Force/Position Control of Robot Manipulator via Motion Dynamics

모션 다이나믹스를 이용한 로봇 매니퓰레이터의 힘/위치 제어

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요 약 : 본 논문에서는 디스트리뷰션 인테그랄 서브매니폴드에 의하여 표현되는 마찰이 없는 면을 따라 강체 로봇 매니퓰레이터의 모션 제어에 대한 새로운 힘/위치 제어법칙을 제안한다. 제안된 제어법칙은 힘/위치가 제어되는 방향으로 투영된 앤드 이팩트의 비선형 항을 정확하게 상쇄 하도록 설계하였으며, 미분기하학을 이용하여 스무스 디스트리뷰션의 인테그랄 서브매니폴드 상에서의 새로운 모션 방정식을 제안하고 제안된 힘/위치 제어법칙에 대한 타당성을 컴퓨터 시뮬레이션을 통하여 검증한다.

Keywords : motion dynamics, frictionless, submanifold, distribution, tangential force

Ⅰ. Introduction

In this paper, we address the problem of controlling a robot manipulator which is to track a path on a frictionless surface while exerting a force (tangential and normal force) on the surface. Many manufacturing tasks such as deburring and polishing a surface require such constrained motion execution.

An interesting and informative historical perspective on some past work in the area of robotic force control was presented by Whitney[1]. Considerable number of researchers have studied the problem of constrained motion control of a rigid robot manipulator along a frictionless surface. Early researchers in this area, such as, Paul and Shimano[2] proposed decomposing the manipulator into types of joints those which contribute to the force control and those which contribute to the motion of the end effector along the surface. Mason[3] partitioned the compliant motion control problem into that of performing force control and position control in a global world coordinate frame. Raibert and Craig[4] pursued this subdivision of tasks into position and force control framework and developed a hybrid force/position control law, where each manipulator joint provides a hybrid torque which affects both the end effector force and position.

However detailed stability analysis of the hybrid control law were not performed. Anderson and Spong[5] combined the notion of impedance control with hybrid force/position control to allow for precise force servoing which might be required in many applications. Anderson and Spong's hybrid impedance control strategy allowed for implementation of higher order than second order impedance model suggested by Hogan and Kazerooni[6]. Compensation of the nonlinear manipulator dynamics were also accounted for in the hybrid impedance control scheme. The dynamic equation of the manipulator along an algebraic constraint surface were identified as a singular system of differential equations (or descriptor systems) by McClamroch. Khrisnan and McClamroch[7] also considered the design of a hybrid/force position control scheme for flexible joint robots by linearizing them about an operating point. Ham[8-10] proposed adaptive nonlinear control of one-link flexible arm and adaptive control based on explicit model of robot manipulator.

In this paper, we derive motion equation that can be used to make the trajectory of end-effector move from an initial point to the desired point on the constrained task plane(or hyperplane) by using the mathematical tools concerning vector fields of manifold. We also consider the robust motion equation that can be used even in the presence of uncertainties. We suggest a new force/motion control scheme of robot manipulator based on the proposed motion equation.

The paper is organized as follows. In section II, we describe the concepts of submanifold and determistic and robust motion equations. In section III, we formulate the dynamics of the rigid robot manipulator with an end-effector. In section IV, we design a new force/position control law. In section V, the feasibility of the proposed a new force/position control scheme is verified through a computer simulation.

Ⅱ. Mathematical Tools

In this section, we briefly discuss the concepts of submanifolds and derive motion equations based on that concepts.

Definition 1 : Let $\triangle$ be a distribution defined on the manifold N. A submanifold S of N is said to be integral submanifold of the distribution $\triangle$ if, for every $p \in S$, the tangent space T_pS at p coincides with the subspace $\triangle(p)$ of T_pN.

Definition 2 : A maximal integral submanifolds of $\triangle$ is a connected integral submanifold S of $\triangle$ with the property that every other connected integral submanifold of $\triangle$ which contains S coincides with S.

