ANALYTICAL SCIENCE & TECHNOLOGY (Journal of the Korean Society of Analytical Sciences)
Vol. 8, No. 4, 1995
Printed in the Republic of Korea

# Formulation Of Some Mathematical Models For The Estimations Of The Most Probable Salts Derived From The Major Mineral Constituents In Natural Water

#### Dr. Md. Raisuddin Miah

Khulna Power Station, Bangladesh Power Development Board, Bangladesh.

#### Abstract

By extensive studies with the quantitative status of the mineral constituents of thousands of water samples, it was found that almost in all natural waters irrespective of the surface or sub-surface sources, minimum 99.5% of the total amount of the cationic constituents are generally the contributions of 3 commonly present parameters viz. Ca, Mg and Na and that of the anionic species are same and contributed by HCO<sub>3</sub>, Cl and SO<sub>4</sub> only. In the field of water works, all these major mineral substances are conventionally measured as their individual ions. But till now, no reliable and generalised methods or rules have been developed for the determination of the exact kinds of the individual salt components and their amounts from these ionic constituents normally present in water. As salt content, only the TDS (Total Dissolved Solids) parameter is frequently measured by evaporation of the water sample. But TDS can tell nothing about the kinds and amounts of the individual salts present in it. Considering the analytical importance of the estimation of the mineral substances as their individual salts, some generalised mathematical models have been developed by this research which are based on the 'hypothetical order of chemical combinations' as may occur among the ionic constituents. With the help of these models, one can easily assume the most probable salts with approximate quantities derived from the ionic constituents. In addition, approximate amount of Na content can also be estimated mathematically with simultaneous verification of the correctness of the water analysis results. The models are stated in this paper with practical illustrations and descriptions of the method of applications.

#### Introduction

All natural waters irrespective of the surface and sub-surface sources contain various kinds of

dissolved impurities, of which, the universal presence of different mineral substances such as Ca, Mg, Na, K, Fe, HCO<sub>3</sub>, Cl, SO<sub>4</sub>, NO<sub>3</sub>, SiO<sub>2</sub> etc. are quite common. Besides, more other elements may also be present. But

they are very small in quantities and hence normally ignored in industrial water treatment field. By extensive studies on the quantitative status of the mineral substances, it was found that almost in all natural waters, minimum 99.5% of the total amount of the cationic species are the summation of only 3 particular constituents which are Ca, Mg, and Na [1]. Similarly, 99.5% of the total anionic species are the contributions of only HCO3, Cl and SO4 [1]. These 6 parameters are known as major minerals of natural water significantly encountered in industrial water analysis and Deionization Process [2,3]. Absence of any one of these major constituents in water analysis report is a big shortfall and for which it is difficult to ascertain the quality characteristics the water and to verify the correctness of the analytical results [4].

All dissolved mineral constituents exist in water in ionic state and which are also conventionally estimated as ions. From these ionic results, it is not easy for one to conclude – what are the exact kinds and quantities of the individual salt components present [4]? Only TDS parameter is measured by conventional evaporation method but it can tell nothing about any individual salts and their quantities. Whereas identification of the individual salt components is a significant aspect of water analysis.

No reliable and generalised methods or rules have been developed for finding out the above. Although some attempts were taken long ago by some water analysts to solve the problem mathematically [5], the method found valid only for some specific cases. Considering the application limit of the method and analytical importance, some generalised mathematical models have been developed and formulated by this research in the light of the aforesaid quantitative status of the major minerals. The most fundamental principles of these models are based on the "hypothetical order or priority of the chemical combination" assumed to be occurred among the major ionic constituents. The

models can be used as 'thumb rules' for the following purposes of water analysis.

- Quantitative derivation of the most probable salts
- Estimation of the approx. amount of Na content mathematically.
- Verification and assurance of the correctness or accuracy of the test result of the said major mineral constituents.

#### **Principles**

# (a) For Sodium (Na<sup>+</sup>) Estimation.

Total Salts (meq/l) = 
$$\sum M_c = \sum M_a$$
 ...............(A)  

$$\sum M_c = \sum M_a$$
 .........................(B)  

$$Q_{Na} = \sum M_a - \text{TH}$$
 ......................(C)  
where,  $M_c$  - major cations (i.e.  $Ca^{+2}$ ,  $Mg^{+2}$  and  $Na^+$ ).  

$$M_a - \text{major anions}$$
 (i.e.  $HCO_3^-$ ,  $CI^-$  and  $SO_4^{-2}$ ).  
TH - total hardness (i.e.  $Ca^{+2} + Mg^{+2}$ ).  

$$Q_{Na} - \text{quantity of } Na^+$$
.

[ In industrial water treatment field, Na is very often expressed as Na&K together [7] or vice versa. Since K is normally absent or trace level in water except sea source, hence it is ignored and only Na is encountered.]

