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PRIMORDIAL BLACKHOLE AS A SEED FOR THE COSMIC MAGNETIC FIELD

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ABSTRACT

We present a model that rotating primordial blackholes(PBHs) produced at the end of inflation generate the random, non-oriented primordial magnetic field. PBHs are copiously produced as the Universe completes the cosmic phase transition via bubble nucleation and tunneling processes in the extended inflation hypothesis. The PBHs produced acquire angular momentum through the mutual tidal gravitational interaction. For PBHs of mass less than $10^{13}g$, one can show that the evaporation (photon) luminosity of PBHs exceeds the Eddington limit. Thus throughout the lifetime of the rotating PBH, radiation flow from the central blackhole along the Kerr-geodesic exerts torque to ambient plasma. In the process similar to the Bierman's battery mechanism electron current reaching up to the horizon scale is induced. For PBHs of Grand Unified Theories extended inflation with the symmetry breaking temperature of $T_{GUT} \sim 10^{10}$ GeV, which evaporate near decoupling, we find that they generate random, non-oriented magnetic fields of $\sim 10^{-11}G$ on the last-scattering surface on (the present comoving) scales of $\sim O(10)$ Mpc.

Key Words : cosmology, primordial magnetic field, large-scale structure

I. INTRODUCTION

The Universe is endowed with magnetic fields. Clarifying field configuration and strength are hardly an easy task. However, we have ample evidence that magnetic field plays an important role in understanding the major structure of the Universe. We well aware that magnetic field of strength $\sim 10^{12}G$ plays an important role in lighting pulsars. In interstellar space magnetic field of strength $\sim \mu G$ is present. The strength of magnetic fields exceeding galactic dimensions varies from a few μG in galactic gaseous corona (Welter, Perry & Kranberg 1984) to $\sim O(1/10)\mu G$ on scales of galactic cluster (Lawler & Dennison 1982; Kim et al. 1990). On scales greater than this, various upper limits of $10^{-9}G \sim 10^{-11}G$ are available at the moment (Sofue, Fujimoto & Wielebinsky 1986; Vallee 1983).

The origin and evolution of the relic magnetic field has been a subject of active research in astronomy. Magnetic fields can be generated locally: via supernovae explosion, plasma transport by (inter) stellar winds, stellar plasma ejection, or stellar rotation and convection. A gigantic supernova explosion is a good source of turbulence, vortices in the interstellar medium, and interstellar plasma winds driven by the shockwave can generate magnetic fields of various configuration and strengths. Historically the more sound mechanism is the Bierman's battery effect (Bierman 1950; Kemp 1982). It is a model based on stellar rotation and thermoelectric conduction of ionized plasma.

These mechanisms however have limitation in applications; notably, they can not explain large-scale uniform magnetic fields of several spiral galaxies (Ruzmaikin, Shukorov & Sololoff 1988). Instead some astronomers believe that observed bisymmetric magnetic lines of galaxies $M51$, $M87$, $NGC6946$, and our own Galaxy could be evidence for the existence of the primordial field in the Universe. (For a review of the large-scale magnetic field in spiral galaxies, see Sofue et al. 1986). The point is that there is a direct resemblance between the observed bisymmetric magnetic lines of the galaxies and a uniform magnetic field which has been twisted and stretched by galactic rotation (Ruzmaikin et al. 1988; Sawa & Fujimoto 1980). If this indeed holds truth, then observed large scale magnetic fields $B \sim 10^{-6}G$ may not be significantly different from the relic field (Piddington 1964; Ohki, Fujimoto & Hitotuyanag 1964). Another argument comes from the observation of microgauss fields in damped Lyα clouds at high redshifts, which indicates that magnetic fields were almost certainly present in the protogalactic environment (Wolfe, Lanzetta & Oren 1992). Although these arguments are not without criticism, it would be an extremely interesting issue to find a mechanism for the origin of the primordial magnetic field.

