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Determination of Weighting Factor in the Inverse Model for Estimating Surface Velocity from AVHRR/SST Data

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AVHRR/SST로 부터 표층유속을 추정하기 위한 역행렬 모델에서 가중치의 설정

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The inverse method has been used to estimate a surface velocity field from sequential AVHRR/SST data. In the model, equation system was composed of heat equation and horizontal divergence minimization and the velocity field contained in the advective term of the heat equation, which was linearized in grid system, was estimated. A constraint was the minimization of horizontal divergence with weighting factor and introduced to compensate the null space(Menke, 1984) of the velocity solutions for the heat equation. The experiments were carried out to set up the range of weighting factor and the matrix equation was solved by SVD(Singular Value Decomposion). In the experiment, the scales of horizontal temperature gradient and divergence of synthetic velocity field were approximated to those of real field. The neglected diffusive effect and the horizontal variation of heat flux in the heat equation were regarded as random temperature errors. According to the result of experiments, the minimum of relative error was more desirable than the minimum of misfit as the criteria of setting up the weighting factor and the error of estimated velocity field became small when the weighting factor was order of 10^{-1}

연속된 AVHRR/SST 자료를 이용한 표층유속의 추정에 역행렬법이 이용되어 왔다. 본 모델에서 방정식체계는 열방정식과 제한요소로서 가중치가 있는 발산최소화이다. 제한요소는 열방정식의 속도해중에서 null space (Menke, 1984)에 해당하는 해를 구하기 위하여 도입되었으며 이 식들은 격자화한 영역에서 AVHRR/SST의 수온경사에 의해 선형화된다. 실험은 열방정식에 대한 발산최소화의 상대적 중요성을 나타내는 가중치의 크기를 설정하기 위하여 수행하였으며 행렬식은 SVD(Singular Value Decomposion)에 의해 해를 구했다. 실험에서 가상온도분포의 수온경사와 가상유속장의 발산의 크기는 실제해역에 근사시켰다. 열방정식은 확산의 효과를 무시하고 열속이 공간적으로 일정한 것으로 가정하여 구성하였으며 이와같은 가정에 의한 오류를 고려하기 위하여 가상 온도자료에 무작위오류를 도입하였다. 실험결과에 의하면 가중치를 설정하는 기준으로서 상대오차 최소화가 잔차최소화보다 바람직한 것으로 나타났으며 가중치가 10^{-1}의 크기일 때 추정유속의 오류가 가장 작은 것으로 나타났다.

INTRODUCTION

NOAA/AVHRR(Advanced Very High Resolution Radiometer) data is useful in research of oceanic surface flow for its simultaneity and periodicity.

The methods estimating oceanic element such as surface velocity from NOAA/AVHRR data has been developed, in addition to qualitative des-

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cription of AVHRR image. Estimations of surface velocity field from NOAA/AVHRR data are distinguished into two types: the maximum cross correlation method (Emery *et al.*, 1986; Whal and Simpson, 1990), the inverse method(Kelly, 1989; Kelly and Strub, 1992). We used inverse method because the maximum cross correlation method was known to be not related to ocean dynamics directly(Kubota and Shirota, 1993). Kelly used the heat equation to estimate the surface velocity and constraints were minimization of horizontal divergence, relative vorticity and kinetic energy with weighting factor. But the last two constraints may be not necessary, because they make resulting solutions be smaller and smoother than the real velocity field respectively.

The minimization of horizontal divergence is proper constraint for real oceanic flow. Its weighting factor represents relative importance to the heat equation in case that the velocity field is estimated on the system of equation composed of divergence minimization and heat equation. However, the criteria of determining weighting factor remains ambiguous. This paper is about a try to set up the criteria of determining weighting factor of divergence minimization.

Here, from known temperature gradient, equation system became linearized and a matrix form (**Gm=d**, **G**: coefficient matrix, **m**: unknown velocity vector, **d**: data vector) in grid system.

Experiments were designed to estimate a velocity field inversely from synthetic temperature fields advected by known synthetic velocity field. Then, the procedure were applied to the AVHRR/SST data set. The synthetic velocity field was made by two dimensional tidal model (Flather and Heaps, 1975). Weighting factors were chosen in the range where relative error and misfit became minimum as weighting factor increased from order of 10^{-4} to 10^{4}. Relative error was defined by the difference between synthetic tidal flow and estimated velocity field,

$$\text{relative error} = |\mathbf{m}_{act} - \mathbf{m}_{est}| / |\mathbf{m}_{act}| \times 100$$

where $\mathbf{m}_{act}$ is the actual velocity vector based on the results from the tidal model and $\mathbf{m}_{est}$ is the estimated velocity vector.

