출력 미분값의 추정에 의한 선형 시불변 시스템의 로버스트 출력 궤환 제어

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Robust Output Feedback Control of LTI System Using Estimated Output Derivatives

李 建 馥*
(Gun-Bok Lee)

Abstract - This work is concerned with the estimation of output derivatives and their use for the design of robust controller for linear systems with system uncertainties due to modeling errors and disturbances. It is assumed that a nominal transfer function model and quantitative bounds for system uncertainties are known. The developed control schemes are shown to achieve regulation of the system output and ensures boundedness of the system states without imposing any structural conditions on system uncertainties and disturbances. Output derivative estimation is first conducted through restructuring of the plant in a specific parameterization. They are utilized for constructing robust nonlinear high-gain feedback controller of a SMC(Sliding Mode Control) type. The performances of the developed controller are evaluated and shown to be effective and useful through simulation study.

Key Words : Uncertainty Bound, Relative Degree, Minimum Phase, Nominal Model, Generalized Observer, Output Derivative Estimation, Robust Nonlinear High-gain Feedback Control

1. Introduction

In the past decades, an extensive effort has been made to improve dynamic performances of linear systems using a number of feedback control schemes. The concept of variable structure systems or the Sliding Mode Control (SMC) belongs to one of them. The idea is to find an attractive hyperplane in the state space such that the motion there is independent of system uncertainties or disturbances(invariance property). Then it is possible to study the asymptotic behavior of the system in this hyperplane using the usual linear technique. Since attractiveness is a local property, one has to add conditions that guarantee either reaching the attractive hyperplane or approaching the origin outside it. Another is the Ultimate Boundedness Control(UBC). The two nonlinear high-gain feedback controllers, SMC and UBC achieve state regulation in the presence of system uncertainties and/or disturbances. They require a nominal state space model and quantitative bounds on system uncertainties and disturbances for controller design. In addition the design also requires the so-called "matching conditions", i.e., system uncertainties be structured in a particular form and the disturbances be in the same channel as the control input[1, 2, 5, 8].

Another drawback is that they require full state measurement, restricting their applicability severely in practice. This requirement, naturally, needs state estimation. An observer design for state estimation imposes a different structural condition on system uncertainties from the matching conditions so that observer and controller design can be done independently.

In view of these discussions, here, we set up a new control objective and propose a control scheme which ensures system stability, guarantees satisfactory system output performance and can be applied to a large class of systems through less restrictive conditions imposed on system uncertainties. In order to achieve the new control objective, a scheme is developed to estimate system output derivatives. With the estimated output derivatives, a sliding surface dynamics is constructed on which the dynamics of system output are governed by an asymptotically stable system subject to a small input and the boundedness of the remaining system states follows from the assumption of minimum phase system. The effectiveness of the proposed scheme is demonstrated through a couple of numerical examples.

2. Estimation of Output Derivatives

In this section, we establish the schemes for estimating system output derivatives when states are not directly accessible and only system output is measured. They are based on restructuring of the system dynamics which simplify calculations and avoid accumulations of system

*正 會 員 : 崇實大 工大 機械工學科 副教授・工博
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uncertainties.

2.1. Reparameterization of System Dynamics

The system is considered to be represented by the equation

$$(A(s)+\Delta A(s))y_p=(B(s)+\Delta B(s))u+d \tag{1}$$

where u is a control input, y_p is the output and d is a bounded external disturbance

$$|d(t)|\le D \tag{2}$$

A(s) and B(s) constitute a nominal system model as described below

$$A(s)=s^n+a_{n-1}s^{n-1}+\cdots+a_0$$

$$B(s)=b_m s^m+b_{m-1}s^{m-1}+\cdots+b_0$$

ΔA(s) and ΔB(s) describe the parameter uncertainties as follows

$$\Delta A(s)=\Delta a_{n-1}s^{n-1}+\cdots+\Delta a_0$$

$$\Delta B(s)=\Delta b_m s^m+\cdots+\Delta b_0$$

The following upper bounds on parameter uncertainties are assumed to be known

$$\begin{aligned}&|\Delta a_i|\le\delta a_i\ ,\quad i=0,1,\cdots,n-1\\&|\Delta b_j|\le\delta b_j\ ,\quad j=0,1,\cdots,m\end{aligned}\tag{3}$$

Before establishing the output derivative estimator, restructuring of system dynamics and new signal generation are necessary and introduced below.

We define new signals w_1, w_2 and w_3 as follows. First of all, signal generators for w_2, w_3 are considered in a state-space representation. Choose $\Lambda\in R^{n-1\times n-1}$ and $b_\lambda\in R^{n-1}$ such that (Λ, b_λ) is in a controllable form and det(sI-Λ)= λ(s).

$$\Lambda=\begin{bmatrix}0&1&0&\cdots&0\\0&0&1&\cdots&0\\\cdot&\cdot&\cdot&&\cdot\\\cdot&\cdot&\cdot&&\cdot\\\cdot&\cdot&\cdot&&\cdot\\0&0&0&\cdots&1\\-\lambda_0&\cdot&\cdot&\cdots&-\lambda_{n-2}\end{bmatrix}\qquad b_\lambda=\begin{bmatrix}0\\\cdot\\\cdot\\\cdot\\\cdot\\0\\1\end{bmatrix}\tag{4}$$

$$w_1=y_p \tag{5}$$

$$\dot{w}_2=\Lambda w_2+b_\lambda y_p \tag{6}$$

$$\dot{w}_3=\Lambda w_3+b_\lambda u \tag{7}$$

where the states w_2 , $w_3\in R^{n-1}$ and their initial conditions are arbitrary.