Under this spirit, we sought a new mechanism for the origin of primordial magnetic fields which can seed present galactic and stellar fields. It is clear that if primordial fields are produced to be present on the last-scattering surface, then they survive to the epoch when large-scale non-linear structures start to form. This is due to the fact that the Universe is a remarkably good conductor. The good conductivity implies that throughout most epochs of the Universe, the diffusion of the magnetic field on present comoving scales greater than a few astronomical unit is utterly negligible (Turner & Widrow 1988; Cheng & Olinto 1994).

Many models for the origin of cosmic magnetic field have been suggested. Most proposed models though have a common characteristic that in order to generate magnetic field of significant amplitude coherent on large scales, they rely heavily on inflation models. That is because inflation naturally provides a mechanism which generates perturbations on scales greater than the particle horizon. Thus these models are more akin to particle physics than classical electrodynamics in that they exploit cosmological phase transitions, or its byproducts such as domain walls or cosmic strings (Tuner & Widrow 1988; Witten 1985; Ostriker, Thompson & Witten 1980). The cosmological phase transitions referred here are the Grand Unified Theories(GUTs) inflation (Guth 1981; La & Steinhardt 1989), or other lower-temperature transitions. For example, Quashnock, Loeb and Spergel (1989) proposed that magnetic fields of strength $\sim 10^{-38}G$ on present scales $\sim Kpc$ could have been produced during the quantum chromodynamics phase transition. Vachaspati (1991) proposed that electroweak phase transition can also produce magnetic field of the strength $\sim 10^{-35}G$ on similar scales. Unfortunately, however, models of this type have common characteristic that either the magnetic field generated is too weak or the models appear rather contrived (Cheng and Olinto 1994; Ratra 1993)

II. MODELS

Harrison's vortex model (Harrison 1969, 1970, 1973ab) and Bierman's battery effect is distinct in that they are based on classical electrodynamics. Harrison noted that if we assume the presence of a cosmic vortex with angular velocity ω in the radiation-dominated epoch, then the magnetic field is induced with a magnitude $B = 2m_p c\omega/e$. Here m_p is the proton mass, c is the speed of light, and e is the electric charge. Thus for a vortex of $\omega \sim (1/t)$ where $t \sim 10^{13}$ sec, which is the recombination time, then

$$B \sim 2 \times 10^{-17}G. \tag{1}$$

This is the required 'seed' relic field strength that can explain the observed intracluster fields at decoupling. This field strength can be amplified via dynamo mechanism up to $\sim 10^{-6}G$ (Zel'dovich & Novikov 1983). The key critic on this model is on the very origin of the vorticity. How $\omega \sim H(t)$ has been originated? In general, in a homogeneous, intrinsic, and incompressible cosmic plasma, it is extremely difficult to generate and keep a vortex motion. Bierman's battery effect is known to generate magnetic fields on stellar scales. The simplest way to understand the mechanism is via the Poynting-Robertson drag that could slow down electrons orbiting around a luminous massive object, while leaving protons almost unaffected. The current produced in this way will produce a weak magnetic field (Bierman 1950; Cattani & Sacchi 1966). In this model, orbital angular momentum of an electron in a circular orbit varies in time as $dlnL/dt = L_s/4\pi R^2(\sigma_T/m_e c^2)$, where L_s is the source luminosity, R is the distance from the source, and σ_T is the Thompson cross-section. Thus the maximum work done on the electron cloud in circular orbit with radius

R is

$$W_{max} \sim L_s \frac{v^2}{c^2} \tau, \tag{2}$$

where v is the orbital velocity of the electron, and τ is the duration time scale in which the battery effect is efficient. If this maximum work goes into building up magnetic field over a scale L (or in a volume $V \sim L^3$), then induced magnetic field is (Harwit 1982)