#### (b) For Salt Derivation.

The newly developed mathematical models for salt derivation are based on 4 fundamental principles viz.

 Most probable priority of chemical combinations according to the following hypothetical order [5].

#### Cations:

$$Na^{+} \leftrightarrow OH^{-} > CI^{-} > HCO3^{-} > SO_{4}^{-2} > NO_{3}^{-}$$
 $Ca^{+2} \leftrightarrow HCO_{3}^{-} > SO_{4}^{-2} > CI^{-}$ 
 $Mg^{+2} \leftrightarrow HCO_{3}^{-} > CI^{-} > SO_{4}^{-2}$ 
 $K^{+} \leftrightarrow CI^{-} > NO_{3}^{-} > SO_{4}^{-2}$ 
 $Fe^{+2} \leftrightarrow HCO_{3}^{-} > CI^{-} > SO_{4}^{-2}$ 

#### Anions:

OH 
$$\leftrightarrow$$
 Na<sup>+</sup> > Mg<sup>+2</sup> > Ca<sup>+2</sup>

CI  $\leftrightarrow$  Na<sup>+</sup> > Mg<sup>+2</sup> > Ca<sup>+2</sup> > K<sup>+</sup>

HCO<sub>3</sub>  $\leftrightarrow$  Ca<sup>+2</sup> > Mg<sup>+2</sup> > Na<sup>+</sup> > Fe<sup>+2</sup>

SO<sub>4</sub>  $\xrightarrow{-2}$   $\leftrightarrow$  Mg<sup>+2</sup> > Ca<sup>+2</sup> > Na<sup>+</sup>

NO<sub>3</sub>  $\xrightarrow{-4}$   $\leftrightarrow$  K<sup>+</sup> > Na<sup>+</sup> > Ca<sup>+2</sup>

- Maintenance of the ionic balance i.e. total cation concentration must be equal to total anion concentration [2,5,6] in the same unit (meq/1 or ppm as CaCO<sub>3</sub>).
- Total salinity concentration must be equal to total cation or anion concentration in the same unit.
- No residual or negative balance of any cationic or anionic species should exist after computing for the salts resulting from the above order of the chemical combinations assumed.
- (c) Rules for Salt Derivation and Estimation.

RULE - 1 (WHEN TH 
$$\geq$$
 M-ALK.):

# 1. Estimation of Alkali Compound as NaOH (if p-alk present in water sample).

Then, 
$$R_{Na} = Q_{Na} - Q_{p-alk}$$
 .....(1.2)

where, R<sub>Na</sub> is the residual quantity of Na which will combine with chloride (Cl) to produce NaCl either as in eq. 2.1 or 2.3.

#### 2. Estimation of CI-salt as NaCl.

(i) Case-I: when Q<sub>Cl</sub> ≥ R<sub>Na</sub>

$$Q_{NaCl} = R_{Na} \text{ or } Q_{Na} \text{ (if p-alk absent)} ... (2.1)$$

where, Q<sub>NaCl</sub> is the quantity of NaCl and R<sub>Na</sub> is the first residual quantity of Na after combining with p-alk as in eq. (1.2).

Then, 
$$R_{Cl} = Q_{Cl} - R_{Na}$$
 ............. (2.2) where,  $Q_{Cl}$  – initial quantity of  $Cl^-$  in water sample.

 $R_{Cl}$  - first residual quantity of Cl<sup>-</sup> after combining with  $R_{Na}$  as in eq. (2.1) and which will combine with Mg to produce MgCl<sub>2</sub> salt as in eq. (3.1) or (3.3).

(ii) Case – II : when  $Q_{Cl} < R_{Na}$  or  $Q_{Na}$  (if p-alk absent).

$$Q_{NaCl} = Q_{Cl}$$
.....(2.3).

Here, second residual quantity of Na after combining with  $Q_{CI}$  is calculated as follows:

$$R'_{Na} = R_{Na} - Q_{Cl}$$
.....(2.4)

where,  $R'_{Na}$  is the second residual quantity of Na which will now combine with  $SO_4$  and/or  $NO_3$  as given by eq. (9.1) and/or eq. (10).

# 3. Estimation of CI - sait as MgCl 2.

(i) Case - 1: when 
$$Q_{Mg} \ge R_{Cl}$$
.

 $Q_{MgCl_2} = R_{Cl}$ .....(3.1).

where,  $Q_{MgCl_2}$  - quantity of  $MgCl_2$ .

 $R_{Cl}$  - first residual quantity or  $Cl$  after combining with  $R_{Na}$  or  $Q_{Na}$ .

Here, the residual quantity of Mg after combining with Cl is calculated as follows.

$$R_{Mg} = Q_{Mg} - R_{Cl}$$
 .....(3.2).

where,  $R_{Mg}$  - first residual quantity of Mg which will combine with  $SO_4$  as in eq. (4.1) or (4.3).

