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ON THE DERIVATION OF THE STRATIFICATION OF SOLAR VECTOR MAGNETIC FIELDS BY STOKES PROFILE ANALYSIS*

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I. INTRODUCTION

Before the present method makes its appearence, the most and the most effective mehtods used in the literature depend on the line formation depth theory, which determines the so called '*formation depth*' for one or more selected spectral lines, in the language of the contribution function or response function. The successful one is presented by Ruiz Cobo and del Toro Iniesta (1992).

Our method is different. It consists of four operations which have different actions on the profiles downwards depth by depth. Executing the first one on the surface profiles, we obtain the information of surface parameters of the vector magnetic field **H**, the damping constant a, the Doppler width $\Delta\lambda_D$, the line source function S_l as well as the velocity along the line of sight v_{los}; By the second operation, the opacity ratio $r_0 = \kappa_c/\kappa_0$ at the surface and the continuum source function S_c formulated as in Eq.(6) below is recovered. The third operation computes the Stokes profiles unobservable under the surface while the fourth, instead of the second, to extract r_0 below the surface. Then carrying out these last three operations cyclically to a certain depth, one can achive the goal. Thus the location of the extracted parameters is automatically obtained, no line formation depth theory is needed. In the following, we deal with non-LTE case, which makes the method with relation to the theory very complicated. The description of each operation is given in the following.

II. THEORY

The polarized radiative transfer equations are

$$\frac{d\mathbf{I}}{d\tau} = \mathbf{KI} - \mathbf{j}, \tag{1}$$

without atomic polarization or scattering (*cf.* Rees *et al.*,1989, hereafter RMD),where $\mathbf{I} = (I, Q, U, V)^\dagger$ is the Stokes vector, and matrices $\mathbf{K}, \mathbf{J}$ describe the absorption and magneto-optical effect, and ejection, respectively. The expressions of these matrices, whose elements contain all the free parameters, can be found in Rees *et al.*

(a) Inference of $\mathbf{H}(H, \gamma, \chi)$, v_{los} and Determination of a, $\Delta\lambda_D$, S_l at Every Grid Point of Depth

Assuming that the shifts of the positions of the extrema in Stokes profiles along the depth, due to the gradients of the profile parameters , are discrete,*i.e*, keep constant within each span of suitablely selected grid depths, respectively

$$\frac{dS'}{d\tau}|_{v=v_{S,extrem}} = 0. \quad S = I, Q, U, V \tag{2}$$

Where the superscript ' $'$ ' means the derivative with respect to v (the wavelength displacement from line center normalized by the Doppler width). Thus from Eq.(1)

$$(\mathbf{K}'\mathbf{I} + \mathbf{KI}' - \mathbf{j}')_{v=v_{s,extrem}} = 0. \tag{3}$$

Such equations can easily be solved by Newtonian iteration method.

(b) Determination of the Ratio r_0 at the Surface and the Formulated Continuum Source Function

The ratio r_0, eliminated in Eqs.(3), is needed to be known for further steps. Another necessary quantity is S_l. Following Lites *et al.* (1988), it is formulated as

$$S_c = B_0 + B_1\tau_0, \tag{4}$$

whereas

$$d\tau_0 = -\kappa_0 dz. \tag{5}$$

To achieve the purpose, the analytic solutions to the transfer equations (also see Lites *et al.*, 1988, set μ=1, and hereafter) are employed

$$\mathbf{I}(0) = \{B_0\mathbf{1} + B_1(r_0\mathbf{1} + \tilde{\mathbf{\Phi}})^{-1} - A_1[(r_0+\epsilon_1)\mathbf{1} + \tilde{\mathbf{\Phi}}]^{-1}\tilde{\mathbf{\Phi}}\}\mathbf{e}_0. \tag{6}$$

For simplicity, we keep only one exponential term in their expression of S_l.

(c) Derivation of Stokes Profiles of Any Depth below the Surface

With the inferred vector field and thermodynamics parameters as well as velocity along the line of sight at $\tau_0(k)$(k is the grid number of the optical depth), the right-handed side of the transfer equations (1) can be easily calculated. Then utilizing the Taylor expansion to the first-order of τ_0

$$\mathbf{I}(\tau_0(k) + \Delta\tau_0(k)) = \mathbf{I}(\tau_0(k)) + \frac{d\mathbf{I}(\tau_0(k))}{d\tau_0}\Delta\tau_0(k) + \ldots, \tag{7}$$

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Table 1. Inference of the Stratification Above f the Depth $\tau_0 = 23.80$