$$B \sim \sqrt{\frac{8\pi L_s \tau}{L^3}} (\frac{v}{c}). \tag{3}$$

In this work, we argue that the PBHs produced at the end of extended inflation can generate magnetic field on scales greater than the present intergalactic distance. The idea is quite simple. If a rotating blackhole is present in the radiation-dominated epoch, then in a way similar to the Bierman's battery mechanism, the co-rotation of cosmic matter around the rotating blackhole generates the magnetic field of sufficient strength. This field can be further amplified via galactic dynamo effect up to $\sim 10^{-6} G$. (The classical galactic dynamo effect may not that efficient than previously believed (Vainshtein & Rosner 1991; Kulsrud & Anderson 1992). However the result of this work will not critically depend on the Dynamo effect.) We note that around a rotating blackhole, while evaporating, there always exist an angular momentum exchange along the curved Kerr geodesic of the blackhole. Shapiro (1974) investigated steady-state accretion of matter onto blackholes with arbitrary angular momentum. The simplest case considered is the one which interstellar gas is at rest at infinity with respect to the blackhole. In this case, cosmic fluid itself possesses no intrinsic angular momentum. Rather as it flows inward, the gas corotates with the geometry and acquires a different angular velocity as seen by an observer at infinity. This process is understood as the 'dragging of inertial frames' associated with the blackhole's rotation. The 'angular motion' or 'angular momentum' is just a manifestation of the motion along this curved geometric structure. (Of course this effect vanishes at asymptotically large distances from the blackhole in precisely the same manner as the Kerr metric approaches the Schwarzschild metric.) In our case, we consider an inverse situation that photons and particles emitted from the central blackhole are traveling along the geodesic which corresponds to the path of the 'dragged inertial frames'. Thus at a large distance from the blackhole, emitted photons and particles from central blackhole impart torque to ambient cosmic matter. In this way, material surrounding a rotating blackhole becomes co-rotating (Misner, Thorn & Wheeler 1973; Leahy & Vilenkin 1981; Vilenkin & Leahy 1982). At sufficiently far distance from the rotating blackhole, the scattering angle between an emitted photon and an ambient particle at rest can be approximated by the 'aberration angle': $\Theta \equiv (v_{bh}/c)$. Here v_{bh} is the blackhole's rotational velocity. The idea of rotating blackhole within the context of the generation of the cosmic magnetic field was first discussed by Leahy and Vilenkin (1981). Their idea is based on the parity violation in weak interaction. It is assumed that if a rotating blackhole is present in radiation-dominated epoch. Then along the blackhole's rotation axis, the evaporating blackhole will emit left-handed neutrinos and right-handed antineutrinos. Since their scattering cross-sections to electrons are smaller than protons, a proton current antiparallel to that of the blackkole's angular momentum results. However, their work appears to contain a flaw that the required proton current will not arise. Note that the current is a quantity defined as product of the density of the scattered particle and its velocity induced by the scattering. The former quantity is proportional to the number density of the target particle, which is proportional to the scattering cross-section. The latter quantity is inversely proportional to the mass of the target particle. Thus even if the neutrino-proton scattering cross-section is a thousand times greater than that of neutrino-electron as claimed, this effect will be cancelled by the slower velocity of proton whose mass is thousand times greater than electrons. Of course this argument does not apply to photon-electron scattering, since the Thompson scattering cross-section between photon-electron is thousand times greater than photon-proton scattering.

III. FORMATION OF THE ROTATING PBH

Now we discuss the main issue. We first consider rotating blackholes produced at the end of the extended inflation (La & Steinhardt 1989; Hawking, Moss & Stuart 1982). The PBHs are formed at overlapping corners of colliding true vacuum bubbles. Therefore it is highly unlikely that they are born with a perfect spherically symmetric shape. Thus within a Hubble time of their birth, they acquire angular momentum via mutual tidal gravitational interaction

(Zel'dovich & Novikov 1983). Without loss of generality, one can assume that the rotating axis of the PBHs are random, and non-oriented. While rotating, the blackholes evaporate. They radiate mostly massless particles, such as neutrinos and photons (Hawking 1974; Hartle & Hawking 1976). In this way, Thompson scattering interaction between photons from the central blackhole and surrounding electrons sets in. What is the most important to be noted in the process is that the photon luminosity of evaporating PBHs (which are massive enough to survive until decoupling) always exceed the Eddington limit. This implies that the size of the region with rotational perturbation produced around the blackhole will grow continuously. The maximum scale of the perturbation can be estimated by the sound speed since cosmic matter will not move greater than the sound speed in the cosmic plasma $v_s = c/\sqrt{3}$. Thus the maximum rotational perturbation scale will be $\sim O(1/10)H^{-1}(t)$.

