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A Transportation and Production Model with Depot System

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Abstract

In automobile industry, the depot (distribution center) system is utilized to adjust the inventory and to supply the demands to the firm timely. In case of the small lot demands of short delivery cycle and the long distance from the parts manufacturer to the firm, the depot system is very important.

In this paper, a model to minimize the sum of costs of holding, inventory and transportation, is proposed to determine the optimal quantities of production and transportation in JIT system. Finally, computational results that verify the effectiveness of the proposed model are demonstrated.

I. Introduction

In recent years, the products have become more various with the incorporation of new technologies and customer's needs. But, these trends make to increase the costs of production and transportation. Therefore, a great deal of attention has been focused on logistics by managers and researchers because of the total material flow.

Logistics management considers the process of planning, implementing and controlling the cost effective flow and storage of raw materials, in process inventory, finished goods, and related information from the point of origin to the point of consumption for the purpose of

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confirming customer requirements. The manufacturer's objective is to obtain optimum performance of these functions at minimum total cost [1].

In automobile industry, Just In Time (JIT) system has received a great deal of attention on manufacturing and logistics operations. The primary goals of JIT system are to minimize inventory, to improve production quality, to maximize production efficiency, and to provide optimal customer service level. It is the system of supplying parts and materials just at the very moment they are needed in the factory production process to those parts and materials are instantly put to use [2-5].

Although JIT systems are useful in automobile industry, there are some problems associated with utilizing JIT system [6].

(1) The first problem related to JIT is the supplier production schedules. The success of JIT system depends on supplier's ability to provide parts in accordance with the firm's production schedule. Higher production setup costs due to the large number of small lot quantities produced are incurred to suppliers.

(2) Supplier locations can be a second problem. As distance between the firm and its suppliers increases, delivery times may become more erratic and less predictable. Then, shipping cost is also increased and transit time may cause inventory stockouts that disrupt production scheduling. When this factor is combined with higher delivery costs, total cost will be greater than savings in terms of inventory costs.

Transportation becomes an even more vital component of logistics under JIT system. To resolve this problem, the depot system is utilized in automobile industry. Actually, in the case of the small lot demands of short delivery cycle and the long distance from the parts manufacturer to the parent firm, depot system serves an important role in a firm's logistics system. Depot has three basic functions : transportation, storage, and information transfer. Generally, the depot in automobile industry will perform these activities [6] :

(1) Transportation and receiving from supplier

(2) Storage

(3) Order selection (picking)

(4) Supply the parts to the firm timely

In this paper, we propose a mathematical production and transportation model by considering depot on JIT system. To date, there have been several mathematical production and transportation studies on JIT system. There are several mathematical models under JIT system in the literature.

With mathematical approach to the JIT production system, Bitran and Chang [7] first proposed a model to determine the number of Kanbans. In this model, they employed a nonlinear programming model. Next, they transformed the resulting model to a mixed integer programming model and then to a linear programming model.

Bard and Golany [8] proposed another mathematical programming model to determine the number of Kanban considering the concept of setup and lead time for the multi-product production system. The cost factor is included in the objective function.

Hong and Fukukawa [9] described the model to determine the number of Kanbans in JIT production system with the mixed integer goal programming. They considered following factors : stock on hand process, inventory and labor costs, vendors supplying capacity, and work load. They verified the effectiveness of mathematical model in JIT production system.

Watanabe, Ahn and Hiraki [10] proposed a production and transportation model for lot production process. They presented an approximation procedure to obtain a suboptimal solution in short time by using mathematical programming package.

But, the developed mathematical models for pull type production system only consider matters inside the firm. Therefore, the mathematical model to minimize the holding cost of the depot system, the inventory cost to parts-manufacturers and transportation between parts-manufacturers and depot, is provided in this paper. The characteristics presented in this paper are as follows.

(1) Formulation of a new mathematical production and transportation model.

(2) Features with transportation lead time, the limit of minimum and maximum transportation quality, and space limit in depot.

(3) Computational results that verify the effectiveness of the proposed model are shown.