Example 1 : Consider the following distribution, defined on R^2

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$$\triangle\{(x_1, x_2)\} = span\{(-x_2, x_1)^T\}.$$

Let S be maximal integral submanifold of the above distribution $\triangle$. Then

$$S = \{(x \in R^2) \mid x_1^2 + x_2^2 = c \text{ for } some\ c\}.$$

Let $\triangle$ be a nonsingular and has dimension d, in a neighborhood U^o of x^o. Then the following lemma is satisfied.

Lemma 1 : Let $\triangle$ be a smooth distribution and x^o a regular point of $\triangle$. Suppose $\dim(\triangle(x^o)) = d$, then there exist an open neighborhood U^o of x^o and a set $\{f_1, \cdots, f_d\}$ of smooth vector fields defined on U^o with the property that

i) the vectors $f_1(x), \cdots, f_d(x)$ are linearly independent at each x in U^o.

ii) $\triangle(x) = span\{f_1(x), \cdots, f_d(x)\}$ at each x in U^o.

Remark 1 : Moreover, every smooth vector field τ belonging to $\triangle$ can be expressed, on U^o, as

$$\tau(x) = \sum_{i=1}^{d} c_i(x) f_i(x)$$

where $c_1(x), \cdots, c_d(x)$ are smooth real-valued function of x, defined on U^o.

Based on the above mathematical tool, we suggest two theorems concerning motion equation under the following assumption 1. One is for a deterministic motion equation which has no uncertainty and the other one is for a motion equation which has some uncertainties.

Let x_o, x_1 be two points on a maximal submanifold S of nonsingular d-dimensional smooth distribution $\triangle(x)$ and define a set B_δ contained in S as follows

$$B_\delta = \{x \mid \|x - x_o\| < \delta, x \in S\}.$$

Assumption 1 : The absolute angle between T_xS and T_yS is less than $\pi/2$ for any points $x, y \in B_\delta$ where $\delta = \|x_1 - x_o\|$.

Theorem 1 : (without uncertainty)

Let x_o and x_1 be the points on a same maximal integral submanifold S of the distribution $\triangle$. Then there exist some time functions $k_1(t), \cdots, k_d(t)$ such that the following motion equation move the state trajectory from x_o to x_1 without deviating from the integral submanifold S.

$$\dot{x}(t) = k_1(t) f_1(x) + \cdots + k_d(t) f_d(x) \tag{2.1}$$

where $f_1(x), \cdots, f_d(x)$ are independent and selected such that

$$\triangle(x) = span\{f_1(x), \cdots, f_d(x)\}. \tag{2.2}$$

Proof : The proof is based on the Lyapunov-like function as follow

$$V(x) = (x_1 - x)^T (x_1 - x)/2. \tag{2.3}$$

Taking the time derivative of V along the trajectory of (2.1) yields

$$\begin{aligned}\dot{V}(x) &= -(x_1 - x)^T \dot{x} \\ &= -(x_1 - x)^T (k_1(t) f_1(x) + \cdots + k_d(t) f_d(x))\end{aligned} \tag{2.4}$$

Let us define

$$k_i(t) \cong (x_1 - x)^T f_i(x) \beta_i(t)$$

where $\beta_i(t) > 0$, $i = 1, 2, \cdots, d$.

Then, we can write as follow

$$\begin{aligned}\dot{V}(x) = -[&((x_1 - x)^T f_1(x))^2 \\ &+ ((x_1 - x)^T f_2(x))^2 + \cdots \\ &+ ((x_1 - x)^T f_d(x))^2]\beta_i(t) \le 0.\end{aligned} \tag{2.5}$$

Let Ω_1 be the largest invariant set of contained in set Ω_2 where $\Omega_2 = \{x \in B_\delta : \dot{V}(x) = 0\}$, then we can find $\Omega_1 = x_1$. By using LaSalle's theorem, we can find $x(t)$ approaches x_1 as time goes to infinity. ■

Lemma 2 : If $\dot{x}(t) = f(x, t)$ is a stable, then

$$\ddot{x}(t) = F(x, \dot{x}, t),$$

where

$$\begin{aligned}F(x, \dot{x}, t) = &\frac{\partial f(x, t)}{\partial x} \dot{x}(t) \\ &+ \frac{\partial f(x, t)}{\partial t} - \gamma(\dot{x}(t) - f(x, t)),\end{aligned}$$

is also stable for any constant $\gamma > 0$.