In order to estimate the typical size of the PBHs first produced it is necessary to estimate the typical size of bubbles when they percolate the Universe. (Here, we refer the colliding bubbles as the ones nucleated within each others' event horizon near the end of extended inflation). Guth and Weinberg show that at time t_1, the probability of a given point x_1 being in the false phase is $p_f(t_1, t_B)$, where t_B denotes the time when the bubble nucleates (Guth & Weinberg 1982; La, Steinhardt & Bertschinger 1989). Then consider a point x_2 that at time t it lies just outside the boundary of a bubble which is nucleated at space point x_1 at some (earlier) time $t_1 < t$. Then to a good approximation, one can safely assume that the point x_2 lines on a future null geodesic of x_1. In this case, the conditional probability that x_2 does not lie inside the radius of an another bubble, given that x_1 did not lie inside a bubble prior to time t_1, is

$$\frac{p_f(t, t_B)}{p_f(t_1, t_B)} = exp[-\frac{\pi\epsilon_o\omega}{3}((H_B t/\omega)^4 - (H_B t_1/\omega)^4)], \tag{4}$$

where we have used the fact that the causal past of x_2 includes nearly the entire past of x_1. The quantity ϵ_o is a dimensionless bubble nucleation parameter and ω is the Brans-Dicke parameter (or the non-minimal coupling parameter $\xi \equiv 1/8\omega$ with ω constrained as $1.5 \leq \omega \leq 25$ for a successful extended inflation).

Now one can compute the proper radius of the bubble as $x(t, t_1) \equiv R(t)r(t, r_1) \approx H^{-1}(t)(t/t_1)^{\omega}$, where $H(t)$ is the Hubble parameter at t. Using the expression of $p_f(t/t_B)$, one can substitute t by x to obtain

$$P(X > x) \approx exp[-\frac{\pi\epsilon(t)\omega}{3}(1 - (1 + H(t)x)^{-4/\omega})]. \tag{5}$$

According to Hawking et al.(1982), the quantity $P(X > x)$ represents the probability that a bubble grows beyond the radius of $X > x$ in a given direction before it collides with another bubble. It is readily seen that when $\omega \gg 1$, this expression takes a simpler form:

$$\frac{p_f(t, t_B)}{p_f(t_1, t_B)} = P(X > x) \approx [1 + H(t)x]^{-\beta}, \tag{6}$$

where

$$\beta \approx \frac{4\pi\epsilon(t)}{3[1 + H(t)x]^{4/\omega}}. \tag{7}$$

For the de Sitter inflation, we will aware that the quantity $\epsilon(t)$ is exponentially small and time-independent. Thus the quantity β is very small, causing that the distribution $P(X > x)$ is a very flat function of x. For example, a co-moving region occupied by a galaxy at the present epoch corresponds to $Hx \approx 10^{22}$ for $\rho_F \approx (10^{17}Gev)^4$. When the GUT temperature is $\sim 10^{15}GeV$, and the present horizon is $\sim 3000h^{-1}Mpc = 4.7 \times 10^{41}h^{-1}GeV^{-1}$ with the normalize Hubble parameter $0.4 \leq h < 1$. Since the temperature of the cosmic background radiation $T_{2.7} = 2.3 \times 10^{-13}GeV$, the present horizon scale at the end of inflation $L(t_e) = 4.7 \times 10^{41}(T_{2.7}/T_{GUT}) \approx 1.1 \times 10^{14}h^{-1}GeV^{-1}$. Therefore $H(t_e)L(t_e) \approx 10^{25}h^{-1}$. In the extended inflation, the quantity $\epsilon(t)$ is time-growing, and it exceeds unity near the end of inflation. Thus $\beta > 3$ at this time and the distribution of the size of bubbles that fill space of the observable Universe. The probability of growth of a given bubble to a size equal to the co-moving radius of a galaxy is exponentially small. Instead, most bubbles have sizes $H(t_e)x \sim O(1)$. This implies that near the end of extended inflation, almost all bubbles have comparable sizes $\sim H^{-1}(t_e)$.

