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THE CHARACTERIZATIONS OF WEAKLY PRIME IDEAL ELEMENTS IN $\vee e$–SEMIGROUPS

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1. Introduction.

Recently, Kehayopulu([1]) proved the characterizations of a weakly prime ideals(Theorem 2, [1]) and weakly semiprime(Theorem 4, [1]) of a $\vee e$-semigroup S:

THEOREM A(THEOREM 2, [1]). *Let S be a $\vee e$-semigroup and t an ideal element of S. The following are equivalent:*

i) *t is weakly prime.*
ii) *If $a, b \in S$ such that $aeb \leq t$, then $a \leq t$ or $b \leq t$.*
iii) *If $a, b \in S$ such taht $r(l(a))r(l(a)) \leq t$, then $a \leq t$ or $b \leq t$.*
iv) *If x_1, x_2 are right ideal elements of S such that $x_1x_2 \leq t$, then $x_1 \leq t$ or $x_2 \leq t$.*
v) *If y_1, y_2 are left ideal elements of S such that $y_1y_2 \leq t$, then $y_1 \leq t$ or $y_2 \leq t$.*
vi) *If x is a right ideal element, y a left ideal element of S and $xy \leq t$, then $x \leq t$ or $y \leq t$.*

THEOREM B(THEOREM 4, [1]). *Let S be a $\vee e$-semigroup and t an ideal element of S. The following are equivalent:*

i) *t is weakly semiprime.*
ii) *For every $a \in S$ such that $aea \leq t$, we have $a \leq t$,*
iii) *for every $a \in S$ such that $(r(l(a)))^2 \leq t$, we have $a \leq t$.*
iv) *For every right ideal element x of S such that $x^2 \leq t$, we have $a \leq t$.*

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v) *For every left ideal elements y of S, such that $y^2 \leq t$, we have* $a \leq t$.

The aim of this paper is to give some improvement of Theorem A. By some modifications of the proofs of our main result, we have Theorem B, easily.

DEFINITION 1[2, 3]. *A $\vee e$-semigroup S is a semilattice under $\vee$ with the greatest element e and at the same time a semigroup such that*

$$a(b \vee c) = ab \vee ac \quad \text{and} \quad (a \vee b)c = ac \vee ba.$$

NOTATION. *In $\vee e$-semigroup S,*

$$l(a) = ea \vee a \ \text{ and } \ r(a) = ae \vee a.$$

DEFINITION 2[4]. *An element t of a $\vee e$-semigroup S is said to be a left(resp. right) ideal element if $et \leq t$ (resp. $te \leq t$) for the greast element $e \in S$.*

An element t of a $\vee$-semigroup S is said to be an ideal element if it is a right and left ideal element.

DEFINITION 3[2, 3]. *An element t of a $\vee e$-semigroup S is said to be weakly(resp. left weakly, right weakly) prime if for any pair a, b of ideal (resp. left ideal, right ideal) elements of S such that $ab \leq t$, then we have $a \leq t$ or $b \leq t$.*

DEFINITION 4[1, 3]. *An element t of a $\vee e$-semigroup S is said to be weakly semiprime if for any ideal element a of S such that $a^2 \leq t$, we have $a \leq t$.*

THEOREM 1. *Let S be a $\vee e$-semigroup and t be an ideal element of S. The following are equivalent:*

(1) *t is weakly prime.*

(2) *If $a, b \in S$ such that $aeb \leq t$, then $a \leq t$ or $b \leq t$.*

(3) *If* $a, b \in S$ *such taht* $r(l(a))r(l(b)) \leq t$, *then* $a \leq t$ *or* $b \leq t$.
(4) *If* x *is a right ideal element of* S *such that* $xy \leq t$ *for any element* $y \in S$, *then* $x \leq t$ *or* $y \leq t$.
(5) *If* y *is a left ideal element of* S *such that* $xy \leq t$ *for any element* $x \in S$, *then* $x \leq t$ *or* $y \leq t$.
(6) *If* x_1, x_2 *is right ideal elements of* S *such that* $xy \leq t$, *then* $x_1 \leq t$ *or* $x_2 \leq t$.
(7) *If* y_1, y_2 *is a left ideal elements of* S *such that* $xy \leq t$, *then* $y_1 \leq t$ *or* $y_2 \leq t$.
(8) *If* x *is a right ideal element and* y *is a left ideal elemnt of* S *such that* $xy \leq t$, *then* $x \leq t$ *or* $y \leq t$.