Proof : Let $z(t) = \dot{x}(t) - f(x, t)$, then from the above equation we obtain

$$\dot{z}(t) = -\gamma z(t).$$

Because $\gamma > 0$, the above system is stable. So for any $z(0)$

$$\lim_{t\to\infty} z(t) = 0$$

and therefore we can see that

$$\lim_{t\to\infty} x(t) = 0$$ ■

Example 2 : Consider the following distribution, defined on R^3

$$\triangle(x) = span\{(0, -x_3, x_2)^T, (x_2, -x_1, 0)^T\}$$

Let S be integral submanifold of the above distribution $\triangle$. Then

$$S = \{(x_1, x_2, x_3) \mid x_1^2 + x_2^2 + x_3^2 = c \text{ for } some\ c\}$$

Consider the following points $x_o = (1, 0, 0)^T$, $x_1 = (0, 1, 0)^T$ and we choose $k_1(t), k_2(t)$ as follows

$$\begin{aligned}k_1(t) &= (x_1 - x)^T f_1(x)(1 - e^{-3t}) \\ k_2(t) &= (x_1 - x)^T f_2(x)(1 - e^{-3t})\end{aligned} \tag{2.6}$$

then, from theorem 1, we obtain the motion equation that move the state trajectory from x_o to x_1 as follows

$$\dot{x}(t) = f_1(x)(x_1 - x)^T f_1(x)(1 - e^{-3t}) + f_2(x)(x_1 - x)^T f_2(x)(1 - e^{-3t}). \quad (2.7)$$

The state trajectory of above system is shown in Fig.1. From Fig. 1, we can check the validness of theorem 1.

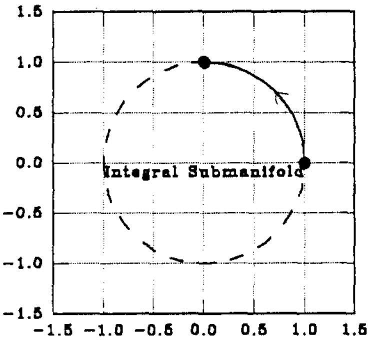

Fig. 1. The state trajectory of motion equation ($x_3 = 0$).

Theorem 3 : (with uncertainty)
Let x_o and x_1 be the points on a same maximal integral submanifold S of the distribution $\triangle$. Then there exist some time functions $k_1(t), \cdots, k_d(t)$ such that the following motion equation move the state trajectory from x_o to x_1 without deviating from the integral submanifold S.

$$\dot{x}(t) = k_1(t)f_1(x) + k_2(t)f_2(x) + \cdots + k_d(t)f_d(x) + \eta_1(t)f_1(x) + \eta_2(t)f_2(x) + \cdots + \eta_d(t)f_d(x) \quad (2.8)$$

where $|\eta_i(t)| < \delta_i, i = 1, 2, \cdots, d$ and $f_1(x), \cdots, f_d(x)$ are independent and selected such that

$$\triangle(x) = span\{f_1(x), \cdots, f_d(x)\}. \quad (2.9)$$

Proof : The proof is based on the Lyapunov-like function as follows

$$V(x) = (x_d - x)^T (x_d - x)/2. \quad (2.10)$$

Taking the time derivative of V along the trajectory of (2.8) yields

$$\dot{V}(x) = -(x_d - x)^T \dot{x} = -(x_d - x)^T \sum_{i=1}^{d} (k_i(t) + \eta_i(t)) f_i(x). \quad (2.11)$$

Let us choose

$$k_i(t) = (x_d - x)^T f_i(x) + K_i \tanh(s_i) \quad (2.12)$$

where $s_i = (x_d - x)^T f_i$ and $K_i > 0$, $i = 1, 2, \cdots, d$.
Let us define $\dot{V}_i(x)$ as follows

$$\dot{V}_i(x) = -(x_d - x)^T (k_i(t) + \eta_i(t)) f_i(x)$$

then, by substituting (2.12) into (2.11), we obtain

$$\begin{aligned}\dot{V}(x) &= \sum_{i=1}^{d} \dot{V}_i(x) \\ &= \sum_{i=1}^{d} \{ -(x_d - x)^T f_i(x)(x_d - x)^T f_i(x) - (x_d - x)^T f_i(x)\eta_i(t) - (x_d - x)^T f_i(x) K_i \tanh(s_i) \} \\ &= \sum_{i=1}^{d} (-s_i^2 - s_i \eta_i(t) - s_i K_i \tanh(s_i)) \\ &\le 0, \end{aligned} \quad (2.13)$$

