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LQG/LTR 기법을 이용한 불확실한 선형 시스템의 견실한 출력 되먹임 제어기의 설계

장 태 정

A Robust Output Feedback Controller Design for Uncertain Linear Systems Using LQG/LTR

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ABSTRACT

In this paper, a controller design method for uncertain linear systems by output feedback is proposed. This method utilizes the LQG/LTR procedure for systems with uncertainties described in the time domain. It is assumed that the uncertainties satisfy the matching conditions and their bounds are known. First, a robust state feedback controller design method is introduced. Then, it is asymtotically recovered for the output feedback system by the loop transfer recovery(LTR) method under a certain condition.

1. Introduction

The system which has unstructured uncertainties can be made to be stable by the LQG/LTR method[2] when the uncertainties bounds, usually described in the frequency domain, are not too big. This gives the impression that the control system designed by the LQG/LTR procedure is robust against parameter perturbations. But it has been recognized that LQG/LTR method may cause servere robustness problems even though the target LQ regulator is robust against plant parameter variations[5].

The loss of robustness in the LQG/LTR procedure is mainly due to the following two reasons. First, in the LQG/LTR design, the plant modelling errors described in the frequency domain are considered, but not the variations of parameters. It is possible that small variations of the parameters may cause large changes in the frequency domain. Second, the asymptotic recovery is accomplished for the nominal system without considering the plant uncertainties, and thus the robust recovery is not guaranteed for the modelling errors.

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For these reasons, it is necessary to consider the robustness of the target feedback loop for the plant parameter uncertainties and its robust recovery conditions.

For the original LQG/LTR method [2]-[4], in which the plant modelling errors are described in the frequency domain, it is difficult to analyze the effect of the plant parameter variations in the time domain because of the complex relations between the modelling errors represented in the frequency domain and those in the time domain. Since the plant is frequently modelled in the state space, it is important to study uncertain systems with modelling errors in the time domain.

The problem of stabilizing uncertain linear systems using state feedback control has attracted a considerable interest in recent years[6]-[11]. In this paper, an output feedback controller design method is proposed to stabilize uncertain linear systems instead of the state feedback method. This method utilizes the LQG/LTR procedure. The robust state feedback control system is regarded as a target feedback loop of the LQG/LTR procedure. A robust recovery condition, under which the proposed output feedback is possible, is given.

This paper is organized as the following. In section 2, a robust state feedback controller design method is presented. In section 3, an output feedback controller design method is proposed and the condition for a robust loop transfer recovery is derived. Conclusions are given in section 4.

In this paper, x' $[A']$ denotes the transpose of vector x [matrix A]. For two matrices A and B, $A>B$ $[A \geq B]$ represents that the matrix $A-B$ is positive definite [positive semi-definite]. The notation $\lambda_{\max}(A)\,[\lambda_{\min}(A)]$ means the maximum [minimum] eigenvalue of matrix A, and $\sigma_{\max}(A)$ $[\sigma_{\min}(A)]$ means the maximum [minimum] singular value of matrix A defined by $\sigma_{\max}(A)=\{\lambda_{\max}(A'A)\}^{1/2}$ $[\sigma_{\min}(A)=\{\lambda_{\min}(A'A)\}^{1/2}]$. The notation $\det(A)$ stands for the determinant of matrix A.

2. Robust State Feedback Controller Design for Uncertain Systems

Consider the uncertain system described by

$$\dot{x}(t)=(A+\Delta A)x(t)+(B+\Delta B)u(t), \quad (1)$$

where $x(t)\in R^{n}$ is the state, $u(t)\in R^{m}$ is the control, $A\in R^{n\times n}$ and $B\in R^{n\times m}$ are system and input matrices, respectively, and $\Delta A\in R^{n\times n}$ and $\Delta B\in R^{n\times m}$ are their corresponding uncertainties. Assume that the pair (A, B) is controllable, and the uncertainties ΔA and ΔB satisfy the matching conditions with their bounds known, i.e.,

$$\Delta A=BD,\ D'D\leq \alpha^2 I,\ \alpha\geq 0, \quad (2)$$

$$\Delta B=BE,\ E'E\leq \beta^2 I,\ 0\leq \beta < 1. \quad (3)$$

We also assume that $m < n$ and B has full column rank.