It follows that

$$(sI-\Lambda)^{-1}b_\lambda=\frac{1}{\lambda(s)}\begin{bmatrix}1\\s\\\cdot\\\cdot\\\cdot\\s^{n-2}\end{bmatrix}\tag{8}$$

For numerical considerations, we require that λ(s) is a Hurwitz polynomial. We now conduct a reparameterization of the system dynamics resulting in output derivative estimators. We reformulate (1) as

$$A(s)y_p=B(s)u-\Delta A(s)y_p+\Delta B(s)u+d \tag{9}$$

Replacing the uncertainty terms by η, that is, $\eta=-\Delta A(s)y_p+\Delta B(s)u$ and dividing (9) by λ(s), we obtain the first derivative of the output as follows

$$\dot{y}_p=(s\lambda(s)-A(s))y_\lambda+B(s)u_\lambda+\eta_\lambda+d_\lambda \tag{10}$$

where the subscript "λ" denotes the filtered signal by λ(s), 1/λ(s). The right-hand side can be expressed using the signals defined earlier, as follows

$$\begin{aligned}\dot{y}_p&=\theta_1 w_1+\theta_2 w_2+\theta_3 w_3+\eta_1+d_1\\&=\sum_{i=1}^{3}\theta_i w_i+\eta_1+d_1\end{aligned}\tag{11-a}$$

where

$$\theta_1=\lambda_{n-2}-a_{n-1}$$

$$\begin{aligned}\theta_2=[&-a_0+\lambda_0(a_{n-1}-\lambda_{n-2}),\\&\lambda_0-a_1+\lambda_1(a_{n-1}-\lambda_{n-2}),\\&\cdot\cdot,\lambda_{n-3}-a_{n-2}+\lambda_{n-2}(a_{n-1}-\lambda_{n-2})]\in R^{n-1}\end{aligned}$$

$$\theta_3=[b_0, b_1, b_2\cdots b_m, 0,\cdots,0]\quad\in R^{n-1}$$

$$\eta_1=\eta_\lambda$$

$$d_1=d_\lambda$$

$$\begin{aligned}\eta_1&=\Delta\theta_1 w_1+\Delta\theta_2 w_2+\Delta\theta_3 w_3\\&=\sum_{i=1}^{3}\Delta\theta_i w_i\end{aligned}\tag{11-b}$$

with

$$\Delta\theta_1 = -\Delta a_{n-1}$$

$$\Delta\theta_2 = [-\Delta a_0 + \lambda_0\Delta a_{n-1},\ -\Delta a_1 + \lambda_1\Delta a_{n-1}\cdots,\ -\Delta a_{n-3} + \lambda_{n-3}\Delta a_{n-1},\ -\Delta a_{n-2} + \lambda_{n-2}\Delta a_{n-1}]$$

$$\Delta\theta_3 = [\Delta b_0, \Delta b_1, \Delta b_2 \cdots, \Delta b_m, 0, \cdots, 0] \qquad (11\text{-c})$$

Note that from (2),(3) and $\lambda(s)$, η_1 and d_1 are bounded, respectively, by

$$|\eta_1(w)| \le \overline{\eta}_1(w) \equiv \overline{\Delta\theta_i}\ \overline{w}_i$$
$$|d_1(t)| \le \overline{d}_1 \equiv \overline{d}_\lambda \qquad (11\text{-d})$$

where $\overline{\Delta\theta_i}$ denotes a vector whose components are upper bounds of the absolute values of the corresponding components of $\Delta\theta_i$ (i=1, 2, 3) and $\overline{w}_i$ denotes the vector function whose components are the absolute values of the corresponding components of w_i (i=1, 2, 3).

Differentiating equation (11-a) continuously, we obtain the following expressions

$$y_p^{(i)} = \theta_1 y_p^{(i-1)} + \sum_{j=1}^{i-1}\theta_2\Lambda^{j-1}b_\lambda\ y_p^{(i-j-1)} + \theta_2\Lambda^{i-1}w_2 + \theta_3\Lambda^{i-1}w_3 + \eta_i + d_i \qquad (12\text{-a})$$

$$i = 1, 2, \cdots, r-1$$

where r represent the relative degree, n-m, and

$$\eta_i = \Delta\theta_1 y_p^{(i-1)} + \sum_{j=1}^{i-1}\Delta\theta_2\Lambda^{j-1}b_\lambda\ y_p^{(i-j-1)} + \Delta\theta_2\Lambda^{i-1}w_2 + \Delta\theta_3\Lambda^{i-1}w_3 \qquad (12\text{-b})$$

$$d_i = d_\lambda^{(i-1)}$$

η_i and d_i are bounded by

$$|\eta_i| \le \overline{\eta}_i(w) \equiv \overline{\Delta\theta_1}\ \overline{y}_p^{(i-1)} + \sum_{j=1}^{i-1}\overline{\Delta\theta_2}\ \overline{\Lambda^{j-1}}b_\lambda\ \overline{y}_p^{(i-j-1)} + \overline{\Delta\theta_2}\ \overline{\Lambda}^{i-1}\ \overline{w}_2 + \overline{\Delta\theta_3}\ \overline{\Lambda}^{i-1}\ \overline{w}_3 \qquad (12\text{-c})$$

$$|d_i| \le \overline{d}_i \equiv \overline{d}_\lambda^{(i-1)}$$

$$i = 1, 2, \cdots, r-1$$

The objective is to develop the estimators for the output derivatives, $y_p^{(i)}$. To do so, naturally, the uncertain terms $\eta_i + d_i$ in equation (12-a) have to be estimated. To estimate $y_p^{(i)}$, we propose the following estimators with uncertainty estimators, β_i

$$\dot{q}_i = \theta_1 y_p^{(i-1)} + \sum_{j=1}^{i-1}\theta_2\Lambda^{(i-1)}b_\lambda\ y_p^{(i-j-1)} + \theta_2\Lambda^{(i-1)}w_2 + \theta_3\Lambda^{(i-1)}w_3 + \beta_i \qquad (13\text{-a})$$

$$e_i = y_p^{(i-1)} - q_i \qquad (13\text{-b})$$

$$\beta_i = \gamma e_i + M_i(w)\ h(e_i) \qquad (13\text{-c})$$

$$M_i(w) > \overline{\eta}_i(w) + \overline{d}_i \qquad (13\text{-d})$$

where

$\dot{q}_i$ is the estimator of $y_p^{(i)}$

e_i is the estimation error

β_i is the estimator of $\eta_i + d_i$

γ is an arbitrary positive constant

$\overline{\eta}_i(w)$, $\overline{d}_i$ are defined in (12-c).