Ⅱ. Problem statement

In this paper, we consider the parts-manufacturers and the depot. Each parts-manufacturer has a multi-stage production process and multiple products are produced in JIT system. The objective of the model presented is to determine how much is produced by the parts-manufacturer and when it is transported to the depot.

A schematic diagram of depot system in automobile industry is shown in Figure 1. We assume that transportation lead time is $L1$ and demands are supplied to firm from the depot timely.

The model consists of K parts-manufacturers. Let $k \in \{1, 2, \cdots, K\}$ be an index of the parts-manufacturers. Each manufacturer has N stages with the condition that $n=1$ stands for the final stage. Each stage $n \in \{1, 2, \cdots, N\}$ means a production process and an immediately succeeding inventory point is included in each stage. Let $t \in \{0, 1, \cdots, T\}$ be an index of the time periods with the condition that the planning horizon starts at the beginning of period 1 and finishes at the end of period T.

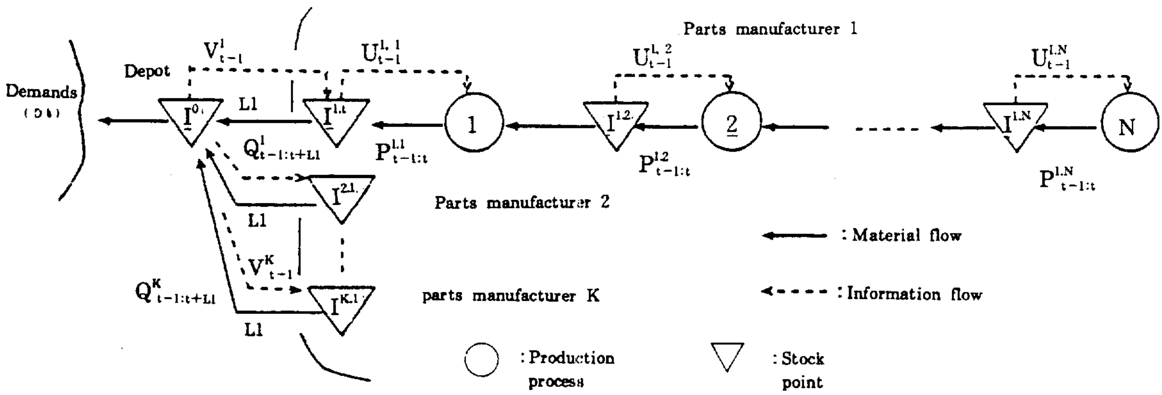

Figure 1. A schematic diagram of depot system

The assumptions used in this paper are as follows.

(1) Demand for parts in each period is deterministic process and suggested by customers (firm).

(2) Parts-manufacturers produce M types of items and let $i \in \{1, 2, \cdots, M\}$ be an index of items.

(3) The quantity of production and transportation ordered for each item in each stage is calculated at the end of each period.

(4) The processing time for each item at each stage is known and fixed in the planning horizon.

(5) Item i at each stage is exactly required to make item i at the immediately succeeding stage.

(6) The transportation (between parts manufacturer and depot) lead time is $L1$.

(7) There is the limit of freight rate to transport.

(8) There is a space limit at depot for each parts manufacturer.

(9) Each production process has the limit of minimum production quantity at each period.

2. 1. Notation

$L1$: transportation lead time between depot and parts-manufacturers

$J_t^{k,n}$: operation time at production process of stage n in period t on parts-manufacturer k

$D_t^{k,(n)}$: demand for the final product i in period t on parts-manufacturer k

$PI_t^{k,n(i)}$: minimum production quantity at production process of stage $n$in period t on parts-manufacturer k

$a^{k,n(i)}$: processing time required to make item i at production process of stage n on parts-manufacturer k

$I_t^{k,0(i)}$: initial inventory quantity ($t=0$) and inventory quantity of item i in depot on parts-manufacturer k

$I_t^{k,n(i)}$: initial inventory quantity ($t=0$) and inventory quantity of item i in the inventory point n at the end of period t on parts-manufacturer k

$S_t^{k,0(i)}$, $S_t^{k,n(i)}$: inventory level at the end of period t of item i in depot and parts-manufacturer k

$C_t^{k,0(i)}$, $C_t^{k,n(i)}$: holding cost of item i in depot for parts-manufacturer k, inventory cost of item i in the inventory point n on parts-manufacturer k.