Under these assumptions, we are going to design a state feedback controller, which stabilizes the uncertain system, to get a target feedback loop for the LQG/LTR procedure. The following definition has been frequently used by many authors [8]-[11].

Definition [11]: The uncertain system (1) is said to be *quadratically stabilizable (via linear control)* if there exists a constant matrix $K_c \in \mathrm{R}^{m\times n}$, a positive definite symmetric matrix $P \in \mathrm{R}^{n\times n}$, and a constant $\lambda > 0$ such that, for any admissible uncertainties ΔA and ΔB which satisfy the conditions (2) and (3), the closed-loop system with the state feedback control law $u(t) = -K_c x(t)$ and the Lyapunov function $V(x) = x'Px$ has the following property:

$$L(x,t) = \frac{dV(x)}{dt}$$
$$= x'[(A+\Delta A)'P + P(A+\Delta A)]x$$
$$-2x'P(B+\Delta B)K_c x \le \lambda \|x\|^2, \qquad (4)$$

for all pairs $(x,t) \in \mathrm{R}^n \times \mathrm{R}$. $\|\cdot\|$ denotes the standard Euclidean norm. □

It is well known that if the inequality (4) is satisfied, the closed loop system is uniformly asymptotically stable at the equilibrium point $x = 0$, for any given admissible uncertainties.

Theorem 1: Let $Q \in \mathrm{R}^{n\times n}$ be a given symmetric matrix, such that $Q > \lambda I$, $\lambda > 0$. Then, for constants ε and μ which satisfy $0 < \varepsilon < 1-\beta$ and $0 < \mu < 2(1-\beta-\varepsilon)$, the Riccati equation

$$A'P + PA - \mu PBB'P + \frac{\alpha^2}{2\varepsilon} I + Q = 0 \qquad (5)$$

has a positive definite solution P, and the system (1) is quadratically stabilizable with the control law

$$u(t) = -K_c x(t) = -B'Px(t). \qquad (6)$$

Fact: For any matrices X and Y with appropriate dimensions and for $\gamma > 0$, we have

$$\pm(X'Y + Y'X) \le \gamma X'X + \frac{1}{\gamma} Y'Y. \qquad (7)$$

Proof of Theorem 1: Let $R_0 = \mu^{-1} I$ and $Q_0 = (\alpha^2/2\varepsilon)I + Q$. Since $R_0 > 0$, $Q_0 > 0$, and the pair (A, B) is controllable, the Riccati equation (5) always has a positive definite solution P [12]. Let the Lyapunov function be given by $V(x) = x'Px$. By (2), (3), (5) and (7), the Lyapunov derivative corresponding to the system with $u(t) = -B'Px(t)$ satisfies the following inequality:

For $0 < \beta < 1$,

$$\frac{dV(x)}{dt} = x'\{A'P + PA + D'B'P + PBD$$
$$-2PBB'P - PBE'B'P - PBEB'P\}x$$
$$\le x'\{A'P + PA + 2\varepsilon PBB'P + (2\varepsilon)^{-1}D'D$$
$$-2PBB'P + \beta PBB'P + \beta^{-1}PBE'EB'P\}x$$
$$\le x'\{A'P + PA - 2(1-\beta-\varepsilon)PBB'P$$
$$+ (\alpha^2/2\varepsilon)I\}x$$
$$\le x'\{A'P + PA - \mu PBB'P + (\alpha^2/2\varepsilon)I\}x$$
$$= -x'Qx \le -\lambda\|x\|^2.$$

For $\beta = 0$, i.e., $\Delta B = 0$,

$$\frac{dV(x)}{dt} = x'\{A'P + PA + 2\varepsilon PBB'P$$
$$+ (2\varepsilon)^{-1}D'D - 2PBB'P\}x$$
$$\le x'\{A'P + PA - \mu PBB'P + (\alpha^2/2\varepsilon)I\}x$$
$$\le -\lambda\|x\|^2.$$

Hence, the uncertain system (1) is quadratically stabilizable with the control law $u(t)=-B'Px(t)$. □

Remark 1: One possible choice for the constants α and β is $\alpha=\sigma_{\max}(D)$, and $\beta=\sigma_{\max}(E)$.