 $Q_{Mg}$  - initial quantity of Mg in water sample.

(ii) Case II: When  $Q_{Mg} < R_{Cl}$ .

$$Q_{MgCl_2} = Q_{Mg}$$
 .....(3.3).

Here, the second residual quantity of Cl after combination with Mg is calculated as follows.

$$R'_{Cl} = R_{Cl} - Q_{Mg}$$
 ..... (3.4).

where,  $R'_{Cl}$  - second residual quantity of Cl after combination with Mg (in eq. 3.4) which now further combines with Ca to produce  $CaCl_2$  (as in eq. 7). In such case, the Ca content in water would be higher than  $CO_3$  and  $HCO_3$  constituents (i.e. mainly  $HCO_3$  - alkalinity). Otherwise a 4th cationic constituent say, Fe'Cu/Al/Zn or uncommon cationic species (if any) is to be found by a quantity equivalent to that of  $R'_{Cl}$ 

# 4. Estimation of SO<sub>4</sub> - salt as MgSO<sub>4</sub>.

(i) Case - I: When 
$$Q_{SO_4} \ge R_{Mg}$$
.  
 $Q_{MgSO_4} = R_{Mg}$  ......(4.1).

where,  $Q_{MgSO_4}$  - quantity of MgSO<sub>4</sub>.

 $R_{Mg}$  - quantity of first residual Mg after combining with  $R_{Cl}$  as in eq. (3.2).

Here, the residual quantity of  $SO_4$  after combine with  $R_{Mg}$  is calculated as follows.

$$R_{SO_4} = Q_{SO_4} - R_{Mg}$$
 ..... (4.2).

where,  $R_{SO_4}$  - first residual quantity of  $SO_4$  which will combine with Ca as in eq. (6.1) and/or (6.3).

 $Q_{SO_4}$  – the initial quantity of  $SO_4$  constituent in the water sample.

(ii) Case II : when  $Q_{SO_4} < R_{Mg}$ .

$$Q_{MgSO_4} = Q_{SO_4}$$
 ..... (4.3)

Here, the second residual quantity of Mg after combination with  ${\rm SO}_4$  is calculated as follows.

$$R'_{Mg} = R_{Mg} - Q_{SO_4}$$
 ..... (4.4)

where,  $R'_{Mg}$  is the second residual quantity of Mg which will be combining with  $R_{HCO_3}$  as in eq. (81.).

# 5. Estimation of HCO<sub>3</sub> - Salt as Ca(HCO<sub>3</sub>)<sub>2</sub>

(i) Case - I: When Q<sub>Ca</sub> ≥ m-alk (in absence of p-alk).

$$Q_{Ca(HCO_3)_2} = Q_{HCO_3}$$
 .....(5.1)

where, Q<sub>Ca(HCO<sub>3)2</sub> - the quantity of</sub>

Ca(HCO<sub>3</sub>)<sub>2</sub> salt present in the water sample.

 $Q_{HCO_3}$  - the initial quantity of m-alk. (only  $CO_3$  &  $HCO_3$ ).

Here, the residual quantity of Ca is then calculated as follows.

$$R_{Ca} = Q_{Ca} - Q_{HCO_3}$$
 ..... (5.2).

where,  $R_{Ca}$  - first residual quantity of Ca which will be combining with  $R_{SO_4}$  and/or  $R'_{Cl}$  as in eq. (6.1) and/or eq. (6.3).

Formulation Of Some Mathematical Models For The Estimations Of The Most Probable Salts Derived 763

From The Major Mineral Constituents In Natural Water

 $\mathbf{Q}_{\mathbf{Ca}}$  - the initial quantity of  $\mathbf{Ca}$  measured in water sample.

(ii) Case - II: when,  $Q_{Ca} < m$ -alk (if p-alk absent).

$$Q_{Ca(HCO_3)_2} = Q_{Ca}$$
 .....(5.3).

Here, the residual quantity of m-alk (only CO<sub>3</sub> & HCO<sub>3</sub>) after combining with Ca is calculated as follows.

$$R_{HCO_3} = Q_{HCO_3} - Q_{Cl}$$
 .....(5.4)

where,  $R_{HCO_3}$  is the first residual quantity of m-alk (i.e.  $CO_3$  &  $HCO_3$ ) which will be combine with the last residual quantity of Mg (i.e.  $R'_{Mg}$  obtained in eq. (4.4) as in eq. (8.1).

# 6. Estimation of the SO<sub>4</sub> - salt as CaSO<sub>4</sub>.

(i) Case – I: When  $R_{Ca} \ge R_{SO_a}$ .

$$Q_{CaSO_4} = R_{SO_4}$$
 .....(6.1).  
=  $R_{Ca}$  (if  $R_{SO_4} = R_{Ca}$ )

where,  $Q_{CaSO_4}$  - quantity of  $CaSO_4$   $R_{SO_4}$  - first residual quantity of  $SO_4$  after combination with Mg (obtained in eq. 4.1).