$h(e_i)$ is a smooth function of any form having the following properties, i.e., $h(-e_i) = -h(e_i)$,and their values increasing monotonically from -1 to +1.

Theorem 1 : Consider the output derivative estimators given by (13) for the system (1). If the system input u meets the regularity conditions, then given any $\varepsilon_i > 0$, there exist a sufficiently large M_i and a finite time T_i such that

$$|\dot{e}_i| = |\dot{q}_i - y_p^{(i)}| < \varepsilon_i,\quad i = 1, \cdots, r-1 \quad \textit{after} \quad t \ge T_i$$

(Proof)
See Appendix.

2.2 Realizable Output Derivative Estimator

In implementation of the estimators (13), the estimation error e_i is not available except for $i=1$. However, estimates for $y_p^{(i-1)}$ are obtained from previous estimators. Therefore, we define a new set of estimation errors and replace the estimators (13) by the followings.

$$\dot{q}_i = \theta_1\dot{q}_{i-1} + \sum_{j=1}^{i-1}\theta_2\Lambda^{j-1}b_\lambda\ \dot{q}_{i-j-1} + \theta_2\Lambda^{i-1}w_2 + \theta_3\Lambda^{i-1}w_3 + \beta_i \qquad (14\text{-a})$$

$$\overline{\eta}_i = \overline{\Delta\theta_1}\ \overline{\dot{q}}_{i-1} + \sum_{j=1}^{i-1}\overline{\Delta\theta_2\Lambda^{j-1}b_\lambda}\ \dot{q}_{(i-j-1)} + \overline{\Delta\theta_2\Lambda^{i-1}}\ \overline{w}_2 + \overline{\Delta\theta_3\Lambda^{i-1}}\ \overline{w}_3 \qquad (14\text{-b})$$

$$\hat{e}_1 = e_1 = y_p - q_1 \tag{14-c}$$

$$\hat{e}_i = \dot{q}_{i-1} - q_i \tag{14-d}$$

2.3 Simulation Study

Performance of the proposed output derivative estimator is simulated for two example systems whose relative degrees are 2 and 3, respectively.

System 1 (r=2) :

$$A(s) = s^3 + s^2 + 13s + 10 \quad , \quad B(s) = s+2$$

$$\Delta A(s) = 8s^2 + 10s + 5 \quad , \quad \Delta B(s) = s+3$$

$$\lambda(s) = s^2 + 3s + 2$$

$$d(t) = \left(\frac{d}{dt} + 3\right)(2\sin 0.2t)$$

$$u(t) = 3\cos 0.5t$$

We use the estimators given by (14). For given system and system uncertainties, the estimator and the corresponding parameters are as follows.

$$\mathrm{M}_1(w) = 8\,|\,y_p\,| + 11\,|\,w_1^{\,1}\,| + 14\,|\,w_1^{\,2}\,| + 3\,|\,w_2^{\,1}\,| + |\,w_2^{\,2}\,| + 8$$

$$e_1 = y - q_1$$

$$h(e_1) = \frac{e_1}{|\,e_1\,| + \delta_1}$$

$$\beta_1 = \gamma e_1 + \mathrm{M}_1(w)\, h(e_1)$$

The simulated responses for the system described are shown in Fig.1 and Fig.2. Fig.1 shows that an accurate estimate of $\dot{y}_p$ is obtained in a very short time. In Fig.2, we can see that uncertainty estimator β_1 also tracks the system uncertainty $\eta_1 + d_1$ reasonably well..

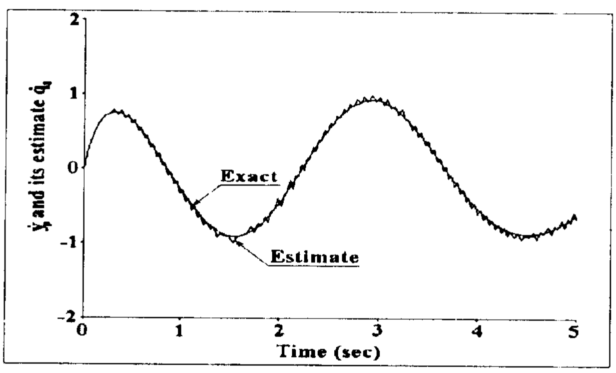


Fig. 1 Estimation of output derivative (Relative degree = 2)

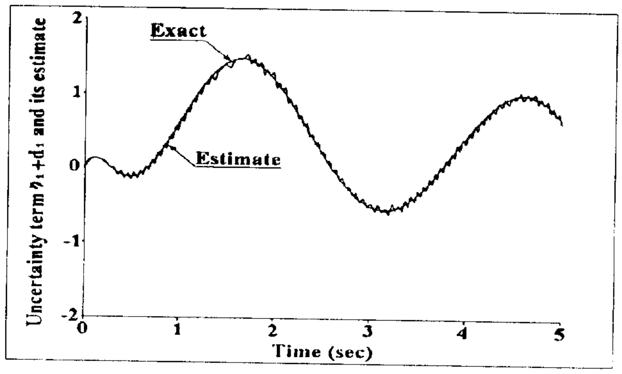


Fig. 2 Estimation of uncertainty term (Relative degree = 2)

System 2 (r= 3)

$A(s)$, $\Delta A(s)$, $\lambda(s)$, $d(t)$, $u(t)$; *same as case* 1

$$B(s) = 1 \quad , \quad \Delta B(s) = 10$$

We estimate $\dot{y}_p$ and $\ddot{y}_p$ with the following parameters

$$\mathrm{M}_1(w) = 8\,|\,y_p\,| + 11\,|\,w_1^{\,1}\,| + 14\,|\,w_1^{\,2}\,| + 10\,|\,w_2^{\,1}\,| + 8$$

