Design of A New Sliding Mode Controller for Uncertain Multivariable Systems Using Continuous-time Switching Dynamics

연속 시간 스위칭 다이나믹을 이용한 불확실한 다변수 계통에 대한 새로운 슬라이딩 모드 제어기 설계

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요 약 : 슬라이딩 모드 제어기의 실현시 발생하는 고주파 chattering 현상을 제거하기 위하여 본 논문에서는 스위칭 표면 행렬의 Range-space 동특성을 이용한 연속 시간 스위칭 다이나믹을 제안한다. 전체 폐루프 시스템은 정칙 변환에 의해 빠른 부시스템과 느린 부시스템으로 분해되어, 결국 전체 폐루프 시스템의 고유치는 각각 스위칭 표면 행렬의 Range-space와 Null-space의 동특성을 지배하는 부시스템들의 고유치들로 구성됨을 보인다. 그리고, 정합된 불확실성이 존재하는 경우 제안된 스위칭 다이나믹을 가진 제어 시스템의 응답은 일정 시간이 경과된 후 스위칭 평면의 임의의 영역으로 균일하게 유계된다는 것을 증명한다. 또한, 제어 입력의 크기에 대한 제약성을 만족하면서도 전체 제어 시스템의 정상 상태 오차를 감소시키기 위하여 두개의 스위칭 다이나믹을 도입한 수정된 슬라이딩 모드 제어기를 제안한다.

Keywords: sliding mode control, continuous-time switching dynamics, uniform ultimate boundedness, uncertain system, slow and fast subsystems

I. Introduction

In this paper, we develop a simple design methodology for state-feedback controller for uncertain systems using sliding mode control theory. Sliding mode control(SMC) based in variable structure system (VSS) is a kind of adaptive scheme, especially characterized as a passive adaptive control which does not employ any identification mechanism[1]. The salient feature of SMC is that the so-called sliding mode occurs on a sliding surface. While in the sliding mode, the system has invariance properties, yielding motion which is independent of certain parameters and disturbances.

However, undesirable high-frequency chattering occurs essentially due to the switching logic of SMC scheme. Some authors proposed the boundary layer in the vicinity of sliding surface to reduce the chattering[2]-[7]. But, these methods have difficulties in the parameter selection for the boundary layer.

Therefore, we develop the switching dynamics in the range-space of switching surface matrix C to eliminate the chattering completely. The dynamic behaviour of the uncertain systems with this switching dynamics will be described by the vector differential equations. It will be shown that the eigenvalues of closed-loop system are composed of those of the fast and the slow subsystems, i. e., the systems which govern the dynamics off the sliding surface and in the sliding surface. It will be also shown that every response of the uncertain system with the proposed switching dynamics in the presence of matched uncertainties is uniformly ultimately bounded within arbitrarily small neighborhoods of the switching surfaces. Finally, the modified robust controller with two switching dynamics is also proposed in order to reduce the steady-state error in view of control input constraints.

In section II, fundamental theory of SMC, sliding mode controller design using continuous-time switching dynamics and its stability analysis are discussed. In the last section, the results of this paper are summarized.

II. Sliding Mode Controller with Switching Dynamics

1. Fundamental Theory of Sliding Mode Control

Let us consider the following uncertain multivariable system:

$$\begin{aligned}\dot{x} = \ & (A+\Delta A(q(t)))x \\ & +(B+\Delta B(q(t)))u+Fv \\ & x(t_0)=x_0\end{aligned} \tag{1}$$

where $x \in R^n$ is the state, $u \in R^m$ is the control, $v \in R^p$ is the disturbance and $q(t) \in Q$ represents the parameter uncertainty of the system. It is assumed that the pair (A, B) is completely controllable, $\Delta A(\cdot)$ and $\Delta B(\cdot)$ are continuous, Q is a compact subset in R^s and $q(\cdot)$ is Lebesgue measurable

$$\dot{x} = Ax + Bu \quad x(t_0) = x_0 \tag{2}$$

The nominal system - the system without uncertainties - is described as follows.

If some conditions are satisfied (primary among which are the so-called matching conditions [8]), then all

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uncertain elements can be lumped and the system is described by

$$\dot{x} = Ax + Bu + Be \quad x(t_0) = x_0 \tag{3}$$

where e is the lumped element such that the absolute value of the i-th component e_i is bounded by a nonnegative constant f_i, $|e_i| \le f_i$.