Here, the second residual quantity of Ca is calculated as follows.

$$R'_{Ca} = R_{Ca} - R_{SO_4}$$
 ..... (6.2).

where, R' is the second residual quantity of Ca which will be combining with  $R'_{Cl}$  (obtained in eq. 4.4) as in eq. 7.

(ii) Case – II : When 
$$R_{Ca} < R_{SO_A}$$

$$Q_{CaSO_A} = R_{Ca}$$
 ..... (6.3).

where, R<sub>Ca</sub> is the first residual quantity of Ca as obtained in eq. (5.2).

Here, the second residual quantity of SO<sub>4</sub> is to be calculated as follows.

$$R'_{SO_4} = R_{SO_4} - R_{Ca} \cdot (6.4)$$

where,  $R'_{SO_4}$  is the second residual quantity of  $SO_4$  which will be combining with  $R'_{Na}$  (obtained in eq. 2.4) as in eq. 9.1

# 7. Estimation of Cl-salt as CaCl<sub>2</sub>.

$$Q_{CaCl_2} = R'_{Cl} - R'_{Ca}$$
 .....(7).

 $R'_{Cl}$  - second residual quantity of Cl as left in eq. (3.4).

 $R'_{Ca}$  - second residual quantity of Ca as obtained in eq. (6.2).

# 8. Estimation of $HCO_3$ -salt as $Mg(HCO_3)_2$

(i) Case – I: 
$$R_{HCO_3} = R'_{Mg'}$$

$$Q_{Mg(HCO_3)_2} = R'_{Mg} = R_{HCO_3}$$
 .....(8.1).

where,  $R'_{Mg}$  - second residual quantity of Mg as obtained in eq. 4.4.

 $R_{HCO_3}$  - first residual  $HCO_3$  as left in eq. (5.4).

(ii) Case - II : When  $R_{HCO_3} > R'_{Mg'}$ 

In such case, the second residual HCO<sub>3</sub> would be--

$$R'_{HCO_3} = R_{HCO_3} - R'_{Mg}$$
 ..... (8.2).

where, 
$$R'_{HCO_3}$$
 is the second residual  $HCO_3$ .

If second residual  $HCO_3$  is found to left in any water sample analysis, then this will be combining with Na as in eq. (4.2) of Rule – 2. For such type of water, it is not suggested to apply Rule–1 but to be followed Rule–2.

# 9. Estimation of SO<sub>4</sub>-salt as Na<sub>2</sub>SO<sub>4</sub>.

(i) Case-I: When  $R'_{SO_A} = R'_{Na}$ 

$$Q_{Na_2SO_4} = R'_{Na} = R'_{SO_4}$$
 .....(9.1).

where, 
$$Q_{Na_2SO_4}$$
 - quantity of  $Na_2SO_4$ .

 $R'_{Na}$  - the second residual quantity of Na as obtained in eq. (2.4).

 $R'_{SO_4}$  - the second residual quantity of  $SO_4$  as obtained in eq. (6.4).

(ii) Case – II: When  $R'_{SO_4} < R'_{Na}$ .

$$Q_{Na_2SO_4} = R'_{SO_4}$$
 .....(9.2).

But in such case, the third and last residual quantity of Na is calculated as follows.

$$R''_{Na} = R'_{Na} - R'_{SO_A}$$
 .....(9.3).

where,  $R''_{Na}$  is the third and last residual quantity of Na which will be finally combining with  $NO_3$  or other minor anionic species (if any) as in eq. (10).

# 10. Estimation of $NO_3$ -salt as $NaNO_3$

$$Q_{NaNO_3} = R''_{Na} = Q_{NO_3}$$
 .....(10)

where,  $Q_{NaNO_3}$  is the quantity NaNO\_3 subjected to the presence of NO\_3 in the analysed water sample. Which has been termed as  $Q_{NO_3}$ .

#### RULE-2 (WHEN TOTAL HARDNESS < M-ALK)

# 1. Estimation of Alkali Compound as NaOH (if p-alk present in water sample).

$$Q_{NaOH} = Q_{p-alk}$$
 .....(1.1)

where,  $Q_{\mbox{NaOH}}$  is quantity of NaOH.

Then, 
$$R_{Na} = Q_{Na} - Q_{p-alk}$$
 .....(1.2)

where, R<sub>Na</sub> - residual quantity of Na which will be combining with HCO<sub>3</sub>, Cl and SO<sub>4</sub>.

# 2. Estimation of HCO3-salt as Ca(HCO3)2.

(i) Case – I: When  $Q_{HCO_3} \ge Q_{Ca}$ .

$$Q_{Ca(HCO_3)_2} = Q_{Ca}$$
 .....(2.1).

where,  $Q_{Ca(HCO_3)_2}$  - quantity of  $Ca(HCO_3)_2$ .

