equipped with tile drains is essential. Therefore, the purposes of this paper are the following:

(1) To integrate the multiple flow direction algorithm to calculate the topographic index in the GRASS environment.

(2) To evaluate the effect of the tile system on the spatial and statistical distribution of the topographic index.

2. Geographic Information System

A new trend of hydrologic modeling has focused on the representation of spatial heterogeneity in watersheds. The appearance of the Geographic Information System (GIS) provides powerful data management facilities for handling spatial data bases. Furthermore, GIS supports various functions to maintain and analyze spatial and attribute data, integrate information, and display the results of analysis in both a tabular and map format. In this study, one of the most popular raster based GIS systems, Geographical Resources Analysis Support System (GRASS 4.0 user's reference manual, 1991), is used to manipulate the Digital

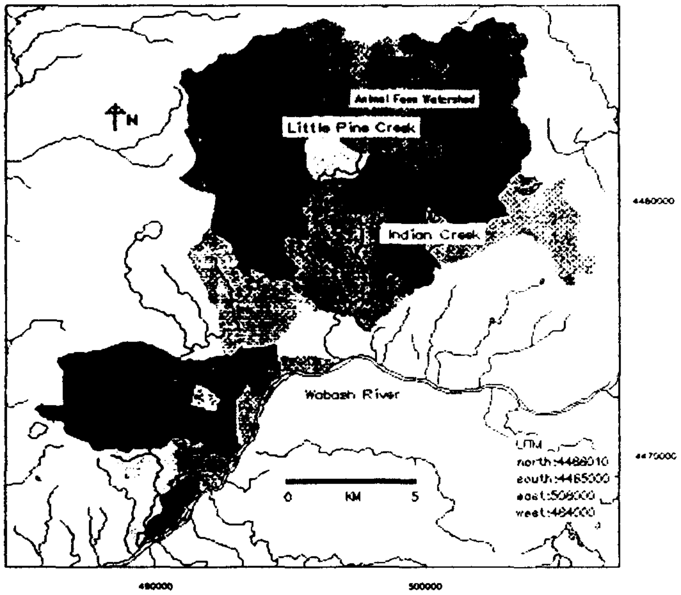

Fig. 1. Subwatersheds in Little Pine Creek and Indian Creek Watershed Forming the Indian Pine Natural Field Station

Elevation Model (DEM).

3. Study Areas

The Animal Science Farm Subwatershed (3.6 km^2) (Fig. 1) is located in a field station, called the "Indian Pine Natural Field Station" composed of two major watersheds, Little Pine Creek (139 km^2) and Indian Resources Creek (67 km^2), near the campus of Purdue University,West Lafayette, Indiana. The slope of the Animal Science Farm Watershed varies from 0 to 4 degrees. This watershed is primarily used for agricultural purposes and is strongly drained by a tile system.

4. Basics of TOPMODEL

The hydrological processes at the hillslope scale vary both spatially and dynamically due to the heterogeneous features of the topography, of the soil hydraulic conductivity, of the vegetation and the unsteadiness of the rainfall inputs. This complexity of the hydrologic system results in several runoff processes such as overland flow generated by either the infiltration excess mechanism (Horton, 1933) or the saturation excess mechanism (Dunne and Black, 1970) and the subsurface flow through the soil matrix. In an attempt to schematize the complex hydrologic processes into a functional and mathematical structure in a manageable way, the following three basic assumptions were introduced in the development of TOPMODEL (Beven et al., 1995).

(1) The temporal–spatial distribution of the saturation tendency can be approximated by a succession of steady state representations.

(2) The local hydraulic gradient can be approximated by the local surface slope, tan β

(3) The saturated hydraulic conductivity decreases exponentially with the depth from the surface.