CT^k : transportation cost between parts-manufacturer k and depot

TD^k : inventory space for parts-manufacturer k in depot

$AK^{k,(i)}$: volume of container i for parts-manufacturer k

TI^k, TX^k : minimum and maximum freight quantity to transport the parts from parts-manufacturer k to depot

$Q_{-j:L1+1-j}^{k,(i)}$: transportation quantity of item i ordered at behind planning horizon for parts-manufacturer $k(j=1, 2,\cdots, L1)$

$Q_{t-1:t+L1}^{k,(i)}$: actual transportation quantity ordered at the end of period $t-L1$ and transported at the period $t+1$ for parts-manufacturer k (decision variable)

$P_{t-1:t}^{k,n(i)}$: actual production quantity in period t of item i at stage n for parts-manufacturer k (decision variable)

$V_t^{k,(i)}$: transportation ordering quantity for parts-manufacturer k at the end of period t and transport at the period $t+1$

$U_t^{k,n(i)}$: production ordering quantity at the end of period t of item i at stage n for parts-manufacturer k

Y_t^k : variable to represent transportation at period t in final inventory point

2. 2. Formulation

$$V_t^{k,(i)}=V_{t-1}^{k,(i)}-Q_{t-1:t+L1}^{k,(i)}+D_t^{k,(i)}$$
$$(k=1,\ 2,\cdots,\ K\ ;\ i=1,\ 2,\cdots,\ M\ ;\ t=1,\ 2,\cdots,\ T) \quad (1)$$

$$U_t^{k,(i)}=U_{t-1}^{k,1(i)}-P_{t-1:t}^{k,1(i)}+Q_{t-1:t+L1}^{k,(i)}$$
$$(k=1,\ 2,\cdots,\ K\ ;\ i=1,\ 2,\cdots,\ M\ ;\ t=1,\ 2,\cdots,\ T) \quad (2)$$

$$U_t^{k,(i)}=U_{t-1}^{k,n(i)}-P_{t-1:t}^{k,n(i)}+P_{t-1:t}^{k,n-1(i)}$$
$$(k=1,\ 2,\cdots,\ K\ ;\ i=1,\ 2,\cdots,\ M\ ;\ t=1,\ 2,\cdots,\ T:n=2,\ 3,\cdots,\ N) \quad (3)$$

$$I_t^{k,0(i)}=I_{t-1}^{k,0(i)}-Q_{t-L1-1:t}^{k,(i)}-D_t^{k,(i)}$$
$$(k=1,\ 2,\cdots,\ K\ ;\ i=1,\ 2,\cdots,\ M\ ;\ t=1,\ 2,\cdots,\ T) \quad (4)$$

$$I_t^{k,1(i)}=I_{t-1}^{k,1(i)}-P_{t-1:t}^{k,1(i)}+Q_{t-1:t+L1}^{k,(i)}$$
$$(k=1,\ 2,\cdots,\ K\ ;\ i=1,\ 2,\cdots,\ M\ ;\ t=1,\ 2,\cdots,\ T) \quad (5)$$

$$I_t^{k,n(i)}=I_{t-1}^{k,n(i)}-P_{t-1:t}^{k,n(i)}+P_{t-1:t}^{k,n-1(i)}$$
$$(k=1,\ 2,\cdots,\ K\ ;\ i=1,\ 2,\cdots,\ M\ ;\ t=1,\ 2,\cdots,\ T\ ;\ n=2,\ 3,\cdots,\ N) \quad (6)$$

$$\sum_{i=1}^{M} a^{k,n(i)}P_{t-1:t}^{k,n(i)} \le J_t^{k,n}$$
$$(k=1,\ 2,\cdots,\ K\ ;\ t=1,\ 2,\cdots,\ T) \quad (7)$$