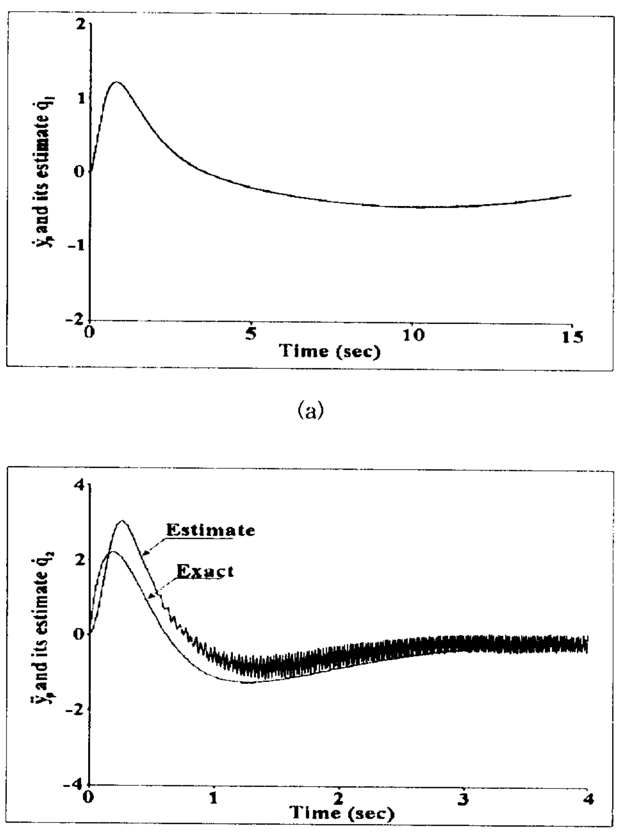
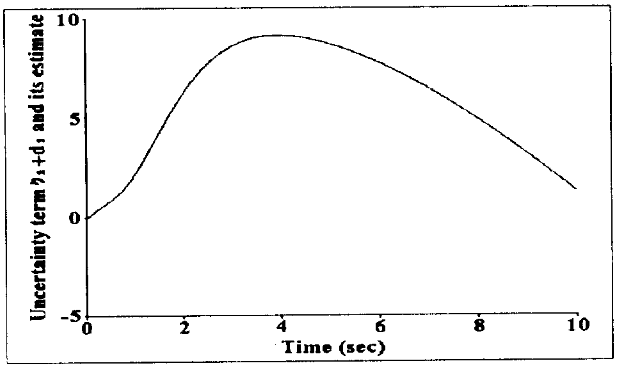
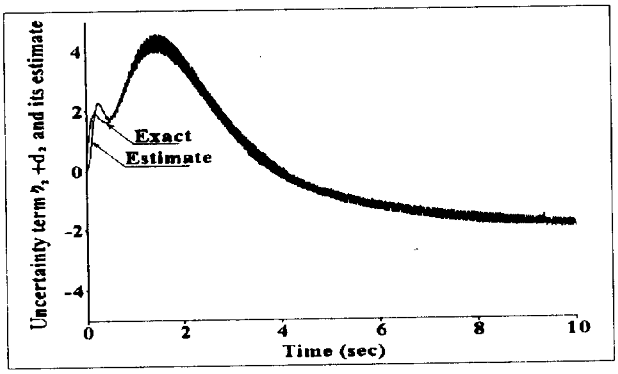


Fig. 3 Estimation of output derivative $\dot{y}_p$ and $\ddot{y}_p$(Relative degree = 3)



(a)



(b)

Fig. 4 Estimation of uncertainty term η_i+d_i(Relative degree=30

$$M_2(w)=8\,|\,\dot{q}_1\,|\,+28\,|\,w_1^{\,1}\,|\,+31\,|\,w_1^{\,2}\,|\,+14\,|\,y_p\,|\,+10\,|\,w_2^{\,2}\,|\,+\,|\,d_2\,|$$

$$\widehat{e_2}=\dot{q}_1-q_2$$

$$h(\widehat{e_2})=\frac{\widehat{e_2}}{|\,\widehat{e_2}\,|+\delta_2}$$

Fig.3 (a) and (b) show that accurate estimates of $\dot{y}_p$ and $\ddot{y}_p$ are achieved within a few seconds. Shown in Fig.4 (a), (b) are the uncertainty terms estimated, resulting in accurate output derivative estimation.

From the simulation results, we can see that the proposed derivative estimator is very effective and has the strong possibility to be used for control purpose.

3. Controller Design

In this section, we synthesize a nonlinear high-gain feedback controller utilizing the estimated output derivatives for systems with a partially known input-output representation and disturbances. The controller is shown to achieve regulation of the system output and ensure boundedness of the system states.

3.1. Robust Nonlinear Output Feedback Controller

Consider a linear system described by

$$(A(s)+\Delta A(s))y_p=K_p(B(s)+\Delta B(s))u+d \tag{15}$$

where K_p is a unknown high frequency gain. However, it is assumed that a nominal system gain K and an upper bound on K_p is known. Also we assume that K_p and K are positive and satisfy

$$K_m \le \frac{K_p}{K} \le K_M \tag{16}$$

The ultimate objective is to design a control to constrain the system dynamics to the following surface

$$S=y_p^{(r-1)}+\alpha_{r-2}y_p^{(r-2)}+\cdots+\alpha_0 y_p \tag{17}$$

Where α_i's are chosen such that when S equals zero, equation (17) defines an asymptotically stable (r-1)[th] order differential equation of y_p. To be implementable, we have to use the output derivative estimators, $\dot{q}_i$ instead of $y_p^{(i)}$ in (17).

