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STABILITY OF ASTEROID MOTIONS

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ABSTRACT

In this paper it is explained how most of asteroids can avoid very close approach to Jupiter, to the earth for earth orbit crossing asteroids, and to Neptune for Kuiper-belt asteroids by mechanisms which work also for Neptune-Pluto system. In fact the mutual distance of the planets cannot become very small as the critical argument librates around 180^0 because of 2:3 mean motion resonance and the argument of perihelion of Pluto librates around 90^0. And it is found that among nearly 40 Kuiper-belt asteroids discovered in recent years 40% have orbits similar to Pluto. For main-belt asteroids the distribution with respect to the semi-major axes has peculiar characteristics and the author tries to explain how their peaks and gaps are created. It is also found that 30% of 80 earth orbit crossing asteroids which have minimum perihelion distances less than 1.04AU have no chance to collide with the earth. Still 30% of them have a few probability to collide with the earth as they have dynamical characteristics of short-periodic comets.

Key Words : solar system, asteroids, motion

I. SECULAR PERTURBATIONS

According to theories of secular perturbations of asteroids the semi-major axes, a, are constant after averaging the disturbing function with respect to fast moving angular variables. And when it is assumed that both the eccentricity, e, and the inclination, i, are very small, by adopting the variables $\xi = e\cos\varpi$ and $\eta = e\sin\varpi$ as well as $p = \sin i\cos\Omega$ and $q = \sin i\sin\Omega$ with the longitudes of the perihelion and the ascending node, ϖ and Ω, the differential equations of motion are reduced to two sets of linear systems with constant coefficients, one for ξ and η and one for p and q. The solutions are expressed as sums of free oscillations and forced ones due to the secular variations of orbital elements of disturbing planets. Therefore, it is concluded that both eccentricity and inclination cannot take any very large values.

However, when the eccentricity and/or inclination are not small, the equations are not linear and there are interaction terms between (ξ, η)- and (p, q)-sets. However, when it is assumed that all the disturbing planets are moving along circular orbits on the same plane, the equations are expressed as a canonical set of one degree of freedom with an energy integral.

In fact when Delaunay variables,

$$L = \sqrt{\mu a}, \quad G = L\sqrt{1-e^2}, \quad H = G\cos i,$$
$$l, \qquad g, \qquad h,$$

are introduced, they satisfy the equations,

$$\frac{dL}{dt} = \frac{\partial F}{\partial l}, \quad \frac{dG}{dt} = \frac{\partial F}{\partial g}, \quad \frac{dH}{dt} = \frac{\partial F}{\partial h},$$
$$\frac{dl}{dt} = -\frac{\partial F}{\partial L}, \quad \frac{dg}{dt} = -\frac{\partial F}{\partial G}, \quad \frac{dh}{dt} = -\frac{\partial F}{\partial H},$$

with the Hamiltonian, F,

$$F = \frac{\mu^2}{2L^2} + \sum m'/\Delta, \tag{1}$$

where Δ is the mutual distance with one of the planets and m' is its mass.

Because of the assumption the Hamiltonian is symmetric around the z-axis after the averaging with respect to the mean anomaly, l, of the asteroid and the mean longitudes, λ', of the disturbing planets. Therefore, the longitude of the ascending node, h, does not appear in the Hamiltonian. That is, H, the z-component of the angular momentum, becomes constant. Also as l has been eliminated, L is constant. Then the equations are reduced to a system of one degree of freedom with the integral F=constant.

Then with the two parameters, L and H, the variable, G, can be expressed as periodic functions of g, the argument of perihelion, in fact of $2g$. Practically, instead of L, H and G,

$$a, \quad \Theta = \sqrt{1-e^2}\cos i, \quad X = \sqrt{1-e^2}, \tag{2}$$

are used and the Hamiltonian is replaced by,

$$F^* = \frac{1}{(2\pi)^2}\sum\int_0^{2\pi}\int_0^{2\pi}\frac{m'}{\Delta}\,dl\,d\lambda'. \tag{3}$$

For given values of a and Θ the values of F^* are computed for various sets of X and $2g$ and equi-F^*-value curves, along which X as well as e and i vary as functions of $2g$, can be drawn. In fact X takes values between 1 and Θ which correspond, respectively, to $e = 0$ and $i = \arccos\Theta$ and to $e = \sqrt{1-\Theta^2}$ and $i = 0$. When Θ is nearly equal to 1, any equi-F^*-curve is almost a straight line parallel to $2g$-axis. However, when

Θ is well below 1, the curves are no longer horizontal lines, and they show that the eccentricity is minimum at $2g = 0$ and 2π and maximum at $2g = \pi$ and vice versa for the inclination.