$$P_{t-1:t}^{k,n(i)} \ge PI_t^{k,n(i)}$$
$$(k=1,\ 2,\cdots,\ K\ ;\ i=1,\ 2,\cdots,\ M\ ;\ t=1,\ 2,\cdots,\ T\ ;\ n=2,\ 3,\cdots,\ N) \quad (8)$$

$$\begin{cases} Q_{t-1:t+L1}^{k,(i)} \le V_{t-1}^{k,n(i)} \\ P_{t-1:t}^{k,n(i)} \le U_{t-1}^{k,n(i)} \end{cases}$$
$$(k=1,\ 2,\cdots,\ K\ ;\ i=1,\ 2,\cdots,\ M\ ;\ t=1,\ 2,\cdots,\ T\ ;\ n=2,\ 3,\cdots,\ N) \quad (9)$$

$$\sum_{i=1}^{M} TA^{k,(i)}\times Q_{t-1:t+L1}^{k,(i)} \ge TI^k,\ \text{if}\ \sum_{i=1}^{M} Q_{t-1:t+L1}^{k,(i)}>0$$
$$(k=1,\cdots,\ K\ ;\ t=1,\ 2,\cdots,\ T) \quad (10)$$

$$\sum_{i=1}^{M} TA^{k,(i)}\times Q_{t-1:t+L1}^{k,(i)} \le TK^k$$
$$(k=1,\cdots,\ K\ ;\ t=1,\ 2,\cdots,\ T) \quad (11)$$

$$Y_t^k=\begin{cases} 0,\ \text{if}\ \sum_{i=1}^{M} Q_{t-1:t+L1}^{k,(i)}=0 \\ 1,\ \text{if}\ \sum_{i=1}^{M} Q_{t-1:t+L1}^{k,(i)}>0 \end{cases}$$
$$(k=1,\cdots,\ K\ ;\ t=1,\ 2,\cdots,\ T) \quad (12)$$

$$\sum_{t=1}^{M} TA^{k,(i)} \times I_t^{k,0n(i)} \leq TD^k$$
$$(k=1, 2, \cdots, K; t=1, 2, \cdots, T) \quad (13)$$

$$\sum_{t=1}^{T} Q_{t-1:t+L1}^{k,(i)} \geq \sum_{t=1}^{T} D_t^{k,(i)} - I_0^{k,n(i)} + S_t^{k,n(i)}$$
$$(k=1, 2, \cdots, K; t=1, 2, \cdots, M) \quad (14)$$

$$I_t^{k,0(i)} \geq S_t^{k,0(i)}, \; I_t^{k,n(i)} \geq S_t^{k,n(i)}$$
$$(k=1, 2, \cdots, K; i=1, 2, \cdots, M; t=1, 2, \cdots, N) \quad (15)$$

$$P_{t-1:t}^{k,n(i)}, \; Q_{t-1:t+L1}^{k,(i)}, \; V_0^{k,(i)}, V_0^{k,n(i)} : \text{nonnegative integer}$$
$$(k=1, 2, \cdots, K; i=1, 2, \cdots, M; t=1, 2, \cdots, N) \quad (16)$$

The constraints (1)-(3) describe the conservation of ordering flow for production and transportation. The constraints (4)-(6) show the conservation of material flow at each inventory point and depot. Constraint (7) indicates that production is restricted by the production capacity. Constraint (8) represents that production quantities are more than predetermined quantities. Constraint (9) indicates that actual quantities of production and transportation are restricted by the ordering quantity. Constraints (10)-(12) stand for transportation between parts-manufacturers and depot. Constraint (13) means that there is a space limit in depot for each parts-manufacturer. Constraint (14) indicates that transportation quantities in this planning horizon are more than total demands. Constraint (15) represents that inventory quantity at the end of each period must keep at least predetermined inventory level. The nonnegative integrality of actual quantity and initial ordering quantity for production and transportation is enforced by constraint (16).