Thus the PBHs of our interest have sizes comparable to the Hubble radius at the end of the extended inflation. Then the mass of the blackhole formed as a result of bubble collision is about (Hawking, Moss & Stuart 1982)

$$M_{BH} \sim \frac{4\pi}{3} T_{GUT}^4 H^{-3}(t_e) = \frac{3}{8\pi} \frac{M_p^2}{H} \sim 10^3 g (\frac{10^{15} GeV}{T_{GUT}})^2. \quad (8)$$

Here $H = 1.92 \times 10^{11} (T_{GUT}/10^{15} GeV)^2 GeV$, and $M_p = 10^{19} GeV$. Therefore if the GUT phase transition occurred when the temperature of the Universe $T_{GUT} = 10^{15}, 10^9$, or $10^4 GeV$, then the mass of the PBH is 10^3, 10^{15}, $10^{25} g$, respectively.

The rate of energy loss from an evaporating blackhole is given by (Hawking 1974; Hartle & Hawking 1976; Shapiro & Teukolsky 1983)

$$L_{BH} \sim \eta 10^{20} ergs^{-1} (\frac{10^{15} g}{M_{BH}})^2. \quad (9)$$

The quantity $\eta \sim O(1/10)$ is the fraction of the energy emitted in the form of photons. Here we considered the case when the emission is maximum, *i.e.*, when it is exploding to disappear. This is because we are interested in blackhole's maximum 'perturbation' on the largest scale.

The temperature of a blackhole is

$$T_{BH} \approx 10^{-7} K (\frac{M_{sun}}{M}), \quad (10)$$

and the estimated evaporation time for a blackhole of mass M is

$$\tau_{BH} \sim 10^{17} (\frac{M}{10^{15} g})^3 sec. \quad (11)$$

Thus for a PBH of mass $M_{BH} \leq 10^{13} g$, $T_{BH} \geq 10^{13} K$.

Now it is reminded that the PBHs of interest evaporate during the period when the temperature of the Universe falls from $\sim 10^9 K$ to $\sim 3000 K$. For temperature greater than $\sim 10^9 K$, pair creation of electron is so frequent that cosmic plasma loses required 'fluid' property. The lower limit corresponds to the decoupling temperature. Since the age of the radiation dominated Universe at temperature T is given as $T \approx 10^{10} K/\sqrt{t} \approx 8 \times 10^{-4} GeV/\sqrt{t}$, the ages of the Universe when $T \sim 10^9 K$ and $3000 K$ are $t \sim 100$ sec and 4×10^5 yr, respectively. These constraints on the mass of PBH

$$10^{10} g \leq M_{BH} \leq 5 \times 10^{13} g. \quad (12)$$

Further, the temperature of the GUT phase transition is constrained as

$$5 \times 10^9 GeV \leq T_{GUT} \leq 3 \times 10^{11} GeV. \quad (13)$$

Note that we are interested in the PBHs which evaporate on the last-scattering surface since they generate the maximum-size perturbation. Thus in this model, we will primarily concern $T_{GUT} \sim 10^{10} GeV$ extended inflationary scenario.

Since an isolated false vacuum (which turns in the PBH) has an intrinsic quadrupole moment, they are subject to the mutual tidal interaction. The interaction time scale and typical separation would be the Hubble time $\sim H^{-1}(t_e)$, and the horizon radius at that epoch. (We assume that the formation of the PBH is a random process so that their mutual tidal interaction is the most efficient only between the nearest neighbors.) Of course, it is possible that the quadrupole moment of the PBH is zero initially, but their mutual gravitational interaction gives rise to a tidal deformation with respect to each other, and hence produces the quadrupole moment. In this way, the acquired angular momentum of an isolated false vacuum in the percolated Universe is (Zel'dovich & Novikov 1983)

$$J \sim \sqrt{G M^3 L}. \quad (14)$$

Here, we set the mass and the physical size L of the false vacuum (which becomes the PBH). At a distance far away from the PBH, the 'aberration angle' is