Assumption (3) is validated by the abundant micropores near the soil surface (Beven, 1984). Based on assumptions (1) and (2), the topographic index, $\ln(a/\tan\beta)$, where $a = A_x/C_x$ (see below) is the area drained per unit length of contour line, was proposed by Kirkby (1975) and used in developing the TOPMODEL by Beven and Kirkby (1976, 1979). The area, A_x $[L^2]$, is upslope from the location that drains past x, $\tan\beta_x$ is the hydraulic gradient at x and C_x [L/T] is the contour length at x traversed by surface flow.

Using the continuity equation and assumption (3), the description of the local watertable depth, Z, from its areal average, $\bar{Z}$, is (Beven et al., 1995):

$$f(\bar{Z}-Z)=\left[\ln\left(\frac{a}{\tan\beta}\right)_x-\lambda\right]-(\ln T_0-\ln T_e) \quad (1)$$

where

$$\lambda=\frac{1}{A}\int \ln\left(\frac{a}{\tan\beta}\right)_x dA \quad (2)$$

$$\ln T_e=\frac{1}{A}\int \ln T_0\, dA \quad (3)$$

$$\bar{Z}=\frac{1}{A}\int Z\, dA=-\frac{1}{f}\ln R-\frac{1}{f}\Lambda \quad (4)$$

$$\Lambda=\frac{1}{A}\int \ln\left(\frac{a}{T_0\tan\beta}\right)_x dA \quad (5)$$

T_0 is the lateral transmissivity and R[L/T] is the recharge rate to the water table.

Eq. (1) is seen to express the deviation of the depth of the water table from its areal average value, scaled by the parameter f, in terms of the deviation of the logarithm of the transmissivity from its areal average value and the deviation in the local topographic index from its areal average value.

Due to the dominant role of topography and the uncertainty of the hydraulic conductivity at the catchment scale, Eq. (1) can be approximated as:

$$Z=\bar{Z}+\frac{1}{f}\left(\lambda-\ln\left(\frac{a}{\tan\beta}\right)_x\right) \quad (6)$$

Eq. (6) indicates that local hydraulic behavior can be expressed in terms of the topographic index. In other words, locations having the same topographic index show very similar hydrologic response patterns. Therefore, the spatial and statistical distribution of this parameter is very important for understanding the hydrologic processes at the hillslope scale.

5. Calculation of Topographic Index

In order to compute the spatial and statistical distribution of the topographic index, either single flow direction or multiple flow direction algorithms can be used. A single flow direction algorithm, which is based on the methods suggested by Jenson and Dominque (1988), assumes that subsurface flow occurs only in the steepest downslope direction from any given point. The multiple flow direction algorithm (Quinn et al., 1991) assumes that subsurface flow occurs in all downslope directions from any given point and allows for flow convergence (several cells draining into one downslope neighboring cell) and flow divergence (one cell draining into multiple downslope neighboring cells). The single flow direction algorithm, in contrast, allows only flow convergence.

Moore (1995) maintains that multiple flow direction algorithms produce better results than single flow direction algorithms in the computation of a topographic index. Research performed by Wolock and McCabe (1995) suggests that although the spatial and statistical distribution of ln(a/tan β) values are affected by the computational algorithm, the aggregated effects on TOPMODEL efficiency and simulated flow paths are minimal. However, if TOPMODEL is being used to simulate spatial patterns of hydrologic characteristics, a multiple flow direction algorithm may be preferred to a single flow direction algorithm. Therefore, the multiple flow direction algorithm was used in the following analysis.

The total area draining into each grid cell (A) as well as the contour length (C) and slope (tan β) along which this area drains out of the cell are calculated in the GRASS environment. To perform the calculation for a given cell, the elevation of a cell was compared to that of its four diagonal and four cardinal neighboring points. The area that drained into the cell(A) was then partitioned into all its downslope neighbors in quantities proportional to tan β and C and added to the previous values of A for these downhill points (Quinn, 1991). The values of ln(A/tan β) are directly related to the topographic likelihood of developing saturated surface runoff producing areas.