We propose the following optimization model in logistics system. The objective function in Equation (17) means the minimization of the sum of holding cost in depot, inventory cost in parts manufacturer and transportation cost between depot and parts-manufacturer. We put equal weight to holding cost, inventory cost and transportation in this paper. Of course, unequal weights will result in different solutions. Notations for various costs are denoted as follows :

F_1^k : the holding cost in depot for parts-manufacturer k
F_2^k : the inventory cost at each stage in parts-manufacturer k
F_3^k : the transportation cost between depot and parts-manufacturer k

$$F_1^k = \sum_{i=1}^{M} \sum_{t=1}^{T} C^{k,0(i)} \times I_t^{k,0(i)} \qquad (k=1, 2, \cdots, K)$$

$$F_2^k=\sum_{i=1}^{M}\sum_{t=1}^{T}\sum_{n=1}^{N}C^{k,n(i)}\times I_t^{k,n(i)} \qquad (k=1,\ 2,\cdots,\ K)$$

$$F_3^k=\sum_{t=1}^{T}CT^kY_t^k \qquad (k=1,\ 2,\cdots,\ K)$$

Minimize

$$F=\sum_{k=1}^{K}(F_1^k+F_2^k+F_3^k) \qquad (17)$$

S.T.(1) – (16)

Ⅲ. Model application

In order to demonstrate the effectiveness of the model developed, we apply the model to an automobile parts-manufacturer and make a numerical experiment by using the mathematical programming package. We assume that there are two parts-manufacturers and the planning horizon starts at the beginning of period 1 and finishes at the end of period T=5. Following data are used in this paper.

1. The parts-manufacturer 1
   (a) transportation lead time : 1 period
   (b) process : 2 stages
   (c) products : 2 items

2. The parts-manufacturer 2
   (a) transportation lead time : 1 period
   (b) process : 2 stages
   (c) products : 1 items

3. 1. Input data

(1) Demand (container)

$t=$	1	2	3	4	5
$D_t^{1,(1)}$	100	110	100	110	120
$D_t^{1,(2)}$	110	120	110	120	130
$D_t^{1,(3)}$	120	130	120	130	140

(2) Operation time at production processes (min.)

$J_t^{1,n}=270,\ J_t^{2,n}=150 \quad (n=1,\ 2\ ;t=1,\ 2,\cdots,\ 5)$

(3) Processing time required to make one container of item i at production processes (min.)

$a^{1n(i)}=1,\ a^{2n(i)}=1 \quad (n=1,\ 2\ ;i=1,\ 2)$

(4) Minimum production level in processes (container)

$PI_t^{1,n(i)}=90,\ PI_t^{2,n(1)}=100 \quad (n=1,\ 2\ ;i=1,\ 2\ ;t=1,\ 2,\cdots,\ 5)$

(5) Initial inventory quantity in the depot and the parts-manufacturer

These data are shown in Table 4 through Table 7 at the end of period 0.

(6) Safety inventory levels and transportation quantities in process

$S_t^{1,0(i)}=20,\ S_t^{1,n(i)}=15,\ S_t^{2,0(1)}=20$

$S_t^{2,n(1)}=15,\ Q_{-1:1}^{1,(1)}=170,\ Q_{-1:1}^{1,(2)}=180$

$Q_{-1:1}^{2,(1)}=200 \quad (n=1,\ 2\ ;i=1,\ 2\ ;t=1,\ 2,\cdots,\ 5)$

(7) Minimum and maximum shipping quantity to transport the parts from the parts-manufacturers to the depot (m^3)

$TI^1=350,\ TX^1=450,\ TI^2=180,\ TX^2=220$

(8) Volume for container (m^3)

$TA^{1,(i)}=1\ (i=1,\ 2),\ TA^{2,(1)}=1$

(9) Holding space for each manufacturer in depot (m^3)

$TD^1=300,\ TD^2=200$

(10) Holding cost in the depot, inventory and transportation costs at the parts-manufacturers ($)

$C^{1,0(i)}=10,\ C^{1,n(i)}=5 \quad (n=1,\ 2\ ;i=1,\ 2)$

$C^{2,0(i)}=12,\ C^{2,n(i)}=8 \quad (n=1,\ 2)$

$CT^1=500,\ CT^2=400$

3. 2. Computational results

By computing input data, we obtained the computational results shown in Table 1 through Table 7.