In conventional sliding mode control theory, the j-th switching surface s_j passing through the state-space origin is defined by

$$s_j(x) = \{ x \in R^n : c^j(x) = 0 \} \quad j = 1, 2, \dots, m \tag{4}$$

where c^j is a row n-vector. The sliding mode occurs when the state lies simultaneously in each of the surfaces s_j. Assembling the rows c^j into an m×n matrix C, the sliding mode is attained when state reaches and remains in the intersection S of the m switching surfaces:

$$S = \bigcap_{j=1}^{m} s_j = \{ x : Cx = 0 \} \tag{5}$$

In geometric terms the subspace is the null space of C, N(C).

A sufficient condition for the existence of the sliding mode on the intersection S is that the following inequality is satisfied [9].

$$S^T\dot{S} < 0 \tag{6}$$

Consider the equivalent system of (2).

$$\dot{x} = [\, I_n - B(CB)^{-1}C \,]\, Ax = A_{eq}x \tag{7}$$

It is easy to see that $B(CB)^{-1}C$ is a projection operator and has rank m. Hence $I_n - B(CB)^{-1}C$ is a projection operator with rank n-m. Therefore, the matrix A_{eq} in the equivalent system can have at most n-m nonzero eigenvalues. Our goal is to choose C so that the nonzero eigenvalues of A_{eq} are prescribed negative real numbers and the corresponding eigenvectors $[\, \omega_1, \omega_2, \dots, \omega_{n-m} \,]$ are to be chosen to lie on the switching surface.

Let $W = [\, \omega_1, \omega_2, \dots, \omega_{n-m} \,]$, where $W \in R^{n\times(n-m)}$. In the sliding mode, the system is described by

$$\dot{x} = A_{eq}x, \quad S(x) = Cx = 0 \tag{8}$$

The order of system is n-m and the solution must be in the null space of C, that is, CW=0. It is well known that complete controllability of the pair (A, B) is equivalent to the existence of a controller of the form u=-Kx so that the eigenvalues of A-BK can be arbitrarily assigned [10]. Our equivalent system has the following form:

$$\dot{x} = Ax - B(CB)^{-1}CAx \tag{9}$$

If we let $K = (CB)^{-1}CA$, we need A-BK to have n-m prescribed negative eigenvalues $\{ \lambda_1, \lambda_2, \dots, \lambda_{n-m} \}$ and n-m corresponding eigenvectors $[\, \omega_1, \omega_2, \dots, \omega_{n-m} \,]$. This is equivalent to

$$(A - BK)W = WJ \tag{10}$$

where J is (n-m)×(n-m) Jordan matrix. Here C is determined directly from the n×(n-m) eigenvector matrix W. Since

$$\mathrm{col}(W) \in N(C) \tag{11}$$

it follows that

$$C = \Lambda W^{\perp} \tag{12}$$

where Λ is an arbitrary nonsingular m×m matrix and $W^{\perp}$ is an annihilator of W ($W^{\perp W} = 0$). If CB is required to assume a certain value H, Λ must be determined from

$$CB = H = \Lambda W^{\perp}B \tag{13a}$$

$$\Lambda = H(W^{\perp}B)^{-1} \tag{13b}$$

The inverse of $W^{\perp}B$ always exists since $R(W) \cap R(B) = \{0\}$. From (12) and (13b) we can design the switching surface matrix C as follows.

$$C = H(W^{\perp}B)^{-1}W^{\perp} \tag{14}$$

In general the sliding mode controller varies its structure depending on the position relative to the switching surface and has the form:

$$u_i = u_{ieq} + u_{in}, \quad u_{in} = \begin{cases} u_{in}^{+} & \text{if } s_i(x) > 0 \\ u_{in}^{-} & \text{if } s_i(x) < 0 \end{cases} \tag{15}$$

where u_{ieq} is the i-th component of the equivalent control part - which is continuous - and u_{in} is the discontinuous or switched control part. Note that there are several possible discontinuous control structures for u_{in}[11].

2. Sliding Mode Controller Design using the Continuous-time Switching Dynamics

From (15) the control inputs are essentially discontinuous due to switching logic and as a result the trajectories chatter along the sliding surfaces resulting in the generation of an undesirable high-frequency unmodeled dynamics of the control system. Therefore, in this section we introduce the following switching dynamics to remove the chattering completely in actual implementation of sliding mode controller.

Let us consider the following switching dynamics.

$$\dot{S} = -\Gamma S \tag{16}$$

where $\Gamma = \mathrm{diag}(\gamma_1, \gamma_2, \dots, \gamma_m)$, $\gamma_i > 0$.