Proof. (1) $\Longrightarrow$ (2). Assume that $aeb \leq t$ for any two elements a and b in S. Since t is an ideal element, we have

$$(eae)(ebe) \leq e(aeb)e \leq ete \leq t.$$

Since eae and ebe are ideal elements, we get

$$eae \leq t \text{ or } ebe \leq t.$$

If $eae \leq t$, then

$$\begin{aligned}(r(l(a))^3 &= (eae \vee ae \vee ea \vee a)^3 \\ &\leq e(eae \vee ae \vee ea \vee a)e \\ &\leq eae \leq t.\end{aligned}$$

Since t is weakly prime, $r(l(a)) \leq t$ or $r(l(a))^2 \leq t$. In any case, $r(l(a)) \leq t$ so $a \leq r(l(a)) \leq t$.

We can easily prove that $ebe \leq t$ implies $b \leq t$ by similar method to above.

(2) $\Longrightarrow$ (3). Suppose that $r(l(a))r(l(b)) \leq t$. Then we have

$$\begin{aligned} aeb \leq & eae^2be \vee eaebe \vee eae^2b \vee eaeb \\ & \vee ae^2be \vee aebe \vee a^2b \vee aeb \\ & \vee eaebe \vee eabe \vee eaeb \vee eab \\ & \vee aebe \vee abe \vee aeb \vee ab \\ = & (eae \vee ae \vee ea \vee a)(ebe \vee be \vee eb \vee b) \\ = & r(l(a))r(l(b)) \leq t. \end{aligned}$$

By hypothesis, $a \leq t$ or $b \leq t$.

(3) $\Longrightarrow$ (4). Let x be a right ideal element of S such that $xy \leq t$ for any $y \in S$. Then we have

$$\begin{aligned} r(l(x))r(l(y)) =& (exe \vee xe \vee ex \vee x)(eye \vee ye \vee ey \vee y) \\ =& exe^2ye \vee exeye \vee exe^2y \vee exey \\ & \vee xe^2ye \vee xeye \vee xe^2y \vee xey \\ & \vee exeye \vee exye \vee exey \vee exy \\ & \vee xeye \vee xye \vee xey \vee xy \\ \leq& exye \vee exy \vee xye \vee xy \\ \leq& ete \vee et \vee te \vee t \\ \leq& t, \end{aligned}$$

since x is a right ideal element and t an ideal element. By hypothesis $x \leq t$ or $y \leq t$.

(3) $\Longrightarrow$ (5). Let y be a left ideal element of S such that $xy \leq t$ for any $x \in S$. Then as the froof of above, we get

$$\begin{aligned} r(l(x))r(l(y)) =& (exe \vee xe \vee ex \vee x)(eye \vee ye \vee ey \vee y) \\ =& exe^2ye \vee exeye \vee exe^2y \vee exey \\ & \vee xe^2ye \vee xeye \vee xe^2y \vee xey \\ & \vee exeye \vee exye \vee exey \vee exy \\ & \vee xeye \vee xye \vee xey \vee xy \\ \leq& exye \vee exy \vee xye \vee xy \\ \leq& ete \vee et \vee te \vee t \\ \leq& t, \end{aligned}$$

since y is a left ideal element and t an ideal element. By hypothesis $x \leq t$ or $y \leq t$.

(4) $\Longrightarrow$ (6) and (5) $\Longrightarrow$ (7). It is obvious.

(6) $\Longrightarrow$ (8), (7) $\Longrightarrow$ (8) and (8) $\Longrightarrow$ (1). It is obvious from Theorem A.

By some modifications of the proof of the above Theorem, we have Theorem B(Theorem 4, [1]) as Corollary to Theorem 1.

COROLLARY 2. *Let S be a $\vee e$-semigroup and t be an ideal of S. The following are equivalent:*

(1) *t is weakly semiprime.*

(2) *If $aea \leq t$ for any $a \in S$, then $a \leq t$.*

(3) *If $(r(l(a)))^2 \leq t$ for any $a \in S$, then $a \leq t$.*

(4) *If $x^2 \leq t$ for any right ideal element $x \in S$, then $x \leq t$.*

(5) *If $y^2 \leq t$ for any left ideal element $y \in S$, then $y \leq t$.*

References

1. N. Kehayopulu, *On weakly prime, weakly semiprime, prime ideal elements in poe-semigroups*, Math Japon., **35(6)** (1990), 381-389.
2. N. Kehayopulu, *On minimal quasi-ideal elements in poe-semigroups*, Math. Japon., **34(5)** (1989), 767-774
3. N. Kehayopulu, *On intra-regular $\vee e$-semigroups*, Semigroup Forum, **19** (1980), 111-121.
4. N. Kehayopulu, *On completely regular poe-semigroups*, Math Japon., **37(1)** (1992), 123-130

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