 $\mathbf{Q}_{\mathbf{Ca}}$  - initial quantity of  $\mathbf{Ca}$  as in water sample.

Here, the residual quantity of HCO<sub>3</sub> may be calculated as follows.

$$R_{HCO_3} = Q_{HCO_3} - Q_{Ca}$$
 .....(2.2).

where,  $R_{HCO_3}$  - first residual quantity of  $HCO_3$  left after combination with Ca and which will be combining with Mg,  $R_{Na}$  and Fe (if any more  $HCO_3$  is present). However, the residual  $HCO_3$  will first combine with Mg as in eq. (3.1).

 $Q_{HCO_3}$  - initial quantity of  $HCO_3$  in water.

(ii) Case – II : When 
$$Q_{HCO_3} < Q_{Ca}$$
 .

$$Q_{Ca(HCO_3)_2} = Q_{HCO_3}$$
 .....(2.3).

Here, the residual quantity of Ca is calculated as follows.

$$R_{Ca} = Q_{Ca} - Q_{HCO_3}$$
 ..... (2.4).

where,  $R_{Ca}$  is the first residual quantity of Ca which will be then combining with Cl and  $SO_4$  shown later on.

# 3. Estimation of HCO3 salt as Mg(HCO3)2

(i) Case – I: When  $R_{HCO_3} \ge Q_{Mg}$ .

$$Q_{Mg(HCO_3)_2} = Q_{Mg}$$
 .....(3.1).

where,  $Q_{Mg(HCO_3)_2}$  - quantity of  $Mg(HCO_3)_2$  salt.

 $Q_{Mg}$  - initial quantity of Mg in water sample.

Here, the second residual quantity of HCO<sub>3</sub> is calculated as follows.

$$R'_{HCO_3} = R_{HCO_3} - Q_{Mg}$$
 ......(3.2).

where,  $R'_{HCO_3}$  - second residual quantity of  $HCO_3$  left after combination with Mg which enters into further combination with  $R_{Na}$  and Fe.

(ii) Case – II : 
$$R_{HCO_3} < Q_{Mg}$$

$$Q_{Mg(HCO_3)_2} = R_{HCO_3}$$
 .....(3.3).

where,  $R_{HCO_3}$  is the first residual quantity of  $HCO_3$  remaining in excess as in eq. (2.2).

Here, the residual quantity of Mg is calculated as follows.

$$R_{Mg} = Q_{Mg} - R_{HCO_3}$$
 .....(3.4).

where, the  $R_{Mg}$  is the first residual quantity of Mg left after combination with the residual HCO<sub>3</sub> as in eq. (3.4) and which will be combining with Cl and  $SO_4$  later on.

# 4. Estimation of HCO3 sait as NaHCO3.

(i) Case – I: When  $R_{HCO_3} \ge R_{Na}$ .

$$Q_{NaHCO_3} = R_{Na} = Q_{Na}$$
 (if p-alk absent).... (4.1).

where,  $Q_{NaHCO_3}$  - quantity of NaHCO<sub>3</sub> salt

 $R_{Na}$  - first residual amount of (Na) as in eq. (1.1).

Q<sub>Na</sub> - initial quantity Na in water sample.

Here, the third residual quantity of  $HCO_3$  is calculated as follows.

$$R''_{HCO_2} = R'_{HCO_2} - R_{Na}$$
 .....(4.2)

where,  $R''_{HCO_3}$  - the third residual amount of  $HCO_3$  constituent which will be combining with Fe and will be balanced at this stage.

R'<sub>HCO<sub>3</sub></sub> - the second residual HCO<sub>3</sub> as left after combination with Mg as in eq. (3.2).

(ii) Case – II: When  $R'_{HCO_3} < R_{Na}$ .

$$Q_{NaHCO_3} = R'_{HCO_3}$$
 .....(4.3).

where,  $R'_{HCO_3}$  is the second residual quantity of  $HCO_3$  as in eq. (3.2).

Here, the second residual amount of Na is calculated as follows.

$$R'_{Na} = R_{Na} - R'_{HCO_3}$$
 .....(4.4).

where,  $R'_{Na}$  is the second residual amount of Na which will be combining with Cl and  $SO_4$  given in eq. (5.1 or 5.3) and eq. (10.1 or 10.3). The residual amount (if any may combine with  $NO_3$  as shown in eq. (11).

#### 5. Estimation of Ci-sait as NaCl.

(i) Case – I: When  $Q_{Cl} \ge R'_{Na}$ .

$$Q_{NaCl} = R'_{Na}$$
 .....(5.1).

where, Q<sub>NaCl</sub> - propable quantity of NaCl salt.

 $Q_{Cl}$  - initial quantity of Cl in water sample.

