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지진양자의 통계적 성질
Statistical Properties of Earthquake 'Quanta'

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요약/Abstract

Sacks와 Rydeleck(1995)에 의해서 제의된 "지진양자"의 개념이 일반형으로 나타낼 수 있다는 것이 보여진다. 대지진에 대해서 응력강하(stress drop)은 거의 일정한 데, 소규모 지진에 대해서 응력강하가 모멘트에 비례하는 성질이 지진양자의 파열기준(failure criteria)과 무관한 것으로 보인다. 지진양자의 물리적 의의는 '사이스몬(Seismon)'이란 관점으로 논의 된다.

It is shown that the concept of 'earthquake quanta' proposed by Sacks and Rydeleck (1995) may be expressed in a more general form. The property that for large earthquakes the stress drop is approximately a constant, while for small events the stress drop is proportional to the moment seems independent of the failure criteria of the *earthquake quanta*. The physical significance of the concept of earthquake quanta is discussed in the perspective of 'seismon'.

Key words : stress drop, earthquake quanta, statistics

Introduction

It was observed that for large earthquakes the stress drop was approximately constant, while for small events with magnitude less than 3.5 the stress drop was proportional to the seismic moment (e.g., Dysart *et al.*, 1988). To model this phenomena, recently Sacks and Rydeleck (1995) suggested a concept of *earthquake quanta*. The linear relation between the logarithm of the number of earthquakes and their magnitudes is commonly ascribed to the distribution of fault region in a self-similar process. Observations indicate not only the linear relation but also the relatively constant stress drop for large earthquakes. Results from computer simulation seems consistent with observations from detailed seismicity studies. In this paper we try to show that the concept may be expressed in a more general way which is independent of the failure criteria of the earthquake quanta.

The Statistical Properties of Earthquake 'Quanta'

Suppose that each earthquake quantum has the seismic moment (Sacks and Rydelek, 1995)

$$M_{0i}=L^3\Delta\sigma_i \quad\text{......(1)}$$

where L is the size of the earthquake quantum and $\Delta\sigma_i$ the stress drop on the i-th quantum. The energy radiated by the quantum can be represented by (Scholz, 1990)

$$E_i=\frac{\Delta\sigma_i}{2\mu}M_{0i}=\frac{L^3}{2\mu}\Delta\sigma_i^2 \quad\text{......(2)}$$

where μ is the shear modulus. The energy radiated by an earthquake being composed of many earthquake quanta is

$$E=\sum_i n_iE_i \quad\text{......(3)}$$

in which n_i is the number of quanta with energy E_i,

$$N=\sum_i n_i \quad\text{......(4)}$$

is the number of quanta, which is not necessarily fixed.

Suppose that for the quanta with energy E_i there are ω_i states, either discrete or continuous, to be chosen. In this case the number of states accessible to the system is

$$\Omega=\frac{N!}{\prod_i n_i!}\prod_i \omega_i^{n_i} \quad\text{......(5)}$$

or in the logarithm form

$$\ln\Omega=\ln N!-\sum_i \ln n_i!+\sum_i n_i\ln\omega_i \quad\text{......(6)}$$

Taking $n_i \gg 1$, one has

$$\ln n_i!\approx n_i(\ln n_i-1) \quad\text{......(7)}$$

thus

$$\ln\Omega\approx N\ln N-\sum_i n_i\ln n_i+\sum_i n_i\ln\omega_i \quad\text{......(8)}$$

Maximizing ln Ω under the constraint of

$$E=\sum_i n_iE_i \quad\text{......(9)}$$

leads to

$$\delta\ln\Omega=-\sum_i \ln\left(\frac{n_i}{\omega_i}\right)\delta n_i=0 \quad\text{......(10)}$$

$$\delta E=\sum_i E_i\delta n_i=0 \quad\text{......(11)}$$

Therefore,

$$\ln\frac{n_i}{\omega_i}+\beta E_i=0 \quad\text{......(12)}$$

or

$$n_i=\omega_i e^{-\beta E_i} \quad\text{......(13)}$$

where β is the Lagrange multiplier.