Particularly, when Θ is smaller than roughly 0.8, a libration region appears, in which the argument of perihelion cannot make any complete revolution but librates around $2g = \pi$ with a certain amplitude, that is, the argument of perihelion never takes the value of 0 and π. For this case both the maximum and minimum values of the eccentricity and the inclination are at $2g = \pi$.

Such properties of the secular perturbations are very important for most of asteroids with high eccentricity and inclination to avoid very close approach to Jupiter as it is explained later (Kozai, 1962; 1979).

II. KUIPER-BELT ASTEROIDS

Mechanisms to avoid very close approach to major planets work also for Pluto. In fact it is known that for Pluto the eccentricity and the inclination are high, 0.249 and $15.^{0}6$ respectively, whereas for Neptune they are small, 0.009 and $0.^{0}7$. And as the semi-major axes of the planets are 30.1 and 39.5AU the perihelion distance of Pluto is less than the mean orbital radius of Neptune. And as the orbital periods are 164.8 and 247.8 years, respectively, they are nearly in the ratio of 2 to 3.

By numerical integrations of the equations of motion it is known that the critical argument, $\theta = 2\lambda - 3\lambda' + \varpi'$, where λ and λ' are the mean longitudes of Neptune and Pluto and ϖ is the longitude of perihelion of Pluto, librate around 180^{0} with the period as long as 20,000 years so that any conjunction takes place only near the aphelion of Pluto as when $\theta = 0$ and $\lambda = \lambda'$, $\lambda' = \varpi' + 180^{0}$.

Therefore, it is concluded that the mutual distance of the two planets cannot take any very small value. Also for Pluto the argument of perihelion librates around 90^{0} so that any conjunction cannot take place near the orbital plane of Neptune as the conjunction takes place always near the aphelion of Pluto and the aphelion never comes on the orbital plane of Neptune. This is another mechanism for Pluto to avoid very close approach to Neptune although its perihelion is inside the orbit of Neptune.

Since the asteroid, 1992 QB1, was discovered in August, 1992, nearly 40 asteroids of size of 100km or less have been found in Kuiper-belt which is between 30 to 50AU heliocentric distance. For 1992 QB1 the semi-major axis is 44.4AU, the eccentricity is 0.11 and the inclination is $2.^{0}2$. In fact as the mass of Pluto is too small to disturb the orbit of Neptune considerably to fit their astrometric observations and most of short-periodic comets with periods less than 20 years have aphelion distances in Kuiper-belt. Therefore, Kuiper proposed that there should be many parent bodies of comets in the region now called Kuiper-belt.

And it is known that 40% of Kuiper-belt asteroids have orbits similar to Pluto, which is now thought to be the largest member in the Kuiper-belt and that such asteroids can avoid very close approach to Neptune by the same mechanisms as Pluto. And for other astroids also they can avoid very close approach to Neptune. Therefore, it is not yet well explained how such asteroids turn out to be short-periodic comets which have dynamical characteristics different from those of most of asteroids as it is explained later.

III. TROJAN TO HILDA GROUPS

The distribution of asteroids with respect to the semi-major axes is known to be not uniform at all as there are several remarkable gaps and groups, one of which is called Trojan group moving along orbits with the semi-major axes around 5.2AU, the value for Jupiter. However, they never make any very close approach to Jupiter as they move only near equilateral triangular points with the sun and Jupiter.

And the difference of the mean longitudes librates around $\pm 60^{0}$ with the period of 200 years by changing the semi-major axes. Therefore, the semi-major axes of Trojan asteroids scatter between 5.0 and 5.4AU and the mean motions also take values around 300" per day, that for Jupiter. And as their eccentricities are not very large whereas the inclinations are generally large there is not any chance for them to make very close approach each other.