Misfit is the residual part of heat equation, as follows

$$\text{misfit} = |\mathbf{d}_{obs} - \mathbf{G}\mathbf{m}_{est}| / |d_{obs}| \times 100$$

where $\mathbf{d}_{obs}$ is the observed data vector.

In accordance with experiments, minimum relative error was more desirable as criteria of weighting factor than the minimum of misfit because relative error appeared small within much narrow range of weighting factor than misfit did.

EXPERIMENT

Method

The resulting velocity to be estimated was contained in the advective term in heat equation written by

$$\partial T/\partial t + U \cdot \nabla T = \kappa \nabla^2 T + Q$$

where T and U represent temperature and velocity respectively, κ and Q are diffusion coefficient and flux of heat respectively.

Divergence minimization used as a constraint is given by

$$\omega \nabla \cdot U = 0$$

where ω is weighting factor.

In the experiment design, diffusive effect of heat equation was small compared with advection based on scale analysis. Using velocity of 10^{-1} ms^{-1} and temperature gradient of 10^{-1} K/km made the order of advective term about 10^{-5} Ks^{-1} and by using the diffusion coefficient of 10^{2} m^2/s, the diffusion term became order of 10^{-6} Ks^{-1}. Spatial variation of heat flux was assumed to be negligable over the mesoscale (order of 10^{5} m) because atmospheric motion was larger than the oceanic motion.

Under assumption and analysis mentioned above, diffusion effect was neglected. Heat flux could be calculated on the difference of spatial mean temperature of sequential SST data.

Finite difference form (trapezoidal implicit scheme) of heat equation was as follows

$$[(u_{i,j+1,n+1} + u_{i+1,j,n+1})/2] \times [(T_{i+1,j,n+1} - T_{i-1,j,n+1}$$

$$+ T_{i+1,j,n} - T_{i-1,j,n})/(4l_x)] + [(v_{i,j+1,n+1} + v_{i,j+1,n+1})/2] \times [(T_{i,j+1,n+1} - T_{i,j-1,n+1} + T_{i,j+1,n} - T_{i,j-1,n})/(4l_x)] = -[(T_{i,j,n+1} - T_{i,j,n-1})/2\Delta t] + Q_D$$

where i,j are grid point of staggered grid system, n is time, l_x, l_y are grid scale and Δt, Q_D are time step and heat flux respectively. Because the temperature gradient was known from sequential SST data, above equation became linear and established on each grid.

The along-isotherm component of estimated velocity is meaningless because the advective term is dot product of velocity and temperature gradient. So unacceptable along-isotherm velocity can be estimated and large velocity may be calculated in the area of small temperature gradient. Divergence minimization with weighting factor was used to reduce these errors and appended at the end of linear equation system of heat equation.

Velocity field was expanded in the form of Fourier series as in Kelly (1989).

$$u(x,y) = \sum_{k,l=0}^{N} [A_{kl} \cos(k\pi x/L_x) \times \cos(l\pi y/L_y) + B_{kl} \sin(k\pi x/L_x) \times \cos(l\pi y/L_y) + C_{kl} \cos(k\pi x/L_x) \times \sin(l\pi y/L_y) + D_{kl} \sin(k\pi x/L_x) \times \sin(l\pi y/L_y)]$$

where A_{kl}, B_{kl}, C_{kl}, and D_{kl} are expansion coefficient, L_x and L_y are the length of whole domain in x and y direction and v(x,y) are defined analogously.

Expansion coefficients became elements of unknown vector of the reformed matrix equation. With given grid system, the resulting matrix form was overdetermined and the unknown expansion coefficients were solved by SVD method of IMSL package (1989).

In the experiments, relative error and misfit were investigated as the weighting factor increased from order of 10^{-4} to 10^{4}. Relative error was the difference between estimated velocity and known synthetic velocity described in the next section.

Data

Data sets used were temperature and velocity data. Synthetic temperature data were three types shown in Fig. 1. Synthetic temperature fields composed of 13×13 grids contained random error (Fig. 1(d)) which were generated by using random number and introduced to consider neglected diffusion, spatial variation of heat flux, and SST error derived from AVHRR data. The number of linear equations were 484 from three temperature data sets of 13×13 grid system and one set of continuity equation. The number of unknown expansion coefficients were 162 from 5th-order Fourier expansion.