Accordingly, we define another sliding surface

$$\hat{S}=\dot{q}_{r-1}+\alpha_{r-2}\,\dot{q}_{r-2}+\cdots+\alpha_0 y_p \tag{18}$$

A control law must be designed to satisfy the fall condition to the sliding surface. Here, we propose the control law of the following form

$$u=-\frac{\Omega\hat{S}+\hat{\sigma}(w)+K\theta_3\Lambda^{r-1}w_3+\hat{M}(w)h(\hat{S})}{K} \tag{19-a}$$

$$\hat{M}(w)=\frac{m_1\,|\,\hat{\sigma}(w)\,|\,+m_2\,\widehat{\eta}_r(w)+m_3}{K_m} \tag{19-b}$$

$$m_1>\kappa\;,\quad m_2>1.0\quad\text{and}\quad m_3>\bar{d}_r$$

where

$$\hat{\sigma}(w)=\theta_1\,\dot{q}_{r-1}+\sum_{j=1}^{r-1}\theta_2\Lambda^{j-1}b_\lambda\,\dot{q}_{r-j-1}+\theta_2\Lambda^{r-1}w_2+\sum_{j=1}^{r-1}\alpha_{j-1}\dot{q}_j \tag{19-c}$$

$$\widehat{\eta}_r(w)=\overline{\Delta\theta_1}\,\overline{\dot{q}_{r-1}}+\sum_{j=1}^{r-1}\overline{\Delta\theta_2\Lambda^{j-1}b_\lambda}\,\overline{\dot{q}_{r-j-1}}+\overline{\Delta\theta_2\Lambda^{r-1}}\,\overline{w_2}+KK_M\overline{\Delta\theta_3\Lambda^{r-1}}\,\overline{w_3} \tag{19-d}$$

$$\kappa = \max(\,|1-K_M|\,,\,|1-K_m|\,)$$

$h(\hat{S})$ is the smooth function defined previously.

Theorem 2 : Let the system (15) be a minimum phase system with a relative degree r and bounded parameter errors and disturbances. If the control input (19) is applied to the system, then the output y_p, the states x and the control input u remain bounded.

Proof : When the output derivative estimators (13) are used, we obtain from theorem 1 that

$$|\dot{q}_i - y_p^{(i)}| < \varepsilon_i \quad , \quad i = 1, \cdots, r-1$$

within a finite period of time and ε_i can be made arbitrarily small if sufficiently large M_i's are used in (13-d).

Define another control input

$$\hat{u} = -\frac{\Omega S + \sigma(w) + K\theta_3 \Lambda^{r-1} w_3 + M(w)h(S)}{K} \tag{20-a}$$

$$M(w) = \frac{m_1|\sigma(w)| + m_2\,\overline{\eta}_r(w) + m_3}{K_m} \tag{20-b}$$

$$\sigma(w) = \theta_1 y_p^{(r-1)} + \sum_{j=1}^{r-1}\theta_2\Lambda^{j-1}b_\lambda y_p^{(r-j-1)} + \theta_2\Lambda^{r-1}w_2 + \sum_{j=1}^{r-1}\alpha_{j-1}y_p^{(j)} \tag{20-c}$$

From (19), (20) and the following relationships

$$e_k = \dot{q}_{k-1} - y_p^{(k)} \tag{21-a}$$

$$\hat{S} - S = \mu_1(e_k) \tag{21-b}$$

$$\hat{\sigma}(w) - \sigma(w) = \mu_2(e_k) \tag{21-c}$$

$$\hat{M}(w) - M(w) = \mu_3(e_k) \quad k = 1, \cdots, r-1 \tag{21-d}$$

We can obtain the following relationships

$$\hat{u} = u + \zeta_1 + \zeta_2 M(w) \tag{21-e}$$

where

$$\zeta_1 = \frac{\Omega}{K}\mu_1(e_k) + \frac{1}{K}\mu_2(e_k) + \frac{1}{K}\mu_3(e_k)\,h(\hat{S})$$

$$\zeta_2 = \frac{1}{K}\mu_1(e_k)\,h'(\underline{S})$$

$$h'(S) = \frac{d}{dS}h(S)$$

$\underline{S}$ is between S and $\hat{S}$

ζ_1 and ζ_2 approach zero when ε_i's approach to zero.

We now show that the control law (19) ensures the reaching condition of the sliding surface S.

Let $V_3 = \frac{1}{2}S^2$ and take the derivative of V_3

Then

$$\begin{aligned}\dot{V}_3 &= S(y_p^{(r)} + \alpha_{r-2}y_p^{(r-1)} + \cdots + \alpha_0\dot{y}_p) \\ &= S(\sigma(w) + K_p u + K_p\theta_3\Lambda^{r-1}w_3 + \eta_r(w) + d_r) \\ &= S(\sigma(w) + K_p\hat{u} + K_p\theta_3\Lambda^{r-1}w_3 + \eta_r(w) + d_r \\ &\qquad - K_p\zeta_1 - K_p\zeta_2 M(w))\end{aligned} \tag{22-a}$$

from the control law (20)

$$\begin{aligned}\dot{V}_3 &= S(-\frac{K_p}{K}\Omega S - \frac{K_p}{K}M(w)h(S) + (1-\frac{K_p}{K})\sigma(w) \\ &\qquad + \eta_r + d_r - K_p\zeta_1 - K_p\zeta_2 M(w)) \\ &\le -K_m\Omega S^2 - K_m M(w)|S|\{(h(|S|) \\ &\qquad - \frac{\kappa|\sigma(w)| + \overline{\eta}_r(w)}{K_m M(w)} \\ &\qquad \frac{+\,\overline{d}_r + KK_M(\overline{\zeta}_1 + \overline{\zeta}_2 M(w))}{}\} \\ &\le -K_m\Omega\xi^2 \quad \text{for } all \;\; S \in L_3{}^c\end{aligned} \tag{22-b}$$

where L_3 is defined by

$$L_3 = \{\; S : |S| < \xi\;, \quad \xi = h^{-1}\{\sup_w(\frac{h|\sigma(w)| + \overline{\eta}_r(w)}{K_m M(w)} \quad \frac{d_r + KK_M(\overline{\zeta}_1 + \overline{\zeta}_2 M(w))}{}\}$$