τ_0	$a(\times 10^{-3})$	$\Delta\lambda_D$	$S_l(\times 10)$	$S_c(\times 10^{-6})$	$r_0(\times 10^{-5})$
	H	γ	χ	$v_{los}(\times 10^{-3})$	$\lvert\delta\chi\rvert(\times 10^{-11})$
1.180E-2	4.8(−18.01%)	41.00(-31.80%)	2.4(38.16%)	1.46(125%)	2.00(38%)
	906.05(−0.56%)	120.21(0.21)	225.76(1.37)	3.28	4.28
3.862E-2	4.8(−19.63%)	41.00(-30.43%)	2.4(37.37%)	1.49(129%)	2.01(56%)
	896.43(−4.11%)	120.41(5.41)	225.77(13.27)	3.24	4.04
7.654E-2	4.8(−20.31%)	41.00(-29.84%)	2.4(36.36%)	1.53(136%)	1.95
	883.25(−6.88%)	120.69(10.69)	225.78(20.09)	55.1	4.32
3.989E-1	4.8(−21.72%)	36.00(-37.29%)	2.9(54.74%)	1.80(177%)	20.3
	1049.21(6.88%)	122.88(17.88)	154.10(-35.09)	2.24	4.89
8.993E-1	4.8(−24.05%)	27.00(-51.54%)	2.9(42.15%)	2.23(244%)	14.2
	1057.93(6.01%)	124.91(24.91)	260.40(79.34)	4.09	2.53
1.705	6.8(1.56%)	26.00(-50.57%)	2.9(26.74%)	2.92(350%)	21.0
	1252.29(8.57%)	123.67(105.41)	80.62(-94.04)	3.24	2.67
3.805	6.8(−11.86%)	35.00(-23.41%)	3.4(20.39%)	4.71(626%)	125.0
	1180.66(−4.29%)	129.45(111.45)	145.09(-21.54)	2340	965.0
6.22	4.8(−43.67%)	36.00(-13.08%)	2.9(−11.82%)	6.78(946%)	231.0
	1139.67(−11.15%)	71.56(55.06)	262.03(100.31)	5480	576.0
9.69	6.8(−26.70%)	46.00(55.06%)	1.4(−62.72%)	9.74(1403%)	809.0
	1439.50(8.46%)	46.76(31.76)	160.63(3.34)	−5680	2680.
23.80	13.8(25.45%)	67.00(107.88%)	4.9(8.88%)	21.79(%)	5480.
	2043.10(44.18%)	120.81(107.31)	176.93(28.63)	−5730	9350.

* Where H in unit of Gauss,γ, χ in degrees; a is the damping normalized by $\Delta\lambda_D$ in $m\AA$; $S_l, |\delta\chi|$ in $ergs/cm^2\ sr\ Hz\ s$, v_{los} in km/s. The numbers in brackets indicate the error. $|\delta\chi|$ represents the minimum of the merit function in corresponding iteration.

one can easily obtain the profiles at depth $\tau_0(k+1)(= \tau_0(k) + \Delta\tau_0(k), \Delta\tau_0(k)$ is the grid span of the depth). However, each $\Delta\tau_0(k)$ should satisfy

$$\Delta\tau_0(k) \ll 2|\frac{d\mathbf{I}(\tau_0(k))}{d\tau_0} / \frac{d^2\mathbf{I}(\tau_0(k))}{d\tau_0^2}|. \tag{8}$$

(d) Determination of r_0 Below Surface

Operation two can only obtain approximate r_0 at the surface. Under the surface the magnitude can be got by the numerical solution which links the Stokes profiles at two adjacent layers. We make use of the DELO method (see Rees *et al*,1989) solution

$$\mathbf{I}(\tau_k) = P_k + I_k\mathbf{I}(\tau_{k+1}), \tag{9}$$

to fulfil the operation by directly solving the equation. The expressions of matrices (P_k of 4×1 and I_k of 4×4) on the right-handed side containing the known and the free parameter r_0 can be also found in RMD.

III. CONCLUSION

As usual, an inversion of the synthetic profiles has been done as the test for the method (See Table 1). We adopted the model atmosphere of sunspot umbra used by RMD. The line is Mg I 5172.7, its Lande factor $g_{eff} = 1.75$. The **H** structure assumed is set as fan-model like, and that of v_{los} is assumed with small magnitude. We employ the DELO method to produce the synthetic Stokes profiles. From the inversion, we conclude:

1. Even under the bad condition of the thermodynamics and velocity determined, the present method can extract the stratification of the vector magnetic fields, especially that of the field strength if the gradients of all the paramaters are not so large and the v_{los} is small.

2. The method is of self-consistence and independent of model atmosphere, and more significantly, of the line formation theory.

3. The error in computed profiles within a reasonable region seems not to influence significantly the recovered stratification of the field strengths, but really does that of the field angles and limits the depth distance the process can go.

4. If the determined stratification of the thermodynamics and the velocity are better, that of the vector magnetic fields will be inferred with better accuracy. The two depend on each other because the profiles are determined by both of them. And such an improving work will be the future work.

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