This method is coded in C language employing library functions supported by GRASS. The synopsis of this tool on a GRASS platform is:

```
r.atb   elv=elevation_map   atb=atb_map
```

where elevation map_refers to the DEM and atb map_refers to the resulting topographic index map layer.

The topographic index in the Animal Science Farm Watershed was calculated using the digital elevation data in GRASS. Due to the flat topography of this watershed and the type of elevation data (integer) in GRASS, the calculation was not successful in representing the expected riparian area. As a methodology to get better Digital Elevation Model, contour lines of 10m resolution were amplified about 100 times and converted to an elevation map. With this map layer the topographic index was recalculated. Although some improvement (i.e., absence of an empty region) can be obtained, it is still insufficient for predicting the riparian area. The fre-

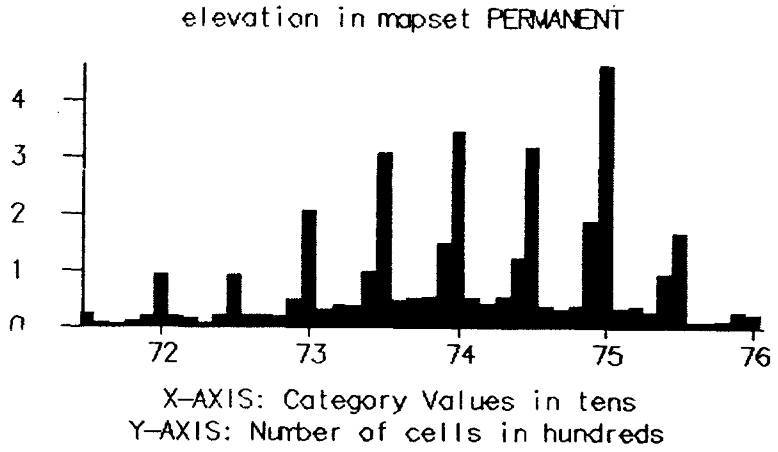

Fig. 2. The Frequency Histogram (F.H.) of the DEM in Animal Science Farm

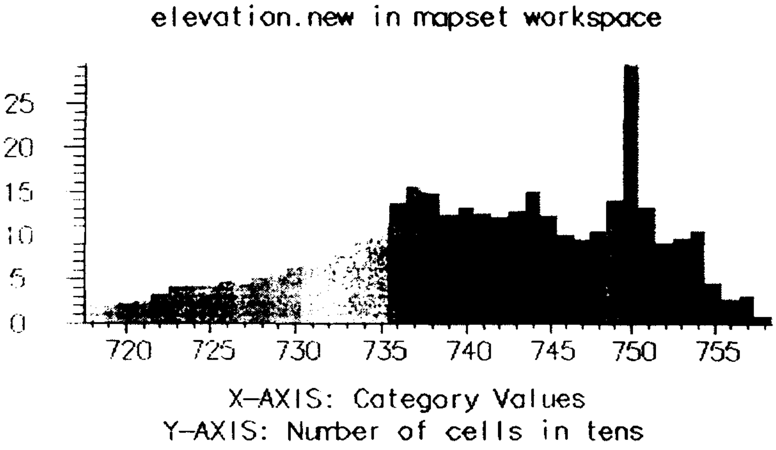

Fig. 3. The F.H. of the DEM after the Application of Laplace's Equation

Fig. 4. The Topographic Index Map in the Animal Science Farm Subwatershed without Tile

quency histogram of DEM (Fig. 2) shows that a few categories are dominant in the statistical distribution of DEM. This can be connected to the existence of the flat regions in DEM, which is extremely detrimental in the calculation of the topographic index.

In order to provide spatially continuous features in DEM, this map layer is exported to an ASCII formatted file and a few successive applications of Laplace's equation to elevation data are performed. In ideal conditions (spatially homogeneous hydraulic conductivity and uniform depth to the bedrock), groundwater flow is governed by Laplace's equation:

$$\frac{\partial^2 h}{\partial x^2}+\frac{\partial^2 h}{\partial y^2}=0 \qquad (7)$$

where *h* is the watertable elevation and *x* and *y* are the orthogonal directions in a horizontal plane. Based on the assumption of the topographic index that the groundwater table is the reflection of topography, Laplace's equation can be used to determine a new digital elevation model.