Equation (22) implies that within a finite period of time, say, $T_3(\le V_3(0)/K_m\Omega\xi^2)$, S enters L_3 and stays there permanently. Then, equation (17) implies that the dynamics of y_p are governed by an asymptotically stable system subject to small input S inside the neighborhood defined by L_3. Therefore $[y_p, \dot{y}_p, \cdots, y_p^{(r-1)}]^T$ is regulated to remain small by the controller. Then, boundedness of the system states follows from the assumption of a minimum-phase system. This concludes the proof. Fig. 5 represents overall structure of the developed control scheme with integrated output derivative estimator and controller.

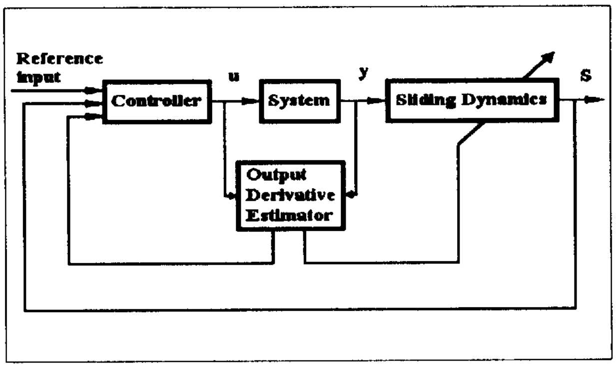
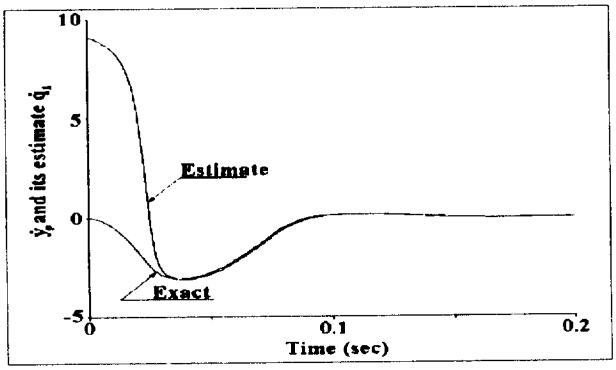


Fig. 5 Robust output feedback control

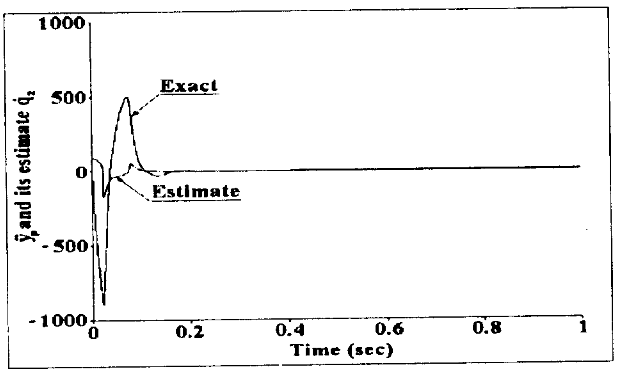
3.2 Simulation Study

Performances of the proposed output feedback sliding mode controller are simulated. As was discussed, sliding mode control requires feedback of output derivatives up to $(r-1)^{th}$ order.

Three performance criteria have been numerically examined : (1) accuracy of output derivative estimation (2) sensitivity to parameter uncertainty (3) sensitivity to disturbance.



(a)



(b)

Fig. 6 Estimation of output derivative $\dot{y}_p$ and $\ddot{y}_p$

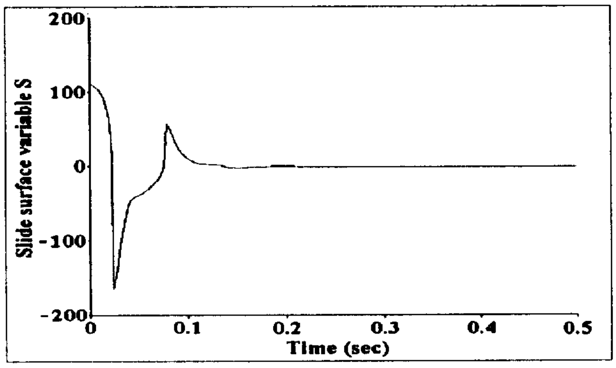


Fig. 7 Sliding surface variable

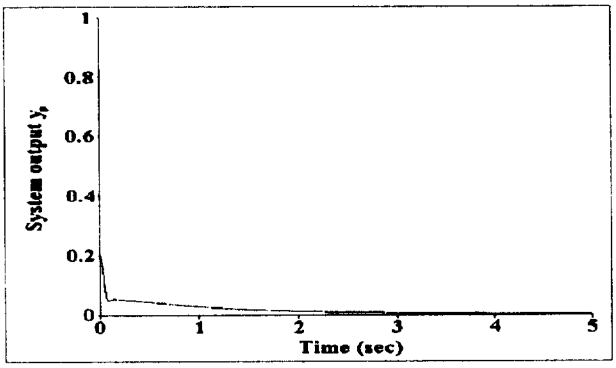


Fig. 8 System output

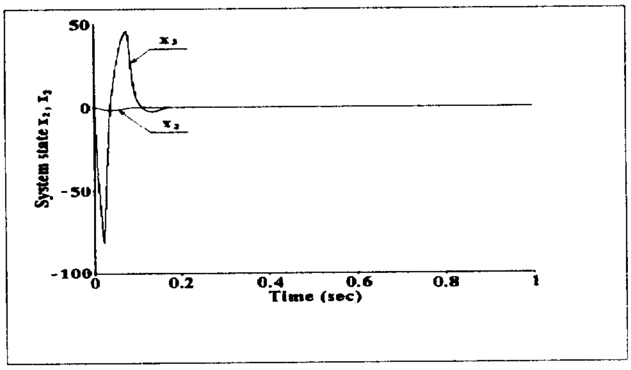


Fig. 9 System states (X_2, X_3)