The condition for the existence of a sliding mode is easily checked as follows.

$$S^T\dot{S} = -S^T\Gamma S < 0 \quad \text{if } S \neq 0 \tag{17}$$

We can construct the controller using the above switching dynamics. Differentiating (5) and using (16) yields

$$\dot{S} = C\dot{x} = C(Ax + Bu) = -\Gamma S \tag{18}$$

Solving for u

$$\begin{aligned} u &= -(CB)^{-1}[\, CAx + \Gamma S \,] \\ &= -(CB)^{-1}CAx - (CB)^{-1}\Gamma S \\ &= u_{eq} + u_{sd} \end{aligned} \tag{19}$$

where u_{eq} is equivalent control and u_{sd} is continuous control with above switching dynamics.

Here let us examine close-loop eigenvalues. Substituting (19) into the nominal system (2) without uncertainties, we obtain the following closed-loop system

$$\dot{x} = [\, A_{eq} - B(CB)^{-1}\Gamma C \,]\, x = A_c x \tag{20}$$

where $A_{eq} = [\, I - B(CB)^{-1}C \,]\, A$.

Theorem 1 : Let us consider the system (2) with a continuous state feedback control (19). Then the eigenvalues of the closed-loop system (20) can be determined as follows :

$$\sigma(A_c) = \{ \lambda : \det(\lambda I_{n-m} - J) \; \det(\lambda I_m + \Gamma) = 0 \} \tag{21}$$

where J is (n-m)×(n-m) Jordan matrix with n-m prescribed negative eigenvalues.

Proof : We introduce a similarity transformation M which decouples the system (20) into the fast and the slow subsystems, that is, the systems which govern the dynamics of the system off the switching surface and on the switching surface.

Let $M \in R^{n\times n}$ be defined by

$$M = \begin{bmatrix} W^g \\ \cdots\cdots \\ C \end{bmatrix} \tag{22}$$

where $W \in R^{n\times(n-m)}$ is the eigenvector matrix and W^g is a generalized inverse of W. Note that M is invertible with $M^{-1} = [\ W \ \vdots \ B\ ]$.

We introduce new coordinates

$$x^{NEW} = Mx \tag{23}$$

Then, in new coordinates the closed-loop system becomes

$$\dot{x}^{NEW} = M[\ A_{eq} - B(CB)^{-1}\Gamma C]\ M^{-1}x^{NEW} = \begin{bmatrix} D_{11} & \vdots & D_{12} \\ \cdots\cdots & \vdots & \cdots\cdots \\ D_{21} & \vdots & D_{22} \end{bmatrix} x^{NEW}$$

$$\begin{aligned} D_{11} &= W^g[\ A_{eq} - B(CB)^{-1}\Gamma C]\ W \\ D_{12} &= W^g[\ A_{eq} - B(CB)^{-1}\Gamma C]\ B \\ D_{21} &= C[\ A_{eq} - B(CB)^{-1}\Gamma C]\ W \\ D_{22} &= C[\ A_{eq} - B(CB)^{-1}\Gamma C]\ B \end{aligned} \tag{24}$$

From $MM^{-1} = I_n$ we obtain the following relations :

$$\begin{aligned} W^g W &= I_{n-m} \\ W^g B &= 0 \\ CW &= 0 \\ CB &= I_m \end{aligned} \tag{25}$$

From the definition of a generalized inverse and the above relations (25) we can obtain the following simplified state-spae model in new coordinates.

$$\dot{x}^{NEW} = \begin{bmatrix} W^g AW & \vdots & W^g AB \\ \cdots\cdots & \vdots & \cdots\cdots \\ 0 & \vdots & -\Gamma \end{bmatrix} \tag{26}$$

Therefore, the closed-loop eigenvalues can be determined by the following characteristic equation :

$$\det(\lambda I_{n-m} - W^g AW)\det(\lambda I_m + \Gamma) = 0 \tag{27}$$

Since from (10) $W^g AW = J$, we obtain the following result.

$$\det(\lambda I_{n-m} - J)\det(\lambda I_m + \Gamma) = 0 \tag{28}$$

where J is (n-m)×(n-m) Jordan matrix with n-m prescribed negative eigenvalues. ■

Consequently, the closed-loop eigenvalues are determined by both the Jordan matrix J and the positive definite diagonal matrix Γ. Note that J and Γ determine the null-space dynamics and the range-space dynamics, respectively. The range-space dynamics are exponential and controlled by a choice of Γ. Therefore, the stability of closed-loop system can always be guaranteed.