■

if $|s_i| < \frac{\delta_i}{1 + K_i}$. From the above equation, the typical plot of $\dot{V}_i$ versus s_i is shown in Fig.2.

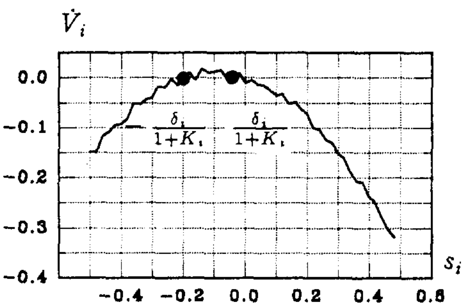

Fig. 2. The typical plot of $\dot{V}_i$ vs s_i.

As we can see in Fig.2, if the absolute value of s_i is greater than $\frac{\delta_i}{1 + K_i}$, then the value of $\dot{V}_i$ is less than zero. Therefore we can guarantee that the following inequality holds as time goes to infinity

$$|s_i| < \frac{\delta_i}{1 + K_i}$$

$$|s_i| = \| (x_d - x)^T f_i(x) \| \cong \| (x_d - x)^T f_i(x_d) \| < \frac{\delta_i}{1 + K_i}. \quad (2.14)$$

Let the angle between $x_d - x$ and $f_i(x_d)$ be θ_i. Then we can obtain the following inequalities

$$\| x_d - x \| \, \| f_i(x_d) \| \cos\theta_i < \frac{\delta_i}{1 + K_i}$$

$$\| x_d - x \| < \min_{i \in I} \frac{\delta_i}{\| f_i(x_d) \| \cos\theta_i (1 + K_i)} \quad (2.15)$$

where $I = 1, 2, \cdots, d$. Therefore if K_i is selected to be large, then $\| x_d - x \|$ becomes small.

Ⅲ. Dynamics of Two-Link Robot Manipulator

In this section, we derive the dynamics of two-link rigid robot manipulator with an end-effector. In derivation of dynamics, we consider end-effector as point-mass.
When we apply Euler-Lagrangian equation to the robot manipulator shown in Fig.3, we obtain the inertia matrix term as follows

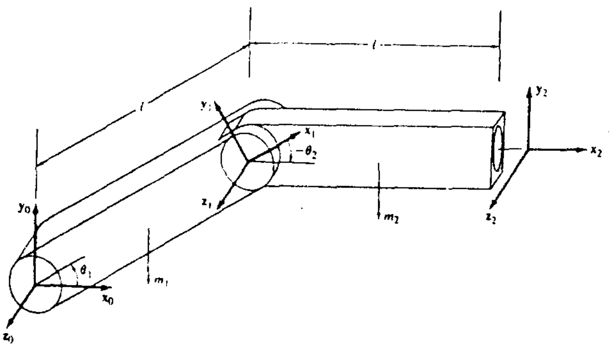

Fig. 3. The two-link robot manipulator.

$$D_{11} = l^2[\frac{1}{3} m_1 + \frac{4}{3} m_2 + 2m_l + m_2 c_2 + 2m_l c_2]$$

$$D_{12} = l^2[\frac{1}{3} m_2 + \frac{1}{2} m_2 c_2 + m_l + m_l c_2]$$

$$D_{21} = D_{12}$$

$$D_{22} = l^2[\frac{1}{3} m_2 + m_l] \quad (3.1)$$

and the Coriolis and centrifugal matrix terms as follows

$$C_{11} = -\frac{1}{2} m_2 l^2 s_2 \dot{q}_2 - m_l l^2 s_2 \dot{q}_2$$

$$C_{12} = -\frac{1}{2} m_2 l^2 s_2(\dot{q}_1 + \dot{q}_2) - m_l l^2 s_2(\dot{q}_1 + \dot{q}_2)$$