In fact it might be unnecessary to repeat the deductions above, which are well known in statistical mechanics. What we are trying to do is to point out that here β, as a Lagrange multiplier, is a description of the 'macroscopic' properties of the system, but not necessarily the 'temperature' of the earthquake quanta system which is hard to define. In fact we prefer that the model be represented by the percolation model, in which β becomes a function of the probability for the quanta to be 'occupied'.

The Stress Drop

Based on the above discussion, the energy of the earthquake can be represented by

$$E=\sum_i \omega_i E_i e^{-\beta E_i} \quad \cdots\cdots(14)$$

and the mean stress drop will be

$$\varDelta\sigma=\frac{\sum_i \omega_i \varDelta\sigma_i \exp(-\beta\frac{L^3}{2\mu}\varDelta\sigma_i^2)}{\sum_i \omega_i \exp(-\beta\frac{L^3}{2\mu}\varDelta\sigma_i^2)} \quad \cdots\cdots(15)$$

When the energy spectra of the 'earthquake quanta' is taken as continuous, the above summation turns into an integration. Assume that there is a correspondence :

$$\sum_i \omega_i \rightarrow S\int dV_x \quad \cdots\cdots(16)$$

in which S is the scaled area of the rupture surface and dV_x is the volumetric element in the x−space

$$dV_x \sim \begin{cases} dx & D=1 \\ xdx & D=2 \\ x^2dx & D=3 \end{cases} \quad \cdots\cdots(17)$$

where D is the dimension of the x−space. Consider the case that $\Delta\sigma$ is a scalar, i.e., the one−dimensional case, the mean stress drop will be

$$\varDelta\sigma=\frac{\int_0^{\varDelta\sigma_M}\exp(-\beta\frac{L^3}{2\mu}x^2)xdx}{\int_0^{\varDelta\sigma_M}\exp(-\beta\frac{L^3}{2\mu}x^2)dx} \quad \cdots\cdots(18)$$

In the above expression $\Delta\sigma_M$ is the maximum stress drop of the earthquake quanta in an earthquake, depending on the moment :

$$M_0=S\int_0^{\varDelta\sigma_M}L^3\exp(-\beta\frac{L^3}{2\mu}x^2)xdx \quad \cdots\cdots(19)$$

As $\Delta\sigma_M$ is small, one has

$$M_0\approx S\int_0^{\varDelta\sigma_M}L^3xdx=SL^3\frac{\varDelta\sigma_M^2}{2} \quad \cdots\cdots(20)$$

thus

$$\varDelta\sigma_M=\sqrt{\frac{2M_0}{SL^3}} \quad \cdots\cdots(21)$$

therefore,

$$\varDelta\sigma\approx\frac{\int_0^{\sqrt{2M_0/SL^3}}xdx}{\int_0^{\sqrt{2M_0/SL^3}}dx}=\frac{1}{2}\sqrt{\frac{2M_0}{SL^3}} \quad \cdots\cdots(22)$$

That means for low magnitudes, the mean stress drop is propotional to the moment density. As the area S does not change very much, the mean stress drop is proportional to the moment.

On the other hand, when $\Delta\sigma_M$ becomes large, one has

$$\varDelta\sigma\approx\frac{\int_0^{\infty}\exp(-\beta\frac{L^3}{2\mu}x^2)xdx}{\int_0^{\infty}\exp(-\beta\frac{L^3}{2\mu}x^2)dx}=\sqrt{\frac{2\mu}{\pi\beta L^3}} \quad \cdots(23)$$

being a constant.

In dynamics, the maximum stress drop on an earthquake is confined by the size of the earthquake. Assume that such a relation may be represented as

$$\varDelta\sigma_M\propto S^{1/2} \quad \cdots\cdots(24)$$

In this case, the number density of 'quanta' which may be accommodated in an earthquake with area S will be

$$N=\int_0^{\sqrt{S}}e^{-\beta\frac{L^3}{2\mu}x^2}dx \quad \cdots\cdots(25)$$

When S is large,

$$N \approx \int_0^{\infty} e^{-\beta \frac{L^3}{2\mu} x^2} dx \quad \cdots\cdots (26)$$