Theoretically, the switching dynamics only guarantees asymptotic hitting to the sliding surface. But, practically it is not unreasonable to be assumed that the states are in the sliding mode already after five times longer than time constant of the switching dynamics. In the remaining of this section we will examine the dynamic behaviors of the uncertain system with switching dynamics.

Let us consider the following range-space dynamics in the presence of matched uncertainties.

$$\begin{aligned} \dot{S} &= C\dot{x} \\ &= C[\ Ax + Bu + Be] \\ &= C[\ Ax + Bu_{eq} + Bu_{sd} + Be] \\ &= CB[\ u_{sd} + e] \\ &= -\Gamma S + CBe \end{aligned} \tag{29}$$

Provided the matrix CB is selected a diagonal matrix $H = \mathrm{diag}(\eta_1, \eta_2, \ldots, \eta_m)$, the solution of range-space dynamics can be described as follows

$$S(t) = \exp[\ -\Gamma(t-t_0)\ ]\ S(t_0) + \int_{t_0}^{t} \exp[\ -\Gamma(t-\tau)\ ]\ He(\tau)d\tau \tag{30}$$

3. Stability Analysis of SMC System with Switching Dynamics.

In this section it will be provided that every response of the matched uncertain system is uniformly bounded and uniformly ultimately bounded within arbitrary small neighborhoods of the switching surfaces.[7]

Let us consider the most simple generalized Lyapunov function candidate for the i-th switching function.

$$V = \sum_{i=1}^{m} V_i(s_i) \tag{31a}$$

$$V_i(s_i) = \tfrac{1}{2} s_i^2 \tag{31b}$$

Differentiating (31b) and using (29) yields

$$\begin{aligned} \dot{V}_i(s_i) &= s_i \dot{s}_i \\ &= s_i(\eta_i e_i - s_i \gamma_i) \\ &\le |s_i| \{ |\eta_i| f_i - |s_i| \gamma_i \} \\ &= f_i |\eta_i| [\ 2\delta(|s_i|)]^{1/2} - 2\gamma_i \delta(|s_i|) \\ &\le 2[\ \varepsilon_i - \gamma_i \delta(|s_i|)\ ] \end{aligned} \tag{32}$$

where $\delta(|s_i|) = \tfrac{1}{2}|s_i|^2$ and $\varepsilon_i = \tfrac{1}{2} f_i |\eta_i| [\,2\delta(|s_i|)\,]^{1/2}$.

Therefore the negative definiteness of $\dot{V}$ can be guaranteed only in the following regions:

$$|s_i| > \frac{|\eta_i| f_i}{\gamma_i} \qquad i = 1, 2, \ldots, m \tag{33}$$

From (33) we can conclude that there exists a compact set $\Delta_i = \{ s_i : |s_i| \le |\eta_i| f_i / \gamma_i \}$ from which no solutions can escape.

Theorem2 : (uniform boundedness) If $s_i(\cdot); [\ t_0, t_1\ ] \to R$, $s_i(t_0) = s_{i0}$ is the i-th solution of (29), then

$$|s_i(t_0)| \le r_i \Rightarrow |s_i(t)| \le d_i(r_i) \quad \forall t \in [\ t_0, t_1] \tag{34}$$

where

$$d_i(r_i) = \begin{cases} r_i & \text{if } r > \xi_i \\ \xi_i & \text{if } r \le \xi_i \end{cases} \tag{35}$$

and

$$\xi_i = |\eta_i|\, f_i \,/\, \gamma_i \tag{36}$$

Proof : Suppose $|s_{i0}| \le r_i$ and $r > \xi_i$.

We define a Lyapunov function candidate for the i-th switching function as follows

$$V_i(s_i) = \tfrac{1}{2}|s_i|^2 = \delta(|s_i|) \quad i = 1, 2, \ldots, m \tag{37}$$

If $s_{i0} \in R^2 \setminus \Delta_i$, the following inequalities are satisfied since $\dot{V}$ is negative definite.

$$\begin{aligned}\delta(r_i) &\geq \delta(\mid s_{i0} \mid) = \tfrac{1}{2} s_{i0}^2 \\ &\geq \tfrac{1}{2} s_i^2(t) = \delta(\mid s_i(t) \mid)\end{aligned} \quad (38)$$

Therefore, since $\delta(\cdot)$ is a monotonous increasing function we can obtain the following results

$$\mid s_i(t) \mid \leq r_i \quad \forall t \in [\, t_0, t_1] \quad (39)$$

Suppose now that $\mid s_{i0} \mid \leq r_i$ but $r_i \leq \xi_i$.