Example System

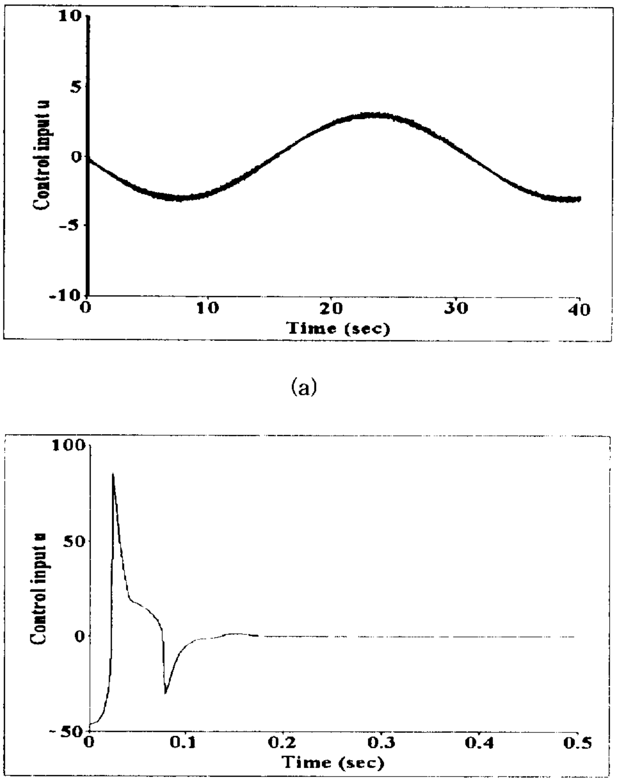
$A(s)=s^3+s^2+13s+10$, $\Delta A(s)=8s^2+10s+5$

$B(s)=1$, $\Delta B(s)=10$

$d(t)=(\frac{d}{dt}+3)\,(2\sin 0.2t)$

Shown in Fig.6 (a) and (b) are the system output derivatives $\dot{y}_p$, $\ddot{y}_p$ and their estimates $\dot{q}_1$, $\dot{q}_2$.

The estimates track the true values in about 0.2 second. Sliding surface variable in Fig.7 shows that system states reach the surface at about 0.2 second. The system output then starts approaching zero asymptotically as seen in Fig.8. The other two states are shown in Fig.9 and remain



(a)

(b)

Fig. 10 Control input (a) Long time view (b) short time view

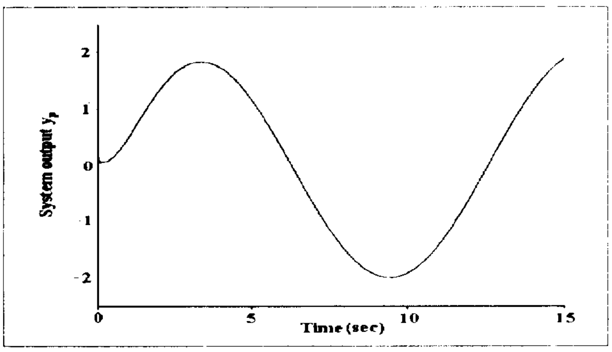


Fig. 11 Tracking control (reference input R(t)=2sin0.5t0

bounded as seen. The control inputs are shown in Fig. 10 (a), (b). When the output is regulated to zero, it is shown that the control inputs are almost chatter-free due to the introduction of smooth function and counteracts the system uncertainties appropriately. Fig. 11 represents the tracking control result for time-varying reference input. We can see that good performance is archieved as expected.

4. Conclusions

Presented in this paper are the schemes to estimate system output derivatives in the presence of system uncertainties. The systems should be described in the frequency domain where only the system output is measured. Based on the estimated output derivatives, a nonlinear high-gain output feedback controller was developed. The controller achieves almost perfect output regulation and ensures boundedness of the system states in the presence of system uncertainties. The assumptions required are that the system be minimum phase and the external disturbances be smooth in terms of having bounded derivatives up to $(r-1)^{th}$ order. Structural conditions, say, matching conditions are not imposed on the system uncertainties. In practical situations, parameter uncertainty bound and disturbance magnitude can be easily obtained to a certain degree of specified accuracy in terms of proper measuraments or from the system characteristics. Therefore, the developed controllers can be applied to a larger class of systems compared with previously proposed SMC or UBC.

Future attention will focus on extending the developed scheme to certain nonlinear systems. In addition, theoretical and experimental work is planned to compare the performance of this output feedback control scheme against an observer based method.

Appendix

Definition : Regularity Conditions (for more details, see ref. [7])

When a signal is expressed as

$$c(t)=\sum_{i=1}^{l} f_i(t)x_i(t)+g(t)=F(t)^T X(t)+g(t) \qquad \text{(A-1)}$$

where

$$F(t)=\begin{bmatrix} f_1(t) \\ \cdot \\ \cdot \\ \cdot \\ f_l(t) \end{bmatrix}, \quad X(t)=\begin{bmatrix} x_1(t) \\ \cdot \\ \cdot \\ \cdot \\ x_l(t) \end{bmatrix}$$

it is "strictly regular" if the following conditions are satisfied

$$\text{(i)} \quad |g(t)| \le G^0, \quad |\dot{g}(t)| \le G^1$$

$$\text{(ii)} \quad \|F(t)\| \le H^0(t), \quad \|\dot{F}(t)\| \le H^1(t) \qquad \text{(A-2)}$$

$$\text{(iii)} \quad \|\dot{X}\| \le a\|X\| + b, \quad a, b > 0$$

from (A-2)

$$|c(t)| \le H^0\|X\| + G^0 \qquad \text{(A-3)}$$

We start with the error dynamics derived from (12-a) (13-a).