Table 1. Quantities of transportation order and actual transportation for parts-manufacturer 1

Order	$i=1$	$i=2$	Actual	$i=1$	$i=2$
$V_0^{1,(i)}$	162	188	$Q_{0:2}^{1,(i)}$	162	188
$V_1^{1,(i)}$	100	110	$Q_{1:3}^{1,(i)}$	0	0
$V_2^{1,(i)}$	210	230	$Q_{2:4}^{1,(i)}$	208	222
$V_3^{1,(i)}$	102	118	$Q_{3:5}^{1,(i)}$	0	0
$V_4^{1,(i)}$	212	238	$Q_{4:6}^{1,(i)}$	180	180

Table 1 shows that transportations between depot and parts-manufacturer are incurred in period 1, 3 and 5. Actually, transportations on planning horizon are 5 times as many as the input data. Using depot system, the frequency of transportation as well as the transportation cost are decreased. Also, the condition to fullfill the reduced cost of transportation is satisfied.

Table 2. Quantities of production order and actual production (process 1) for parts-manufacturer 1

Order	$i=1$	$i=2$	Actual	$i=1$	$i=2$
$U_0^{1,1(i)}$	163	167	$P_{0:1}^{1,1(i)}$	117	133
$U_1^{1,1(i)}$	208	222	$P_{1:2}^{1,1(i)}$	90	90
$U_2^{1,1(i)}$	118	132	$P_{2:3}^{1,1(i)}$	118	132
$U_3^{1,1(i)}$	208	222	$P_{3:4}^{1,1(i)}$	90	90
$U_4^{1,1(i)}$	118	132	$P_{4:5}^{1,1(i)}$	90	90

Tables 2 and 3 show the quantities of production order and actual production at the stages 1 and 2, respectively. In terms of varying quantities to be transported, Tables 2 and 3 describe the constant production level at stages 1 and 2 for parts manufacturer 1 without violating the minimum production level.

Table 4 presents the behaviour of inventory quantities of the depot, inventory point 1 and 2 for the parts-manufacturer 1. The inventory quantities at depot in period 2 and 4 are high level because the parts will be transported to depot in period 2 and 4. The inventory quantities at final inventory point in period 2 and 4 are high level because there are transportations in period 3 and 5. All inventory quantities for the first parts-manufacturer are satisfied to the condition of minimum inventory level.

Table 3. Quantities of production order and actual production (process 2) for parts-manufacturer 1

Order	$i=1$	$i=2$	Actual	$i=1$	$i=2$
$U_0^{1,2(i)}$	93	98	$P_{0:1}^{1,2(i)}$	92	98
$U_1^{1,2(i)}$	118	133	$P_{1:2}^{1,2(i)}$	90	90
$U_2^{1,2(i)}$	118	133	$P_{2:3}^{1,2(i)}$	118	132
$U_3^{1,2(i)}$	118	133	$P_{3:4}^{1,2(i)}$	90	90
$U_4^{1,2(i)}$	118	133	$P_{4:5}^{1,2(i)}$	90	90

Table 4. Behaviour of inventory quantities at the depot, the inventory point 1 and 2 for parts-manufacturer 1

	$t=0$	$t=1$	$t=2$	$t=3$	$t=4$	$t=5$
$I_t^{1,0(1)}$	20	90	142	42	140	20
$I_t^{1,0(2)}$	20	90	158	48	150	20
$I_t^{1,1(1)}$	60	15	105	15	105	15
$I_t^{1,1(2)}$	70	15	105	15	105	15
$I_t^{1,2(1)}$	40	15	15	15	15	15
$I_t^{1,2(2)}$	50	15	15	15	15	15

Table 5 shows that transportations between depot and parts-manufacturer are incurred in period 1, 3, 4 and 5. The condition to fulfill the reduced cost of transportation is satisfied.