If $s_{i0} \in \Delta_i$, the solution $s_i(t)$ can not escape from the compact set Δ_i. Thus, the following inequalities are satisfied

$$\delta(\xi_i) \geq \tfrac{1}{2} s_i(t) = \delta(\mid s_i(t) \mid) \quad (40)$$

Therefore,

$$\mid s_i(t) \mid \leq \xi_i \quad \forall t \in [\, t_0, t_1] \quad (41)$$

■

Theorem 3 : (uniform ultimate boundedness) If $s_i(\cdot) ; [\, t_0, t_1] \rightarrow R$, $s_i(t_0) = s_{i0}$ is the i-th solution of (29) with $\mid s_{i0} \mid \leq r$, then for sufficiently small $\xi_i(<1)$ and given $\bar{d}_i \geq \xi_i$

$$\mid s_i(t) \mid \leq \bar{d}_i \;\; \forall t \in [\, t_0 + T_i(\bar{d}_i, r_i), t_1] \quad (42)$$

where

$$T_i(\bar{d}_i, r_i) = \begin{cases} 0 & , \text{if } r_i \leq \bar{d}_i \\ \dfrac{\delta(\bar{d}_i) - \delta(r_i)}{2\{\varepsilon_i(\bar{d}_i) - \gamma_i \delta(\bar{d}_i)\}} & , \text{if } r_i > \bar{d}_i \end{cases} \quad (43)$$

Proof : Consider $r_i \leq \bar{d}_i$. If $\mid s_{i0} \mid \leq r_i$, then $\mid s_{i0} \mid \leq \bar{d}_i$; hence, in view of the result of Theorem 2,

$$\mid s_i(t) \mid \leq \bar{d}_i \quad \forall t \in [\, t_0, t_1] \quad (44)$$

so that $T_i(\bar{d}_i, r_i) = 0$.

Next consider $r_i > \bar{d}_i$ and suppose that

$$\mid s_i(t) \mid \leq \bar{d}_i \quad \forall t \in [\, t_0, t_1^*] \quad (45)$$

where

$$\begin{aligned} t_1^* &= t_0 + T_i(\bar{d}_i, r_i), \\ T_i(\bar{d}_i, r_i) &= \frac{\delta(\bar{d}_i) - \delta(r_i)}{2\{\varepsilon_i(\bar{d}_i) - \gamma_i \delta(\bar{d}_i)\}} \end{aligned} \quad (46)$$

Then, in view of (32),

$$\begin{aligned} V_i(s_i(t_1^*)) &= \delta(\mid s_i(t_1^*) \mid) \\ &\leq V_i(s_{i0}) + \int_{t_0}^{t_1^*} \dot{V}_i(s_i(\tau))\, d\tau \\ &\leq \delta(\mid s_{i0} \mid) + 2\int_{t_0}^{t_1^*} [\, \varepsilon_i(\mid s_i(\tau) \mid) \\ &\quad - \gamma_i \delta(s_i(\tau))]\, d\tau \\ &\leq \delta(r_i) + 2T_i(\bar{d}_i, r_i) \\ &\quad *[\, \varepsilon_i(\bar{d}_i) - \gamma_i \delta(\bar{d}_i)] \\ &= [\, \delta(r_i) + \frac{\delta(\bar{d}_i) - \delta(r_i)}{\{\varepsilon_i(\bar{d}_i) - \gamma_i \delta(\bar{d}_i)\}}] \\ &\quad *[\, \varepsilon_i(\bar{d}_i) - \gamma_i \delta(\bar{d}_i)] \\ &= \delta(\bar{d}_i) \end{aligned} \quad (47)$$

That is, $\mid s_i(t_1^*) \mid \leq \bar{d}_i$. From this inequality and (45) we conclude that there must be a t_1^* such that $|s_i(t_1^*)| = \bar{d}_i$. therefore, in view of the result of theorem 2

$$\mid s_i(t_1) \mid \leq \bar{d}_i \quad \forall t \in (t_1^*, t_1] \quad (48)$$

■

From above theorems the sliding mode control system with switching dynamics can be guaranteed only the uniform boundedness and the uniformly ultimate boundedness.

4. The Modified Sliding Mode Controller with two Switching Dynamics

From the definition of the compact set Δ_i it is clear that γ_i must be increased in order to reduce the steady-state error. But, in some control problem the control inputs are constrained by physical situation. Therefore we cannot select sufficiently large γ_i.