Velocity data were made by numerical tide model in 30×30 grid system with grid size of 10 km. After calculations, five sets of divergent velocity field composed of 13×13 grids were extracted (Fig. 2). Maximum divergence of tidal flow in the domain was order of 10^{-6} s^{-1} similar to real field (Table 1).

To carry out the experiments using AVHRR/SST, we calculated AVHRR/SST by single channel correction (Kelly and Davis, 1986) and MCSST(Multi Channel Sea Surface Temperature) algorithm (SeaSpace, 1993).

Fig. 3 shows the AVHRR/SST images of data B and 2 in the Table 2.

When AVHRR/SST was used as a temperature field in experiments, Synthetic velocity field was generated by estimating surface velocity from data 2, 3, 4, and 5 and used as the actual velocity in calculating relative error.

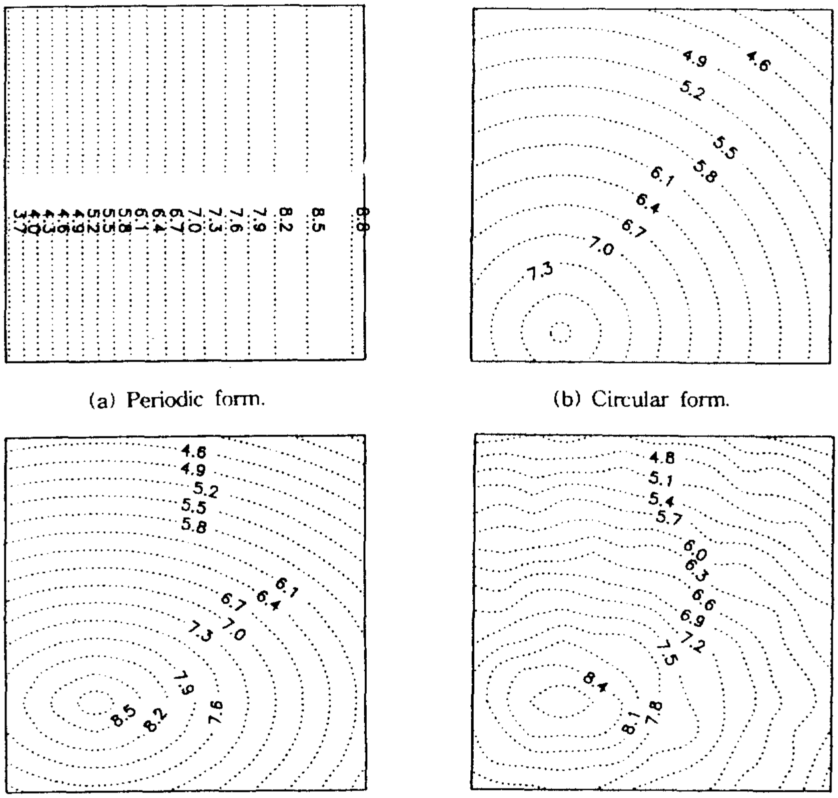
RESULT AND DISCUSSION

Synthetic temperature field

The magnitude of maximum random error were 0.05K, 0.08K, 0.12K, and 0.3K which were chosen arbitarily but 0.12K was NEΔt (Noise equivalent differential temperature) of AVHRR Ch.4 and 0.3K was known as a sensor error by manufacture (Robinson, 1985).

The results of experiment is summarized in Table 3.

Weighting factors at minimum relative error were within narrow range, that is, order of 10^{-1} (the 3rd column in table 3) for each random error and syn-



(a) Periodic form. (b) Circular form.

(c) Elliptic form. (d) Elliptic form with random error. (Max. error 0.3K)

Fig. 1. Synthetic temperature field.

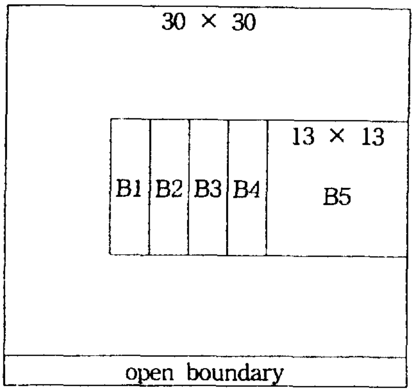


Fig. 2. Grid system for generating divergent flow.

thetic velocity field but weighting factor at minimum misfit were over wide range (the 5th column in table 3).