$$\dot{e_i} = -\beta_i + \eta_i + d_i \quad , \quad i = 1, \cdots, r-1 \qquad \text{(A-4)}$$

Since from equations. (11), (12) and (13), $y_p^{(i)}$ and $\dot{q_i}$ consist of a linear combination of signals, w_i, and filtered disturbance, d_f and its derivatives which are all bounded, they are "strictly regular" by definition. Therefore, we can write $\eta_i + d_i$ into the form of (A-1) through algebraic manipulations.

$$\eta_i + d_i = F(t)^T X(t) + g(t) \qquad \text{(A-5)}$$

Then,

$$\dot{e_i} = -\beta_i + F^T X + g \qquad \text{(A-6)}$$

(Part 1)

Let $V_1 = \frac{1}{2} e_i^2$ and take the derivative along the trajectories (A-6)

$$\begin{aligned} \dot{V_1} &= e_i \dot{e_i} = e_i(-\gamma e_i - M_i(X) h(e_i) + \eta_i + d_i) \\ &\le -\gamma e_i^2 - |e_i| M_i(X) h(|e_i|) \\ &\quad + |e_i| (H^0 \| X \| + G^0) \\ &\le -\gamma e_i^2 - |e_i| M_i(X) \left\{ h(|e_i|) - \frac{H^0 \| X \| + G^0}{M_i(X)} \right\} \\ &\le -\gamma p_i^2 \quad \text{for } all \ e_i \in L_1^c \end{aligned} \qquad \text{(A-7)}$$

where L_1^c is the compliment of L_1

$$L_1 = \left\{ e_i : |e_i| < p_i, \ p_i = h^{-1} \left(\sup_X \frac{H^0 \| X \| + G^0}{M_i(X)} \right) \right\}$$

Equation (A-7) implies that as long as $|e_i| \ge p_i$, the magnitude of e_i is decreasing with a nonzero rate. Therefore, we conclude that e_i enters L_1 within a finite period of time, say $T_1{}' (V_1(0)/\gamma p_i^2)$ and stays inside L_1 thereafter.

(Part 2)

Let $V_2 = \frac{1}{2} \dot{e_i}^2$ and differentiate V_2

$$\begin{aligned} \dot{V_2} &= \dot{e_i} \ddot{e_i} \\ &= \dot{e_i}(-\gamma \dot{e_i} - M_i(X) h'(e_i) \dot{e_i} \\ &\quad - M_i{}'(X) \frac{d}{dt} \| X \| h(e_i) + \dot{F}^T X + F^T \dot{X} + \dot{g}) \end{aligned}$$

using the inequality

$$\left| \frac{d}{dt} \| X \| \right| \le \| \dot{X} \|$$

from (A-2)

$$\begin{aligned} \dot{V_2} &< -\gamma \dot{e_i}^2 - |\dot{e_i}|^2 M_i(X) h'(e_i) \\ &\quad + |\dot{e_i}| M_i{}'(X) \| \dot{X} \| h(|e_i|) \\ &\quad + |\dot{e_i}| H^1 \| X \| + |\dot{e_i}| H^0 \| \dot{X} \| + |\dot{e_i}| G^1 \end{aligned}$$

where

$$h'(e_i) = \frac{d}{de_i} h(e_i)$$

$$M_i{}'(X) = \frac{d}{dX} M_i(X)$$

From Part 1, after $t > T_1$

$$\begin{aligned} \dot{V_2} &\le -\gamma \dot{e_i}^2 - |\dot{e_i}|^2 M_i(X) \bar{h}' \\ &\quad + |\dot{e_i}| M_i{}'(X) \| \dot{X} \| \bar{h} \\ &\quad + |\dot{e_i}| H^1 \| X \| + |\dot{e_i}| H^0 \| \dot{X} \| + |\dot{e_i}| G^1 \end{aligned}$$

where

$$\bar{h}' = \inf_{e_i \in L_1} \left[\frac{d}{de_i} h(e_i) \right]$$

$$\bar{h} = \sup_{e_i \in L_1} h(e_i)$$

Since X satisfies regularity conditions from the signal generator dynamics (6), (7), and (A-2)

$$\begin{aligned} \dot{V_2} &\le -\gamma \dot{e_i}^2 \\ &\quad - |\dot{e_i}| M_i(X) \bar{h}' \left\{ |\dot{e_i}| - \frac{R_1 \| X \| + R_2}{\bar{h}' M_i(X)} \right\} \\ &\le -\gamma \varepsilon_i^2 \quad \text{for } all \ \dot{e_i} \in L_2^c \end{aligned}$$

where

$$L_2 = \left\{ \dot{e_i} : |\dot{e_i}| < \varepsilon_i, \ \varepsilon_i = \sup_X \frac{R_1 \| X \| + R_2}{\bar{h}' M_i(X)} \right\}$$

$$R_1 = a \bar{h} M_i{}'(X) + a H^0 + H^1$$

$$R_2 = b \bar{h} M_i{}'(X) + b H^0 + G^1$$

Now, we can conclude that $\dot{e_i}$ is driven into L_2 after a finite period of time, $T_2(T_1 + V_2(0)/\gamma \varepsilon_i^2)$, and stays there permanently. This concludes the proof.

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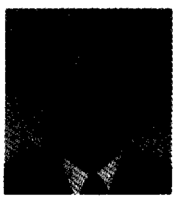
저 자 소 개



이 건 복 (李 建 馥)
1953년 8월 16일생. 1976년 2월 서울대 공대 기계공학과 졸업. 1978년 2월 한국과학기술원 기계공학과 졸업(석사). 1989년 3월 Univ. of California, Berkeley, U.S.A, Ph.D. 1978년 3월~1981년 3월 현대건설. 현재 숭실대 공대 기계공학과 부교수