becomes a constant. That means the probability for any 'quanta' to be a member of an earthquake with size S is the same. In this case the occurrence frequency of an earthquake with size S is reversely propotional to S, leading to the well-known Gutenberg-Richter's relation. On the other hand, as S becomes small, the number density of 'quanta' which may be accommodated in an earthquake with area S will be

$$N \approx \int_0^{\sqrt{S}} dx = \frac{\sqrt{S}}{2} \quad \cdots\cdots (27)$$

Accordingly the probability for any 'quanta' to be a member of an earthquake with size S will be proportional to $N!$. In this case the 'quanta' has a much higher probability to be occupied by an earthquake with bigger S, and seemingly there is a cut-off of the size of earthquake. Such a cut-off value of S may be regarded as the characterized size of the Sacks-Rydeleck quanta. Furthermore, because the earthquake quanta have a characterized size which does not change very much, for each quanta the stress drop is proportional to the moment itself rather than the moment density.

The 'seismon'

It may be noticed that the earthquake quanta here are much smaller in size than the quanta proposed by Sacks and Rydeleck (1995). The Sacks-Rydeleck quanta may be regarded as being composed of many quanta proposed in this paper. Also it may be wondered why the energy of the system does not include the term of the interaction of different earthquake quanta. In fact, to some extent, the concept of earthquake quanta may be understood in the way similar to phonons in physics, in which we use a kind of quasi-particle to describe the properties of the excited state. To show this concept consider a more general case, in which the seismic energy may be represented by

$$E_S = -\Delta U_e - \Delta U_f - \Delta U_S \quad \cdots\cdots (28)$$

where $\triangle U_e$ is the change in internal strain energy, $\triangle U_f$ the work done against friction, and $\triangle U_s$ the surface energy involved in the creation of the crack. If we consider that the earthquake rupture occurred along an existing fault so that the surface energy may be negligible, and assume that during the earthquake sliding the friction stress has some constant value determined by dynamic friction, and further assume that the dynamic friction equals to the final value of the stress after the earthquake (Kostrov, 1974;Husseini, 1977;Scholz, 1990), the radiated energy may be expressed by

$$E_S = \frac{1}{2} \int_{\Sigma} \Delta\sigma(r) \Delta u(r) dS \quad \cdots\cdots (29)$$

in which $\triangle\sigma(r)$ is the stress drop, $\triangle u(r)$ is the dislocation, and the integration is over the rupture surface Σ. The stress drop $\triangle\sigma(r)$ and the dislocation $\triangle\sigma(r)$ depend upon each other. Introducing the dislocation-stress-drop Green's function $T(r-r')$ so that

$$\Delta u(r) = \int_{\Sigma} T(r-r') \Delta\sigma(r') dS \quad \cdots\cdots (30)$$

one has

$$E_S = \frac{1}{2} \int_{\Sigma} \Delta\sigma(r) dS \int_{\Sigma} T(r-r') \Delta\sigma(r') dS' \quad \cdots (31)$$

The coarse-grained version of the above integration may be represented by

$$E_S = \frac{1}{2} \sum_i \sum_j T^{ij} \Delta\sigma_i \Delta\sigma_j \quad \cdots\cdots (32)$$

Since T^{ij} is real and symmetric, there exists a linear transform $\{\triangle\sigma_i\} \rightarrow \{\xi_i\}$ leading to

$$E_S = \frac{1}{2} \sum_i \xi_i^2 \quad \cdots\cdots (33)$$

In this case the radiated energy may be represented by the contribution of 'seismons' having the 'stress drop' ξ_i and energy

$$E_i = \frac{1}{2}\xi_i^2 \quad \cdots\cdots (34)$$

and the 'seismons' themselves may be regarded as indenpendent on each other. Further more, the Sacks–Rydeleck quanta are composed of many 'seismons' proposed in this paper.

One problem of this approach is that in the real case of earthquake rupture often the number of earthquake quanta is not so large, so the fluctuation is not negligible. In this case the computational approach has its advantage in revealing the properties of the system. However, the statistical approach, on the other hand, can at least be used as an aid of the computation and provide some clues which might be useful in understanding the nature of the complexity of earthquake.

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