 $R'_{Na}$  - the second residual quantity of Na according to eq. (4.4).

Here, the residual quantity of Cl is calculated as follows.

$$R_{Cl} = Q_{Cl} - R'_{Na}$$
 ..... (5.2).

where,  $R_{Cl}$  is the first residual amount of Cl which will be combining with  $R_{Mg}$  as left in eq. (3.4).

(ii) Case – II : When  $Q_{Cl} < R'_{Na}$ .

$$Q_{NaCl} = Q_{Cl} \dots (5.3).$$

Here, the third residual quantity of Na is calculated as follows.

$$R''_{Na} = R'_{Na} - Q_{C1} \dots (5.4).$$

where,  $R_{Na}^{"}$  is the third residual quantity of Na which will be combining with  $R_{SO_4}^{'}$  as left in eq. (9.2) according to either of eq. (10.1) or (10.2).

### 6. Estimation of CI - sait as MgCl<sub>2</sub>.

(i) Case – I: When  $R_{Mg} \ge R_{Cl}$ 

$$Q_{MgCl_2} = R_{Cl}$$
 .....(6.1).

where,  $R_{Cl}$  - first residual amount of Cl as obtained in eq. (5.2).

$$Q_{MgCl_2}$$
 - quantity of  $MgCl_2$  salt.

Here, the second residual quantity of Mg is calculated as follows.

$$R'_{Mg} = R_{Mg} - R_{Cl}$$
 .....(6.2).

where,  $R'_{Mg}$  is the second residual quantity of Mg which will be combining with  $R_{SO_4}$  (obtained in eq. 7.2) as shown in the eq. (9.2).

(ii) Case – II : When  $R_{Mg} < R_{Cl}$ .

$$Q_{MgCl_2} = R_{Mg} \dots (6.3)$$

where,  $R_{Mg}$  is the first residual quantity of Mg as in eq.(3.4).

Here, the second residual amount of Cl is calculated as follows.

$$R'_{Cl} = R_{Cl} - R_{Mg}$$
 ..... (6.4).

Formulation Of Some Mathematical Models For The Estimations Of The Most Probable Salts Derived 767
From The Major Mineral Constituents In Natural Water

where,  $R'_{Cl}$  is the second quantity of Cl which will be combining with  $R'_{Ca}$  (as in eq. 7.4) according to eq. (8.1) or (8.2).

# 7. Estimation of $SO_4$ salt as $CaSO_4$

(i) Case – I: When 
$$Q_{SO_4} \ge R_{Ca}$$
.

$$Q_{CaSO_4} = R_{Ca}$$
 .....(7.1).

where,  $Q_{\text{CaSO}_4}$  - quantity of CaSO<sub>4</sub> salt .

 $Q_{SO_4}$  - initial quantity of  $SO_4$  in water .

R<sub>Ca</sub> - first residual quantity of Ca as in eq. (2.4).

Here, the residual quantity is calculated as follows.

$$R_{SO_4} = Q_{SO_4} - R_{Ca}$$
 .....(7.2).

where,  $R_{SO_4}$  is the first residual quantity of  $SO_4$  which will be combining with  $R_{Mg}$  (as in eq. 6.2). and  $R_{Na}$  (as in eq. 5.4) according to eq. (9.1) and (10.1).

(ii) Case – II : When  $Q_{SO_A} < R_{Ca}$ 

$$Q_{CaSO_4} = Q_{SO_4}$$
 .....(7.3).

Here, the second residual quantity of Ca is calculated as follows.

$$R'_{Ca} = R_{Ca} - Q_{SO_4}$$
 ..... (7.4).

where,  $R'_{Ca}$  is the second residual quantity Ca which will be combining with  $R'_{Cl}$  (obtained eq. 6.4) as shown in eq. (8.1) or (8.2).

#### 8. Estimation of Ci-sait as CaCl<sub>2</sub>.

(i) Case –I: When  $R'_{Cl} > R'_{Ca}$ 

$$Q_{CaCl_2} = R'_{Cl} = R'_{Ca}$$
 ..... (8.1).

At this stage of combination, both of the Ca and Cl constituents would be exhausted and balanced.

(ii) Case – II : When  $R'_{Cl} > R'_{Ca}$ 

$$Q_{CaCl_2} = R'_{Ca}$$
 .....(8.2).

where,  $Q_{CaCl_2}$  is the quantity of  $CaCl_2$  salt in

Here, the third residual quantity of Cl is calculated as follows.

$$R''_{Cl} = R'_{Cl} - R'_{Ca}$$
 ..... (8.3).

where, the R"<sub>Cl</sub> is the third residual quantity of Cl which will be combining with Fe or any undetected trace cationic constituent. But possibility of such case is very rare.