Table 1. Veloity field and maximum divergence (unit: × 10^{-6} s^{-1})

velocity field	B1	B2	B3	B4	B5
maximum divergence	4.4	4.7	5.0	5.3	5.6

Table 2. satellite data and observing time

Data	Time	Data	Time
B	1992.11.17 04:23	2	1992. 5.18 19:40
C	11.17 07:32	3	5.19 04:02
D	11.17 15:47	4	5.19 08:30
E	11.17 18:35	5	5.19 15:26
F	11.18 04:11	*	*

Time: local time
B, C, D, E, F: In the Korea Strait
2, 3, 4, 5: In the eastern coast of Korea.

Therefore, error of estimated velocity may become large if the minimum misfit is used as criteria

(a) the Korea Strait (b) The eastern coast of korea (data 2)

Fig. 3. SST Images.

Table 3. The results of experiments on relative error, misfit and weighting factor with random temperature error

velocity field	random error	ω at MRE	ω within 10% of MRE	ω at MM	MRE (%)
B1	0.05	0.5	0.4-1	under 0.0001	27.4
	0.08	0.5	0.4-10	under 0.0001	40.0
	0.12	0.6	0.4-20	0.1	58.0
	0.3	0.7	0.4-10000	0.002	146
B3	0.05	0.2	0.1-0.4	0.2	62.0
	0.08	0.2	0.09-0.4	under 0.0001	92.0
	0.12	0.1	0.06-0.6	under 0.0001	136
	0.3	0.3	0.2-10	0.007	344
B5	0.05	0.3	0.2-0.5	0.07	72
	0.08	0.3	0.2-0.6	0.02	102
	0.12	0.5	0.2-1	under 0.0001	135
	0.3	0.7	0.4-20	0.006	329

MRE: minimum relative error
MM: minimum misfit

of weighting factor.

The relation between misfit and weighting factor for velocity field B3 was shown in Fig. 4. Misfit became small relatively over the weighting factor range from order of 10^{-4} to 10^{-1} for each random error. So the weighting factor range of small misfit was too wide to be chosen. Fig. 5 shows the relation between relative error and weighting factor for velocity field B3. Relative error increased and its change pattern became flatten over the whole of weighting factor range as the divergence of velocity field and random error increased. This means that when the SST error and spatial variation of heat flux became large or as the divergence and diffusion of flow increased, estimated velocity became meaningless and it was difficult to determine a prop-

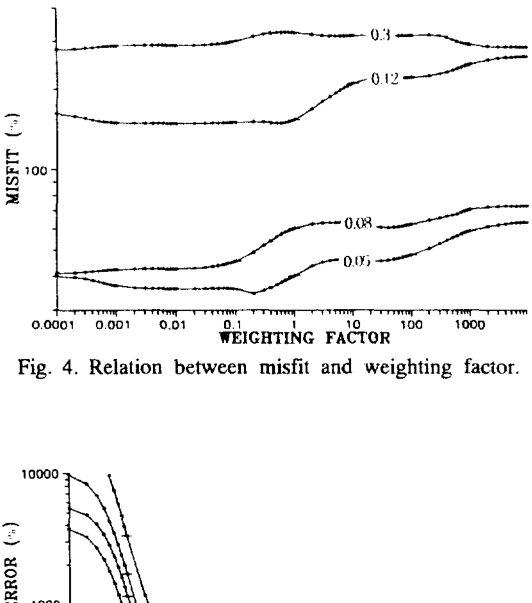


Fig. 4. Relation between misfit and weighting factor.