$$\Theta \propto J. \quad (15)$$

One can conjecture that the PBHs are in supersonic motion, which occurs when false vacuum energy deposited in one of the larger true vacuum bubble is imparted as a kinetic energy of the PBH, then moving PBHs can generate shock wave in cosmic plasma. Also in an incompressible cosmic fluid, in the case when the deviation of the density from the average value $\delta\rho/\rho \neq 0$ quadratic with respect to the amplitude of perturbations occurs, the angular momentum generated differs from zero (Peebles 1969). Thus at percolation, all three mechanism may work together. However, we will only consider the rotational perturbations on cosmic plasma induced by rotating PBHs.

IV. RADIATION BALL PRODUCED BY EVAPORATING BLACKHOLES

We find that due to the strong photon and neutrino fluxes from evaporating blackhole, cosmic matter initially around a blackhole is pushed away up to the horizon scale. Thus as a blackhole evaporates in the radiation-dominated epoch, a radiation filled, matter-devoid sphere (a genuine isocurvature perturbation) is created. The time when a rotating blackhole evaporates corresponds to the moment when the blackhole completely transfers its angular momentum to surrounding cosmic plasma. A massive blackhole emits mostly massless particles like photons and neutrinos, and as the blackhole mass decrease, massive particles such as electron, proton, pion, W and Z bosons, charmed hyperons are emitted. At its final stage of evaporation, an explosive emission of various particle species results. (If the PBH is formed in a region containing three dimensional topological knots, even monopole emission will occur at this stage.) There are three main carriers which transfer blackhole's angular momentum to ambient plasma. Photons emerging out from the 'dragged inertial frame' scatter cosmic plasma (via Thompson scattering) which is located in an asymptotically flat region. Photons scatter preferentially electrons and this produce electron currents in a direction opposite to the rotation of the blackhole. The currents generate magnetic field in a direction antiparallel to the direction of the blackhole's angular momentum. Similarly, neutrinos and massive particles will scatter ambient protons and electrons. In this work, however, we will only discuss the photon-electron scattering process.

Could then photons escape to infinity? We will show that for PBHs of mass less than $10^{13}g$, the photon pressure always exceeds that of the infalling matter. This is because the luminosity of the PBHs exceeds the Eddington limit.

Radiation pressure P_γ of cosmic fluid is $P_\gamma = (1/3)U_\gamma$, where U_γ is the energy density of radiation. Suppose there is an object of mass M radiating with luminosity L. Since the radiation emitted diffuses outward due to many Thompson scatterings, U_γ satisfies a diffusion equation in such a way that the radiation flux F_γ is proportional to ∇U_γ : $F_\gamma = (-c/3\kappa\rho_m)\nabla U_\gamma$. Here, the quality $\kappa \equiv (\sigma_T/m_p)$ and ρ_m is the density of the matter (proton) fluid. Therefore, the net radiation pressure force on a unit area slab of matter of thickness dr is $dP_\gamma = -(\kappa\rho/4\pi c)(Ldr/r^2)$. Equating this to downward gravitational force $-GM\rho dr/r^2$, we find the Eddington limit for an object in cosmic plasma

$$L_{Edd} = \frac{4\pi GMc}{\kappa} \approx 1.4 \times 10^{38}(M/M_{sun})ergs^{-1}. \tag{16}$$

Now let us compare this with the luminosity of an evaporating blackhole:

$$L_{BH} \sim \mu_\gamma 10^{20} ergs^{-1}(10^{15}g/M_{BH})^2. \tag{17}$$

Here the quantity $\mu_\gamma \sim O(1/10)$ is the fraction of energy emitted in photon. Note that $L_{BH} \propto M_{BH}^{-2}$ whereas $L_{Edd} \propto M_{BH}$. Thus for any PBH of mass $M_{BH} < 10^{15}g$, its luminosity exceeds the Eddington limit.