# 9. Estimation of SO<sub>4</sub>-salt as MgSO<sub>4</sub>.

When  $R_{SO_4} \ge R'_{Mg}$ :

$$Q_{MgSO_A} = R'_{Mg}$$
 .....(9.1).

where,  $Q_{MgSO_4}$  - quantity of  $MgSO_4$  salt mater.

 $R'_{Mg}$  - second residual quantity of Mg (as in the eq. 6.2).

Here, the second residual quantity of SO<sub>4</sub> is calculated as follows.

$$R'_{SO_4} = R_{SO_4} - R'_{Mg}$$
 .....(9.2).

where,  $R'_{SO_4}$  is the second residual quantity of  $SO_4$  which will be finally combining and balancing with  $R_{Na}$  (obtained in eq. 5.4) according to eq. (10.1 and 10.3).

# 10. Estimation of SO<sub>4</sub>-salt as Na<sub>2</sub>SO<sub>4</sub>.

(i) Case - I: When 
$$R'_{SO_4} = R''_{Na}$$
.

$$Q_{Na_2SO_4} = R''_{Na} = R'_{SO_4}$$
 .....(10.1).

where,  $Q_{Na_2SO_4}$  - quantity of Na2SO4 salt assumed.

R"Na - the second residual quantity of Na.

(ii) Case – II : When 
$$R'_{SO_4} < R''_{Na}$$
.
$$Q_{Na_2SO_4} = R'_{SO_4} - \dots (10.2).$$

Here, the 4th residual quantity of Na is calculated as follows.

$$R^{\prime\prime\prime}_{Na} = R^{\prime\prime}_{Na} - R^{\prime}_{SO_4}$$
 ...... (10.3). where, the  $R^{\prime\prime\prime}_{Na}$  is the 4th and last residual quantity of Na which will be combining with NO<sub>3</sub> (if any) according to eq. (11).

(iii) Case - III: When all the quantity of Ca and Mg are exhausted by prior combining with HCO<sub>3</sub> and Cl constituents, in such case,

$$Q_{Na_2SO_4} = Q_{SO_4} = R''_{Na}$$
 .....(10.4).

# 11. Estimation of $NO_3$ -salt as $NaNO_3$ .

Satisfying the major anionic constituents (HCO<sub>3</sub><sup>-</sup>, Cl<sup>-</sup> and SO<sub>4</sub><sup>-2</sup>) if any further residual quantity of Na is left, then that quantity may be considered for combining with NO<sub>3</sub><sup>-</sup> constituent (if present) and the probable quantity of the salt is calculated from the relationship as follows.

$$Q_{NaNO_3} = R'''_{Na}$$
 .....(11).

where,  $Q_{NaNO_3}$  is the quantity of NaNO<sub>3</sub> salt present in water sample analysed.

#### **Experiments And Results**

#### Verification of the Validity of the Models by Analytical Results of Typical Water Samples.

Example-1  Water analysis (Type: TH > m-alk).  Cations, meq/l  Anions, meq/l		Example-2  Water analysis (Type: TH < m-alk).  Cations, meq/l  Anions, meq/l		
$Mg^{+2}$ 23.0	m-alk : 3.4	Mg+2 : 2.3	m-alk : 8.5	
TH 30.0	Cl : 130.9	TH : 3.8	Cl : 3.7	
* Na <sup>+</sup> : 112	$50_4^{-2}:7.7$	* Na <sup>+</sup> : 8.7	$50_4^{-2}:0.3$	
$M_{\mathbf{C}} = 142$	$M_{\mathbf{a}} = 142$	$M_{\mathbf{C}} = 12.5$	$M_{\mathbf{a}} = 12.5$	

<sup>\* [</sup>Here, Na<sup>+</sup> has been determined mathematically according to eq. (C) i.e. Na<sup>+</sup> =  $M_a$  – TH i.e. 142-30 = 112 ; 12.5-3.8 = 8.7].

Table-1: Substitution of the analytical results of Example-1 in appropriate equations of Rule-1 and derivation of the salts.

Most probable Salt assumed	Equation applied for calculation	Quantity of salt assumed (meq/l)	Proportion of the assumed salt (%)
NaCl	$Q_{\text{NaCl}} = Q_{\text{Na}} \text{ (eq. 2.1)}$	112	78.87
MgCl <sub>2</sub>	Q <sub>MgCL2</sub> =R <sub>Cl</sub> (eq. 3.1)	18.9	13.31
MgSO <sub>4</sub>	Q <sub>MgSO<sub>4</sub>=R<sub>Mg</sub> (eq. 4.1)</sub>	4.1	2.89
Ca(HCO <sub>3</sub> ) <sub>2</sub>	Q <sub>Ca(HOC<sub>3</sub>)<sub>2</sub>=Q<sub>HCO<sub>3</sub></sub> (eq. 5.1)</sub>	3.4	2.39
CaSO <sub>4</sub>	$Q_{\text{CaSO}_4} = R_{\text{SO}_4} \text{ (eq. 6.1)}$	3.6	2.54

**Table-2:** Substitution of the analytical results of Example-2 in appropriate equations of Rule-2 and derivation of the salts.