The frequency analysis of the new DEM (Fig. 3) shows a statistically more continuous distribution which is more realistic in describing of the natural topography.

The new topographic index map (Fig. 4), which was calculated based on the above elevation map layer, showed a good correspondence with the expected portion of the riparian area (grass waterway). This map layer shows the degree of saturation in the watershed and its potential usage for management purposes in the watershed.

This analysis shows that the DEM in GRASS is not appropriate for representing the spatial patterns of a hillslope hydrological process in this relatively flat region. This problem could be solved by manipulation of the elevation map layer giving the feature of continuity by means of a few consecutive applications of Laplace's Equation.

6. Topographic Index in Tile Drained Watershed

The spatial distribution of the water table in a tiled watershed usually is different from that in an untiled watershed. One reason is that the tile system lowers the water table not only locally but also globally. Another reason is that the groundwater tends to accumulate along the tile drain and does not disperse away from it. Therefore, it is necessary to develop a new method to take into account this particular situation which is common in agricultural watersheds in the Mid-West U.S.

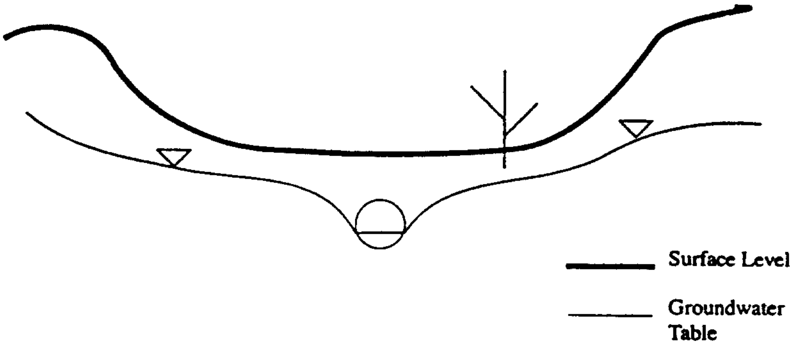

Fig. 5. The Local Water Table hear the Tile System

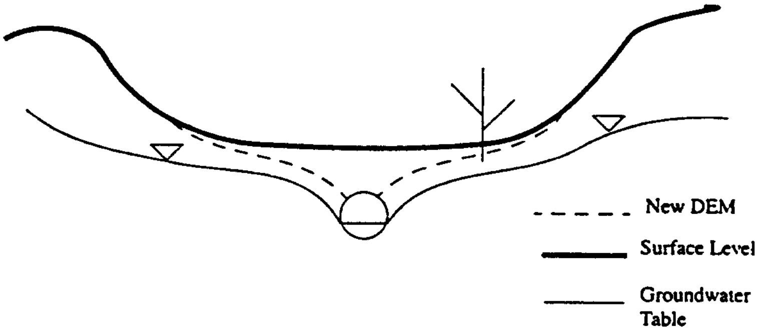

Fig. 6. The New Expected Surface Elevation for the Tile System

This tile effect must be accounted for in calculating the topographic index, and the local water table near the tile system (Fig. 5) should be considered.

Because the original topography no longer plays a determinant role in the water table distribution, it is necessary to modify the original elevation to reflect the effect of tile on the groundwater level. For the purpose of this analysis the tile drain system is considered as a small ditch in the artificial DEM. The dotted line in Fig. 6 shows the new expected surface elevation for the tile system. This artificial DEM surface lowers the water table to the flow level within the tile. From a hydraulic point of view, the water level in the tile acts as a fixed boundary head.

In ideal conditions (spatially homogeneous hydraulic conductivity and uniform depth to the bedrock), groundwater flow is governed by Laplace's equation (Eq. 7).