V. BLACKHOLE SPECTRUM AND ASTROPHYSICAL CONSTRAINTS

One critical condition for the density of blackholes is the energy density of the blackhole should not exceed that of radiation. Otherwise, matter-dominated epoch will appear (since blackholes are particles) in the middle of the radiation-dominated epoch. This is unacceptable since it will greatly shorten the age of the Universe (If the blackholes dominate the energy density of the Universe in the conventional radiation-dominated epoch, the Universe will expand as $R \propto t^{2/3}$ instead of $R \propto t^{1/2}$. In this case, for an adiabatic expansion $RT = const.$ where $T_{GUT} \sim 10^{15}GeV$ and $t_{GUT} \sim 10^{-34}sec$ the age of the Universe becomes $\sim O(10)$ yrs when the temperature of the cosmic background radiation $T_{now} = 3K \sim 10^{-13}GeV$. This is certainly unacceptable.) Corresponding limits on density of primordial blackholes has been summarized by Novikov, Polnarev, and Starobinsky (1979). In order that the energy density of PBHs ρ_{BH} does not exceed that of radiation ρ_γ throughout the radiation-dominated epoch, we should require $\rho_{BH}(t_{dc})/\rho_\gamma(t_{dc}) < 1$ at decoupling $t = t_{dc}$. Thus for $\rho_{BH} \propto R^{-3}$ and $\rho_\gamma \propto R^{-4}$, at the end of extended inflation $t = t_e$,

$$\frac{\rho_{BH}(t_e)}{\rho_\gamma(t_e)} < 10^{-2}\frac{T_{dc}}{T_{GUT}}. \tag{18}$$

The factor 10^{-2} has been included following the analysis of Novikov et al's; otherwise photons from exploding PBHs will distort the Rayleigh-Jeans part spectrum of the cosmic background radiation. Thus one finds that for PBH whose mass $M_{BH} \sim 10^{13}g$ which explodes at $t = t_{dc}$ (This is the one formed when $T_{GUT} \sim 10^{10}GeV$.), $\rho_{BH}(t_e) \sim 10^{-22}\rho_\gamma(t_e)$. This result is very close to that of Novikov et al's.

Unfortunately, the extended inflation model alone cannot predict how many PBHs are formed at the end of inflation. This does not raise a serious problem because the number of the PBHs of astrophysical significance is exponentially small. For example, if there are one $10^{13}g$ PBH over one tenth of the decoupling horizon, there are $\sim 10^3$ PBHs in the present observable Universe. In this case, the ratio of its energy density to that of radiation is $\sim 10^{-73}$. This is quite an insignificant number. We can of course allow more PBHs within the allowed limit $\rho_{BH}(t_{dc})/\rho_\gamma(t_{dc}) < 10^{-22}$, so that their typical (comoving) separation at the present time becomes much less than $\sim O(10)Mpc$. Even in this case, the growing radiation 'voids' will eventually collide with others. Therefore, what will come out from this random collision is the random isocurvature perturbation on the last-scattering surface on the coherent scale $\sim O(10^{-1})H^{-1}(t_{dc})$.

Finally, one may concern that the 'void' isocurvature perturbations may produce unacceptable amount of temperature fluctuations in the cosmic background radiation. Note also that the 'void'-scale corresponds to the arcminute angular separation at the present time. This corresponds to the scale where original in- homogeneities on the last-scattering surface are subject to strong damping because of the gradual recombination process(Zel'dovich & Novikov 1983). Thus one can safely assume that fluctuations due to the radiation 'void' are strongly averaged.

VI. THE ROTATING PBH AS A COSMOLOGICAL BATTERY

Cosmic plasma in the radiation epoch can be viewed as a plasma fluid consisting of positively charged proton fluid and negatively charged electron-photon fluid. General hydrodynamic friction between the two components is relatively small. [This argument holds as long as the temperature of the Universe $T \leq 10^9K$. Otherwise, electron pair production process becomes efficient. In this case, photons, electrons, and protons tend to lock together, failing the required fluid property.]