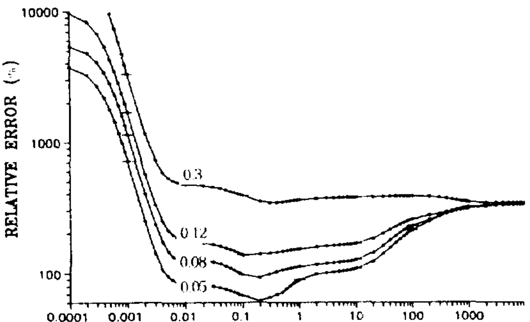
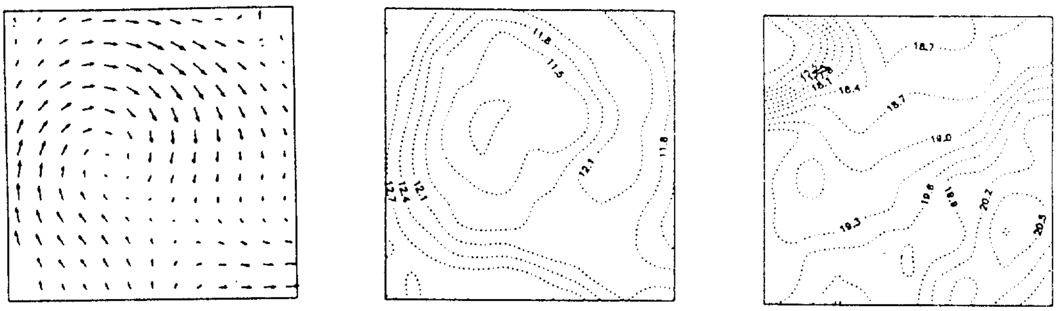
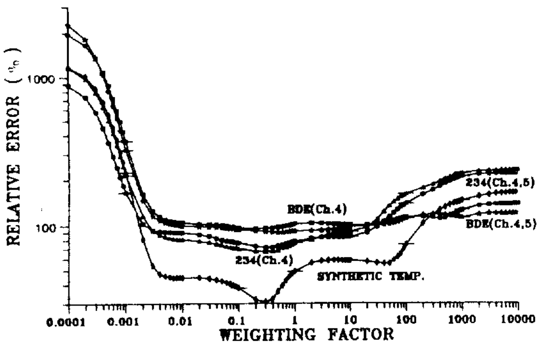


Fig. 5. Relation between relative error and weighting factor.

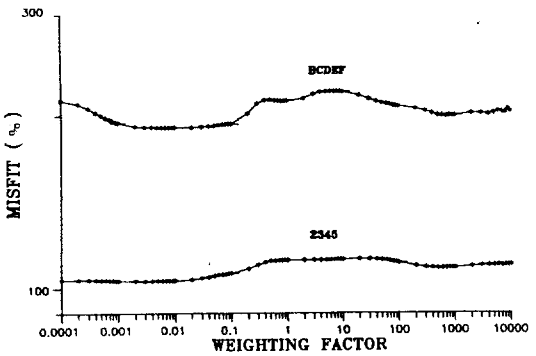


(a) AVHRR/SST-derived velocity field. (b) AVHRR/SST(2 data) (c) AVHRR/SST(E data)

Fig. 6. AVHRR/SST-derived velocity field and AVHRR/SST.



(a) Relation between relative error and weighting factor.



(b) Relation between misfit and weighting factor.

Fig. 7. Results of AVHRR/SST and synthetic temperature experiments.

er weighting factor.

AVHRR/SST data

In the application to the AVHRR/SST data, synthetic velocity field shown in Fig 6(a) was estimated from data 2, 3, 4, and 5 with weighting factor 0.5. Clockwise velocity field of warm eddy was estimated well and maximum divergence of the domain was $1.5\times10^{-5}s^{-1}$. In generating velocity field, divergence in the domain decreased as the weighting factor increased because weighting factor represented relative importance of divergence minimization to heat equation in the linear equation system.

The AVHRR/SST data used are shown partially in Fig. 6(b), (c).

Similar results to synthetic temperature field case are shown in AVHRR/SST data experiment (Fig. 7). The BDE and 234 in Fig. 7(a) corresponds to the result of experiment that AVHRR/SST B, D, E and 2, 3, 4 of Table 2 are advected by velocity field of Fig. 6(a) respectively and estimated inversely. Ch.4 and Ch.4, 5 of Fig. 7(a) correspond to the result using single channel correction data and MCSST respectively. Relative error became minimum in the weighting factor range of 10^{-1}, on the other hand misfit appeared small over wide range of weighting factor (Fig. 7(b)). The relative error changed similarly at the study area in both case of using single channel correction data and MCSST (Lee, 1994).

CONCLUSION

In the estimation of velocity field by an inverse method, the determination of weighting factor for the constraint of divergence minimization is critical because it represents relative importance of divergence minimization to heat equation.

As criteria of determining weighting factor, the results indicate that relative error was more desirable than misfit. Good results were estimated generally when the weighting factor was order of 10^{-1}. When the SST error and divergence of flow increased, the estimated velocity became meaningless and it was difficult to determine a proper weighting factor

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