There are not so many asteroids, for which the semi-major axes are between 4.1 and 5.0AU. However, there are a few which have mean motions nearly equal to $7n'/6$ (a=4.7AU) and $4n'/3$ (a=4.3AU) where n' is Jupiter's mean motion. However, as for them the critical arguments librate around 0 so that any conjunction with Jupiter takes place only the asteroids are near their perihelia they never make very close approach to Jupiter.

And around $a = 4$AU there are about 70 asteroids called Hilda group and for them the mean motions are nearly $3n'/2$. And their eccentricities are not large and the critical arguments librate around 0 except for those which have very small eccentricities and, therefore, their aphelion distances are not so large.

IV. MAIN-BELT ASTEROIDS

As the semi-major axis is decreased beyond the Hilda group the number of asteroids is increased gradually. However, there is a gap for distribution near 3.3AU corresponding to the mean motion of $2n'$. Inwards this gap there are very many asteroids, however, there are gaps at regions with the mean motions of $5n'/2(a = 2.75\text{AU})$, $3n'(a = 2.5\text{AU})$, $4n'(a = 2.1\text{AU})$ and others. In fact there are very many asteroids between $a = 2.1$AU and $a = 3.2$AU, the main-belt of the asteroids.

Table 1. Asteroids with Perihelion Distances larger than 5.2AU

No.	Q_M	a	i_m	i_M	e_m	e_M
1373	5.30	3.40	29	40	0.32	0.56
1922	5.41	3.25	18	40	0.36	0.66
5164	5.54	3.66	17	32	0.25	0.51
5370	0.60	3.35	7	30	0.54	0.67

In the main-belt there are about 100 asteroids, for which the maximum aphelion distances, which correspond to the maximum eccentricities as functions of the argument of perihelion, are larger than 4.0AU and for four of them they are larger than 5.2AU and the data are given for the four asteroids in Table 1.

In the Table the maximum value of the aphelion distance, Q_M, the semi-major axis, a, both in AU, the minimum and maximum values of the inclination referred to the invariable plane, i_m, i_M in degrees, and the minimum and maximum eccentricities, e_m, e_M, are given for each asteroid. Of the four asteroids the argument of perihelion librates around 90^0 for (1373). It should be remembered that the maximum aphelion distances corresponding to the maximum eccentricities are those when the arguments of perihelion are at 90^0 and 270^0 where the mutual distances between the asteroids and Jupiter are not so small because of the inclinations.

Since the heliocentric distance is between 2.34 and 3.05AU when the asteroid (1373) crosses the invariable plane which is almost identical to the orbital plane of Jupiter it never approaches Jupiter very closely even though the aphelion distance is as large as 5.30AU. For the other three asteroids when the arguments of perihelion are at 0 and 180^0 the aphelion distances are, 4.43, 4.56 and 5.14AU, respectively. However, as the mean motion is nearly equal to $2n'$ for (5370), the critical argument librates so that the asteroid cannot make conjunction at its aphelion so that the mutual distance with Jupiter cannot be very small.

In fact there are several asteroids near the gaps of the distribution which have large values of the eccentricities so that their maximum aphelion distances are nearly 5AU. However, when the critical argument librates more eccentric orbit is more stable in the sense that the asteroid can avoid very close approach more easily as any conjunction takes place only near their perihelia, for which heliocentric distances are smaller for larger eccentricities. It is true not only for 2:1 resonance gap but also for 3:1 (a=2.5AU) and 5:2 (a = 2.75AU) resonance cases where any conjunction takes plance always near the midpoint between its perihelion and aphelion.

According to author's idea the gaps at resonance regions were created by the following scenario: Near mean motion resonance region after interactions with Jupiter and other planets orbits become more stable as a critical argument starts to librate and the eccentricity becomes large. However, since the eccentricities become large for many asteroids they collide with other asteroids more easily and, therefore, the gaps were created there. In fact near the gaps more faint and smaller asteroids are discovered.

On the other hand in more distant regions, where Trojan to Hilda groups move, the resonant asteroids are more stable in the sense that in non-resonant regions there is more chance for asteroids with high eccentricities to approach very closely to Jupiter even though the eccentricities change as the arguments of perihelion as the aphelion distance for (1373) in Table 1 shows. Therefore, non-resonant asteroids have moved to more stable regions like Trojan, Hilda and other resonant places.