To overcome this difficulty we introduce the following modified sliding mode controller with two switching dynamics

$$u_i = u_{ieq} + \bar{u}_{isd} \quad i = 1, 2, \ldots, m$$
$$\bar{u}_{isd} = \begin{cases} u_{isd}^l = -(1/\eta_i)\gamma_i^l s_i \\ \quad , \text{if } \mid s_i(t) \mid < \mu_i \\ u_{isd}^s = -(1/\eta_i)\gamma_i^s s_i \\ \quad , \text{if } \mid s_i(t) \mid \geq \mu_i \end{cases} \quad (49)$$
$$(\gamma_i^l \gg \gamma_i^s)$$

Note that the design parameter $\mu_i(>0)$ must be selected such that

$$\mu_i < \frac{\mid \eta_i \mid f_i}{\gamma_i^s} \qquad i = 1, 2, \ldots, m. \quad (50)$$

III. Conclusion

In this paper, we introduced the continuous-time switching dynamics in the range space of switching surface matrix C to remove the chattering in actual implementation of sliding mode controller. Every response of the closed-loop control system with the proposed switching dynamics in the presence of matched uncertainties is uniformly ultimately bounded within arbitrarily small neighborhoods of the switching surface.

The proposed sliding mode controller does not require the complex computations for switching gains of the controller and requires only the selection of Γ in the range-space in order to occur sliding mode. Hence, the simple design procedures will encourage control engineers to implement the proposed sliding mode controller. Despite control input constraints the steady-state errors can be reduced effectively.

References

[1] K. K. D. Young, "Asymptotic stability of model reference systems with variable structure control", *IEEE Trans. Automatic Control,* vol. 22, pp. 279-281, 1977.

[2] G. Ambrosino, G. Celentano and F. Garofalo, "Variable structure model reference adaptive control system", *Int. J. Cont.,* vol. 39, pp. 1339-1349, 1984.

[3] J. J. Slotine and S. S. Sastry, "Tracking control of non-linear systems using surfaces, with application to robot manipulators", *Int. J. Cont.,* vol. 38, no. 2,

pp. 465-492, 1983.

[4] J. A. Burton and A. S. I. Zinober, "Continuous approximation of variable structure control", *Int. J. System Sci.*, vol. 17, no. 6, pp. 875-885, 1986.

[5] Rainer Palm, "Sliding mode fuzzy control", *IEEE Int. Conf. on Fuzzy Systems*, vol. 1, pp. 519-526, 1992.

[6] Fu-Juay Chang, Hsiang-Ju Liao and Shyang Chang, "Position control of DC motors via variable structure systems control: a chattering alleviation approach", *IEEE Trans. Ind. Electronics*, vol. 37, no. 6, 1990.

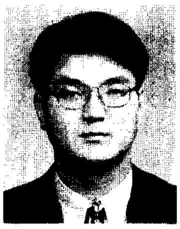
[7] M. J. Corless and G. Leitmann, "Continuous state feedback guaranteeing uniform ultimate boundedness for uncertain dynamical systems", *IEEE. Trans. Automatic Control*, vol. 26, no. 5, 1981.

[8] Gutman, "Uncertain dynamical systems-a Lyapunov min-max approach", *IEEE Trans. Automatic Control*, vol. 26, no. 5, 1981.

[9] V. I. Utkin, "Variable structure systems- present and future", *Automat. Remot. Cont.*, vol. 40, no. 9, pp. 1105-1120, 1983.

[10] W. M. Wonham, *Linear Multivariable Control : A Geometric Approach*, New York, 1977.

[11] R. A. DeCarlo, S. H. Zak and G. P. Mattews, "Variable structure control of non-linear multivariable system : tutorial", *Proc. IEEE*, vol. 76, no. 3, 1988.



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1989년, 1991년 고려대학교 전기공학과에서 학사, 석사학위를 받음. 1992년 ~ 1994년 박사과정 수료. 주요 관심 분야는 비선형 계통에 대한 지능 제어와 로봇 제어등임.

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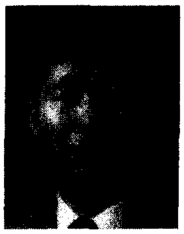
1991년, 1993년 고려대학교 전기공학과에서 학사, 석사학위를 받음. 1993년 ~ 현재 동대학원 박사과정. 주요 관심 분야는 비선형 계통에 대한 강인 제어와 가변구조 제어등임.



박 귀 태

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