Most probable Salt assumed	Equation applied for calculation	Quantity of salt assumed (meq/l)	Proportion of the assumed salt (%)
Ca(HCO <sub>3</sub> ) <sub>2</sub>	$Q_{Ca(HCO_3)_2} = Q_{Ca} \qquad (2.1)$	1.5	12
Mg(HCO <sub>3</sub> ) <sub>2</sub>	$Q_{Mg(HCO_3)_2} = Q_{Mg} \qquad (3.1)$	2.3	18.4
NaHCO <sub>3</sub>	$Q_{\text{NaHCO}_3} = R_{\text{HCO}_3} $ (3.2)	4.7	37.6
NaCl	$Q_{\text{NaCl}} = Q_{\text{Cl}} \qquad (5.3)$	3.7	29.6
Na <sub>2</sub> SO <sub>4</sub>	$Q_{Na_2SO_4} = Q_{SO_4}$ (10.4)	0.3	2.4

#### Discussion

The probable salt derivation and estimation may be done by 2 ways of calculation designated as Rule-1 and Rule-2. Each rule consists of a series of correlative equations termed as 'models' which have been derived from the successive residual quantities of the mineral constituents. Any one of the rules should be followed. Right rule is selected for a particular water sample according to the following quantitative status of the total hardness (TH) and malkalinity (normally  $HCO_3^-$  constituent).

TH>m-alk.

TH=m-alk.

TH<m-alk.

In the case of TH ≥ m—alk, Rule—1 to be applied and when TH < m—alk, then Rule—2 is applicable. The other criteria for the derivation of each individual salts have been explained in the respective equations of the models.

For the application of appropriate models in finding out the most probable salts and their concentrations, the following factors must always be considered.

- Right rule should be selected first according to the above quantitative relation between TH and m-alkalinity.
- 2. Individual equations of the respective selected rule to be applied.

- Appropriate successive equations should be selected and systematically applied according to the equation serial and residual quantitative position of the available ionic concentration.
- During determination of a specific salt component, the condition of predominance between the two constituents taking part in combination must be noted carefully.

After the completion of calculations, if any small residual cationic or anionic constituents still remain(s) surplus, that should be discarded in view of the combination of the undetected trace constituents of opposite charges.

In the equations of Rule-1, the last residual  $HCO_3^-$  (after combining with  $Ca^{+2}$  would have to be balanced with  $Na^+$  to produce  $NaHCO_3$  and with  $Fe^{+2}$  to form  $Fe(HCO_3)_2$ . Such cases frequently occur when m-alk > TH in any water [8,9].

The experimental results of the two typical water sample analyses (Example-1 & 2) have proved the validity of the models. Table-1 and Table-2 give the guide lines for the application procedure of the appropriate Rule and its respective models.

#### Acknowledgment

The author is grateful to the Authority of Khulna Power Station of Bangladesh Power Development Board for giving the opportunity to do this elaborate and extensive research work by providing the laboratory facilities. He is also indebted to Prof. Dr. S.Z. Haider, Dept. of Chemistry, Dhaka University for his constant

guidance, strict work supervision and hard labour in editing this paper incorporated in the Ph. D. thesis.

#### References

- M. R. Miah, Prob. of Indus. Wat. Treat. and Boil. Wat. Condi. in Bangladesh, Ph. D. Thesis, Dhak. Univ., Dhaka (May, 1994).
- R. Kunin, Amber-hi-lites, Tech.Lit., Fluid Proc. Dept., Rohm and Haas Co., Philadelphia, USA, 164, 1(Summ. 1980).
- F. I. Bilan, Wat. Treat. (Trans. Russ-Eng), Mir Publi., Moscow, 24(1985).
- F.T. Morse, Power Plant Engg., Affili. East-West Press, New Delhi, India, Stud. Ed. 521 (1978).
- E.W. Taylor, Exam. of Wat. and Wat. Suppl., J&A Churchill Ltd. London, 6, 18&254(1949).
- G.S. Solt, Optn. in Boil. Feed Wat. Treat., Train.
   Paper, Lorch Foundation, School of Wat. Treat.,
   Highwycomb, England, 16-17 Nov., (1981).
- G. Nikoladze et. al., Wat. Treat. for Pub. Heal. and Ind. Suppl. (Eng. Trans.), Mir Publi., Moscow, 2, 13(1989).
- DEGRIMONT Co. Ion Exch. Mat., Tech. Lit., Paris, France, No 1289L, 14 (Nov. 1981).
- R. Kunin, Amber-hi-lites, Tech. Lit., Fluid Proc. Dept., Rohm and Haas Co., Philadelphia, USA, 131, 3 (Nov. 1972).