Now let us estimate how much angular momentum transfer occurs from an evaporating blackhole. We will only consider the process carried out by photons. Let us first denote the momentum of a photon emitted from the blackhole as p. Then the energy of a typical photon is pc. The number of photons crossing unit area in unit time at a radius R is $L_s/4\pi R^2 pc$. Thus the force per electron is just the rate at which momentum is deposited per unit time, so we multiply the number flux by p and the Thompson cross-section to obtain the force acting on the electron

$$F = \frac{L_s \sigma_T}{4\pi R^2 c}. \tag{19}$$

This expression holds even if photons are not streaming radially. We denoted the momentum of radiation in radial and transverse direction as p_r and p_t, respectively. The torque imparted to an electron at a distance R from the blackhole is $M \equiv R|F|(p_t/p_r + p_t)$. If we set $p_t/p_r + p_t \equiv \Theta$, then

$$M = \frac{L_{BH}\sigma_T}{4\pi R c}\Theta. \tag{20}$$

The angular momentum of the blackhole $M \propto M_{BH} v_{bh} r_{Sch}$. The size of event horizon in Kerr-blackhole is $r_{Kerr} = G_N(M_{BH} + \sqrt{M_{BH}^2 - M^2/M_{BH}^2})$. Here G_N is Newton's constant. We are interested in the epoch when the rotational perturbation of the blackhole is maximum. This occurs when the blackhole explodes with maximum rotation speed.

(Note also in this case that $r_{Sch} \approx r_{Kerr}$). Since there are $N = 4\pi R^2/\sigma_T$ collisions between photons and electrons per unit time, the total rate of angular momentum transfer is $M_{total} = (4\pi R^2/\sigma_T)M$. This quantity is nothing but the maximum total work that can be done on surrounding electron cloud, W_{max}:

$$W_{max} = L_{BH}(\frac{R}{c})\Theta. \tag{21}$$

Thus in a way similar to the Bierman's battery mechanism, one can estimate the maximum work that can go into building up magnetic field on the maximum scale: $O(1/10)$ of the Hubble radius $R(t) \sim O(1/10)cH^{-1}(t)$

$$B \sim \sqrt{\frac{8\pi L_{BH}\Theta}{c}}R^{-2}(t)G. \tag{22}$$

Since $L_{BH} \sim \mu_\gamma 10^{20} ergs^{-1}(10^{15}g/M_{BH})^2$, we find

$$B \sim \frac{10^{10}\sqrt{\mu_\gamma\Theta}}{R(t)}(\frac{10^{15}g}{M_{BH}})G. \tag{23}$$

This is the desired formula. Thus for the PBH of mass $10^{13}g$ exploding at recombination, $O(1/10)cH^{-1}(t_{dc}) \sim 10^{22}cm$, and

$$B(t_{dc}) \sim 10^{-10}\sqrt{\mu_\gamma\Theta}G. \tag{24}$$

The comoving scale of $R(t_{dc})$ at the present time is $\sim 10Mpc$. Thus at the explosion, for a blackhole rotating in relativistic speed, the quantity $\sqrt{\mu_\gamma\Theta}$ will not be significantly different from unity. In this case, the magnetic field generated on $\sim 10Mpc$ comoving scale is

$$B \sim 10^{-11}G. \tag{25}$$

This is the 'seed' field much stronger than that in the Harrison's model. It has a sufficient strength to be amplified up to the present inter/galactic field via the galactic dynamo.

VII. CONCLUSION

We find that PBHs produced at the end of extended inflation can generate the 'seed' magnetic field in the Universe. Isolated false vacuum regions formed in percolated true vacua are not spherically symmetric. Therefore they acquire intrinsic angular momentum via tidal gravitational interaction. For PBHs of mass $M_{BH} \leq 10^{13}g$, it is found that their luminosity exceeds the Eddington limit. Thus as the PBHs evaporate, a matter-devoid radiation sphere of approximately one tenth of the horizon radius is formed in the radiation epoch. In the process, photons scatter ambient plasma with the 'aberration' angle. This produces rotational perturbation, and hence the rotating PBH becomes a version of Bierman's battery. We find that this mechanism can successfully generate the primordial magnetic field of strength $B \sim 10^{-11}G$ on present comoving scales of $\sim 10Mpc$.

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