Table 2. Some of Earth Crossing Asteroids with Encke's Comet

No.	q_m	a	i_m	i_M	e_m	e_M
3200	0.13	1.27	15	45	0.81	0.90
2102	0.21	1.28	37	63	0.22	0.84
2101	0.44	1.87	2	5	0.76	0.76
1866	0.73	1.89	35	50	0.10	0.61
3040	0.87	1.84	36	46	0.17	0.53
4179	0.90	2.51	1	2	0.64	0.64
Encke	0.32	2.21	4	12	0.85	0.86

V. EARTH CROSSING ASTEROIDS

There are not so many asteroids with the semi-major axis less than 2.0AU as in the main belt, however, there are a group of asteroids near a = 1.9AU called Hungaria family and asteroids with smaller semi-major axes were discovered and the smallest semi-major axis known is 0.83AU. And by computing the maximum eccentricities for 7,000 numbered asteroids it is found that 80 of them have minimum perihelion distances less than 1.04AU. For six of them the semi-major axes are less than 1AU and for twenty-five they are larger than 2AU. Among them (3552) has the semi-major axis of 4.23AU, which is the largest and the only one larger than 3AU. Almost all of them the eccentricities and/or inclinations are high.

In Table 2 some of the 80 asteroids are listed with the minimum perihelion distance, q_m, the semi-major axis, a, both in AU, the minimum and maximum inclinations referred to the invariable plane in degrees, i_m, i_M, and the minimum and maximum eccentricities. Also for reference the data for Encke's comet are given.

Among all (3200) has the smallest minimum and present perihelion distances, both less than 0.15AU. Still it is not possible for this asteroid to collide with the earth in near future, as when it crosses the invariable plane its heliocentric distance is computed to be very far from 1AU. However, when the argument of perihelion which is now 321^0 will become 330^0 the orbit will intersect the invariable plane at a point with the heliocentric distance of nearly 1AU, that is, then there is a few chance of collision with the earth. And it is known that a meteor shower is associated with this asteroid.

Of the 80 there are seven asteroids for which the arguments of perihelion librate and two of them, (2102) and (3040), are listed here. For (2102) the asteroid crosses the invariable plane at heliocentric distances of 0.38 and 1.23AU, respectively, when the eccentricity is maximum and minimum. Therefore, it is possible that it will cross the invariable plane near 1AU heliocentric distance. However, since the inclination is high enough the probability seems to be very low.

For (3040) as it is computed that the heliocentric distance when the asteroid crosses the invariable plane is between 1.33 and 1.79AU the probability for collision with the earth seems to be zero. There are three such asteroids among the seven argument of perihelion librators.

Even though the arguments of perihelion do not librate, other about 20 asteroids never intersect the invariable plane with the heliocentric distance less than 1.02AU. One of them is (1866) in the Table, and for this asteroid the heliocentric distance when it crosses the invariable plane and the eccentricity is maximum is 1.10AU and when the eccentricity is minimum it is 1.70AU. By changing the eccentricity in such a way the 20 asteroids can avoid any collision with the earth.

On the hand there are about 30 asteroids of other category, like (2101) and (4179) in the Table, for which their inclinations are very small, less than 10^0, whereas the eccentricities are large. The author claims that this is a typical dynamical property of short-periodic comets like Encke's comet in the Table. Since the eccentricity cannot change so much whereas the inclinations are small they cannot avoid very close approach to the earth. Therefore, they have chance to collide with the earth.

It is concluded that about 30% of the 80 asteroids which have minimum perihelion distances less than 1.04AU have no chance to collide with the earth. However, one third of them have relatively high probability to collide as their dynamical property is similar to that of short-periodic comets. And another third of them have a possibility although it is extremely small.

VI. Conclusion

The author would like to claim that most of asteroids are now in stable orbits after interactions with Jupiter and other planets in many years. Therefore, they never approach to Jupiter. However, there are still several asteroids which have dynamical properties similar to those of short-periodic comets having usually much chance to approach very closely with Jupiter as the comet, Shoemaker-Levi 9, did in 1994.

REFERENCES

Kozai, Y. 1962. Astron. J., 67, 591-598.

Kozai, Y. 1979. In Dynamics of the Solar System, ed. by R.L.Duncombe, 231-237.