Based on the assumption of the topographic index that the groundwater table is a reflection of topography, Laplace's equation can be used to determine a new digital elevation model with the boundary condition at the tile drain location.

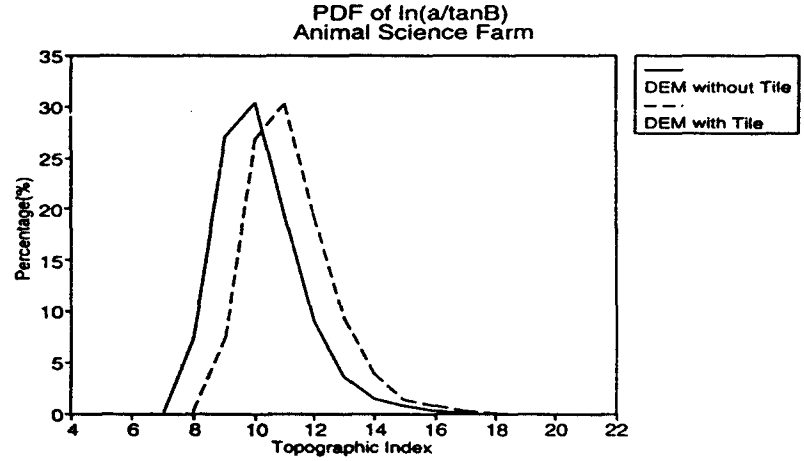

Fig. 7. The Probability Distribution Function of the Topographic Index with and without Tile

Fig. 8. The Topographic Index Map in the Animal Science Farm Subwatershed with Tile

The surface elevations of the artificial DEM are constrained to pass through the top of the tile. These positions act as a fixed boundary condition for the Laplace Equation. The finite difference approximation for Eq. (7) can be simplified to

$$H_{i-1,j}+H_{i+1}+H_{i,j}+H_{i,j+1}-4H_{i,j}=0 \qquad (8)$$

The number of iterations necessary to build the artificial DEM can be related to the capacity of the tile. As the size of the tile gets bigger, the influence of the tile on the groundwater table is expected to expand and number of iterations to represent the tile's influence on the water table is increased. The calculation is propagated from the positions of the tile drainage to the extent of the tile influence. In this study, the extent of the tile influence was assumed to be 100m from the typical modelling scale (Dooge, 1982, 1986) for the hillslope. Hence, the number of iterations to build the artificial DEM with 10m grid size can be determined as 10.

The topographic index of the artificial DEM considering the effects of the tile on the water table was calculated at the Animal Science Farm subwatershed. Fig. 7 compares the probability density functions of the topographic index using the natural topography and the modified topography considering the tile system. This figure shows the PDF of DEM with tile is shifted to the right-hand side from the case without tiles. This means a subwatershed with a tile drainage system provides faster response about a given rainfall event than the case without a tile system. However, the impact of the tile system on the hillsope hydrology can be expressed not only statistically but also spatially. Fig. 8 shows the spatial distributions of the topographic index at the Animal Science Farm subwatershed with a tile system. Compared to the case without a tile system, the topographic index map layer with the tile system shows a higher degree of grass waterway development. This implies that the tile system in this case study provides the feature of continuity in the region of high saturation tendency. However, the impact of a tile system on the hydrologic process in DEM is properly considered in the simulation of hillslope hydrology using TOPMODEL.

7. Conclusion

The topographic index map layer is calculated to predict the riparian area at an agricultural watershed equipped with tile drainage. The incorporation of GIS spatial data manipulation capability with the multiple flow direction algorithm makes this task successful. The impact of a tile system on the spatial variation of the ground water table can be efficiently considered by the application of Laplace's equation to the DEM with fixed boundary conditions at the tile location. The analysis also shows that the watershed with a tile system shows a higher degree of saturation than the case without tile drainage. The predicted riparian area is well fitted to the actual watershed condition.

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