Table 5. Quantities of transportation order and actual transportation for parts-manufacturer 2

Order		Actual	
$V_0^{2,(1)}$	220	$Q_{0:2}^{2,(1)}$	180
$V_1^{2,(1)}$	160	$Q_{1:3}^{2,(1)}$	0
$V_2^{2,(1)}$	290	$Q_{2:4}^{2,(1)}$	180
$V_3^{2,(1)}$	230	$Q_{3:5}^{2,(1)}$	180
$V_4^{2,(1)}$	180	$Q_{4:6}^{2,(1)}$	180

Table 6 represents the quantities of production order and actual production at the stage 1 and 2. There are varying transportation quantities and the production level at stage 1 and 2 for parts-manufacturer 2 is kept to constant.

Table 6. Quantities of production order and actual production for parts-manufacturer 2

Order	$n=1$	$n=2$	Actual	$n=1$	$n=2$
$U_0^{2,1(1)}$	185	115	$P_{0:1}^{2,1(1)}$	135	100
$U_1^{2,1(1)}$	230	150	$P_{1:2}^{2,1(1)}$	100	100
$U_2^{2,1(1)}$	130	150	$P_{2:3}^{2,1(1)}$	130	130
$U_2^{2,1(1)}$	180	150	$P_{3:4}^{2,1(1)}$	150	150
$U_4^{2,1(1)}$	210	150	$P_{4:5}^{2,1(1)}$	150	150

Table 7. Behaviour of inventory quantities at the depot, the inventory point 1 and 2 for parts-manufacturer 2

	$t=0$	$t=1$	$t=2$	$t=3$	$t=4$	$t=5$
$I_t^{2,0(1)}$	20	100	150	30	80	120
$I_t^{2,1(1)}$	70	25	125	75	45	15
$I_t^{2,2(1)}$	50	15	15	15	15	15

Table 7 shows the behaviour of inventory quantities of the depot, inventory point 1 and 2 for the parts-manufacturer 2. The inventory quantities at depot in period 1, 2 and 4 are high level, because the parts were transported to depot in period 1, 2 and 4. The condition of minimum inventory level is satisfied to all inventory quantities for the parts-manufacturer 2.

Ⅳ. Conculsions

In this paper, why depot systems are required in automobile industry is mentioned firstly. A mathematical production and transportation model with depot system is proposed. A proposed model to minimize the sum of costs of holding, inventory and transportation, determines the optimal quantities of production and transportation in JIT system. The effectiveness of the proposed model is verified by the computational results demonstrated.

The contributions of this paper are as follows.

(1) A new model to assist managing the depot system is formulated.

(2) The proposed model considers actual environment factors : transportation lead time between parts manufacturers and depot, the limit of transportation quantity and minimum production quantity at each stage on manufacturers.

(3) A numerical experiment by using a mathematical programming package to verify the effectiveness of the proposed model is demonstrated.

By applying this model to depot system, the decision makers (parts-manufacturers and depot management) can easily determine the quantities of production and transportation in terms of the total cost in depot system.

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Appendix

(1) Constraint for transportation

We reformulate (10), (12) to (12)'

$$\begin{cases} G(Y_t^k-1)+TI^k \le \sum_{i=S}^{M} TA^{k,(i)} \times Q^{k,(i)}_{t-1:t+L1} \le GY_t^k \\ Y_t^k=0 \text{ or } 1 \\ (k=1, 2,,K ; t=1, 2,\cdots, T) \end{cases} \quad (12)'$$

(2) We utilize the following computer system:

(a) Main frame : HITAC M-680/180E

(b) Operating system : HITAC VOS3/AS

(c) MPS : HITAC MIP

The problem size and the result of computation are as follows:

(a) Constraints : 199

(b) Integer Variables : 154 (ten 0-1 variables)

A1. The information of solution

Solution	Time (second)	Iteration (# of nodes)	Objective function
L. P	0.76	295	15,324.20
Integer (1st)	1.59	426 (14)	24,040.00
Integer (2nd)	1.95	471 (21)	24,040.00
Last result	2.26	499 (21)	