$$C_{21} = \frac{1}{2} m_2 l^2 s_2 \dot{q}_1 + m_l l^2 s_2 \dot{q}_1$$

$$C_{22} = 0 \quad (3.2)$$

and the gravity vector terms as follows

$$G_1(q) = \frac{1}{2} m_1 g l c_1 + \frac{1}{2} m_2 g l c_{12} + m_2 g l c_1 + m_l l c_1 g + m_l l c_{12} g$$

$$G_2(q) = \frac{1}{2} m_2 g l c_{12} + m_l l c_{12} g. \quad (3.3)$$

where $c_1 = \cos q_1$, $c_2 = \cos q_2$, $s_2 = \sin q_2$, $c_{12} = \cos(q_1 + q_2)$, m_l denotes point mass of an end-effector. Therefore the resulting dynamic equation of two-link robot manipulator considering the normal force of the end-effector can be expressed as

$$D(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q) + J^T(q)F_n(q) = u \quad (3.4)$$

where $F_n(q)$ is normal force(or contact force) of the end-effector on frictionless surface.

Ⅳ. Controller Design

In this section, we propose new force/position control law for the robot manipulator based on the motion equation discussed in section II. The new force/position control law guarantees that closed-loop system has the uniformly bounded stability. The mathematics that relate the world coordinate to the joint angle coordinate is inherently nonlinear and can be expressed by a nonlinear vector valued function as

$$x = H(q) \quad (4.1)$$

where $x \in R^6$ and $q \in R^n$. The velocity of the end-effector is related to the joint velocity $\dot{q}$ as follows,

$$\dot{x} = J(q)\dot{q}. \quad (4.2)$$

where $J(q)$ is the manipulator's Jacobian. The end-effector and joint accelerations are related by the following equation

$$\ddot{x} = \dot{J}(q)\dot{q} + J(q)\ddot{q} \quad (4.3)$$

Now, we design the force/position control law such that above equation is the same as motion equation suggested in Lemma 2.2, i.e.,

$$\ddot{x} = J(q)\ddot{q} + \dot{J}(q)\dot{q} = F(H(q), J(q)\dot{q}) = F(x, \dot{x}, t) \quad (4.4)$$

From (3.4), we obtain

$$\ddot{q} + D^{-1}(q)C(q, \dot{q})\dot{q} + D^{-1}(q)G(q) + D^{-1}(q)J^T(q)F_n(q) = D^{-1}(q)u. \quad (4.5)$$

If we premultiply $J(q)$, then (4.5) becomes

$$J(q)\ddot{q} + J(q)D^{-1}(q)C(q, \dot{q})\dot{q} + J(q)D^{-1}(q)G(q) + J(q)D^{-1}(q)J^T(q)F_n(q) = J(q)D^{-1}(q)u. \quad (4.6)$$

From (4.3) and (4.6), we get

$$\ddot{x} - \dot{J}(q)\dot{q} + J(q)D^{-1}(q)C(q, \dot{q})\dot{q} + J(q)D^{-1}(q)G(q) + J(q)D^{-1}(q)J^T(q)F_n(q) = J(q)D^{-1}(q)u. \quad (4.7)$$

If we apply (4.4) to above equation, we get

$$F(x, \dot{x}, t) - \dot{J}(q)\dot{q} + J(q)D^{-1}(q)C(q, \dot{q})\dot{q} + J(q)D^{-1}(q)G(q) + J(q)D^{-1}(q)J^T(q)F_n(q) = J(q)D^{-1}(q)u. \quad (4.8)$$

Let us define vector v as follows

$$D^{-1}(q)u = J^T(q)v \quad (4.9)$$

Then, we can find the vector v as follows by using pseudo inverse of $J^T(q)$

$$v = (J(q)J^T(q))^{-1}[F(x, \dot{x}, t) - \dot{J}(q)\dot{q} + J(q)D^{-1}(q)C(q, \dot{q})\dot{q} + J(q)D^{-1}(q)G(q) + J(q)D^{-1}(q)J^T(q)F_n(q)] \quad (4.10)$$

Therefore a new force/position controller of robot manipulator as follows

$$u = D(q)J^T(q)(J(q)J^T(q))^{-1}[F(x, \dot{x}, t) - \dot{J}(q)\dot{q} + J(q)D^{-1}(q)C(q, \dot{q})\dot{q} + J(q)D^{-1}(q)G(q) + J(q)D^{-1}(q)J^T(q)F_n(q)]. \quad (4.11)$$

V. Computer Simulations

Computer simulations are conducted to verify the validity, effectiveness and performance of the proposed a new force/position control scheme. We simulated the proposed a new force/position controller to control the joint angle θ_1, θ_2 of the two-link rigid robot manipulator. Table.1 show the physical parameters of two-link robot manipulator. We set the task plane of end-effector to elliptic function $(x^2+4y^2=2)$ and set the initial angles and velocities of the link 1 and link 2 to $\theta_1=\frac{\pi}{4}, \dot{\theta}_1=0, \theta_2=-\frac{\pi}{2}, \dot{\theta}_2=0$. Hence the initial location and velocity of end-effector is $(\sqrt{2}, 0)$ and 0 respectively. We also set the desired location and velocities of end-effector to $(\sqrt{\frac{2}{5}}, \sqrt{\frac{2}{5}})$ and 0. Fig.4 shows the state trajectory of motion equation. Fig.5 and Fig.6 show the angular displacements of link 1 and link 2. As we already comment in the previous section II, we can see from Fig.4 that end-effector moves along the trajectory from an initial point to the desired point on the constrained task plane which can be represented by elliptic function.

Table 1. Physical parameters of two-link robot manipulator.

parameters	values	units
m_1	2	kg
m_2	1.2	kg
m_l	0.2	kg
l	1	m
g	9.8	N/kg

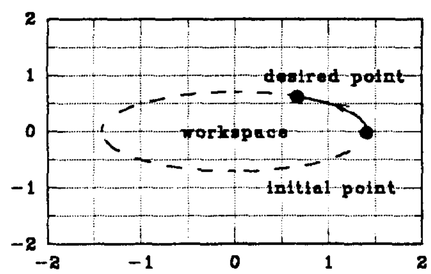

Fig. 4. The state trajectory of motion equation.

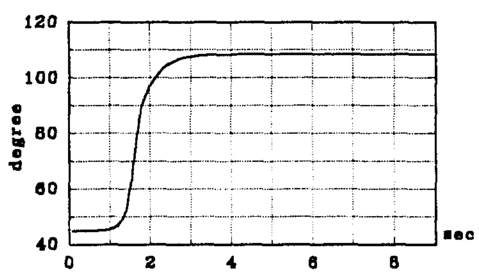

Fig. 5. The joint angle θ_1 of link one.

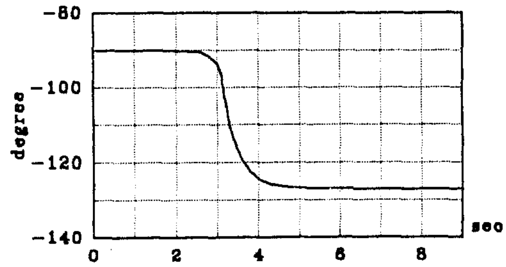

Fig. 6. The joint angle θ_2 of link two.

VI. Conclusions

In this paper, we propose a new force/position control scheme to perform force and position control of the robot manipulator when the end-effector is moving along a frictionless surface represented by algebraic equation. The presented control scheme is based on the motion equation that can be obtained by using differential geometry. The magnitude of vector fields that charaterize the motion equation is designed by using Lyapunov-like function. The uniformly ultimate boundness of the control scheme is guaranteed and has been demonstrated by a simulation. In near future, we will study and develop the adaptive version of the proposed control law which can be applied in real situation when a part of parameters of robot dynamics are unknown.

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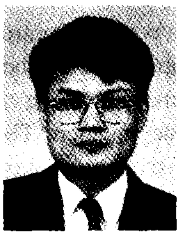

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