Remark 2: For the ordinary LQ regulator, $u(t)=-\mu B'Px(t)$. Therefore, when $\mu\neq 1$, the control law $u(t)=-B'Px(t)$ differs from the control law of LQ regulator. Since $0<\mu<2(1-\beta-\varepsilon)$, we can have $\mu=1$ when $1-2\beta>0$, and this confines the admissible β within $0<\beta<1/2$.

Remark 3: The assumptions and the results given in this section are the same as those proposed by Thorp and Barmish [6]. The only difference between these two is the way of obtaining the stabilizing controller. The design method in this paper is simpler than the method by Thorp and Barmish.

Remark 4: It can be easily shown that Theorem 1 is also valid if the assumption in (3) is replaced by

$$\Delta B=BE,$$
$$\frac{1}{2}\lambda_{\min}(E+E')\geq-\beta,\quad \beta<1. \tag{3'}$$

The assumption in (3)′ is more general than the assumption in (3). In this case, we may choose $\mu=1$ for $\beta<1/2$.

In the following section, we will consider an output feedback control of the uncertain system that asymptotically recovers the properties of the state feedback control. To obtain the loop transfer function of the state feedback system, let us denote

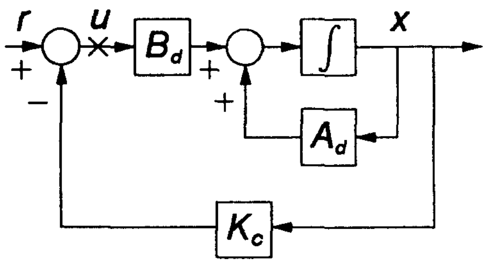

Fig. 1. Closed-loop system with state feedback controller

$$A_d=A+\Delta A,$$
$$B_d=B+\Delta B,$$
$$\Phi_d(s)=(sI-A-\Delta A)^{-1}=(sI-A_d)^{-1}.$$

The structure of the closed-loop system (1) with the state feedback control law in (6) is shown in Fig. 1. If we break the loop of the system at the input, where 'x' is marked, then the loop transfer function $G_{LQ}(s)$ becomes

$$G_{LQ}(s)=K_c\Phi_d(s)B_d. \tag{8}$$

$G_{LQ}(s)$ is the target feedback loop transfer function which we are trying to get by LTR in the next section.

3. A Robust Loop Transfer Recovery Condition for Uncertain Systems

We now consider the uncertain system described by

$$\dot{x}(t)=(A+\Delta A)x(t)+(B+\Delta B)u(t),$$
$$y(t)=(C+\Delta C)x(t), \tag{1'}$$

where $y(t)\in R^m$ is the output,

$C \in \mathrm{R}^{m \times n}$ is the output matrix, $\Delta C \in \mathrm{R}^{m \times n}$ is the output uncertainty, and the other definitions and assumptions are the same as previously given in Section 2. Assume that the pair (C, A) is observable, and C has full row rank. Let us denote $C_d = C + \Delta C$. Then, the transfer function $G(s)$ of the system (1)$'$ is an $m \times m$ matrix given by

$$G(s) = C_d \Phi_d(s) B_d. \tag{9}$$

Let us design an output feedback controller based on nominal model. The state estimate $z(t)$ of the system is constructed using Kalman filter,

$$\dot{z}(t) = Az(t) + Bu(t) + K_f(y(t) - Cz(t)). \tag{10}$$

Here, $K_f \in \mathrm{R}^{n \times m}$ is the Kalman filter gain matrix obtained by $K_f = SC'$, where $S \in \mathrm{R}^{n \times n}$ is the solution of the algebraic Riccati equation

$$AS + SA' - SC'CS + q^2 BB' = 0. \tag{11}$$

The control input $u(t)$ is given by

$$u(t) = -K_c z(t), \tag{12}$$

where $K_c \in \mathrm{R}^{m \times n}$ is given in (6). The loop transfer function $K(s)$ from $y(t)$ to $-u(t)$ is [3]

$$K(s) = K_c(sI - A + BK_c + K_f C)^{-1} K_f. \tag{13}$$

The structure of the closed-loop system (1)$'$ with the output feedback control law in (10) and (12) is shown in Fig. 2. The following theorem gives a condition

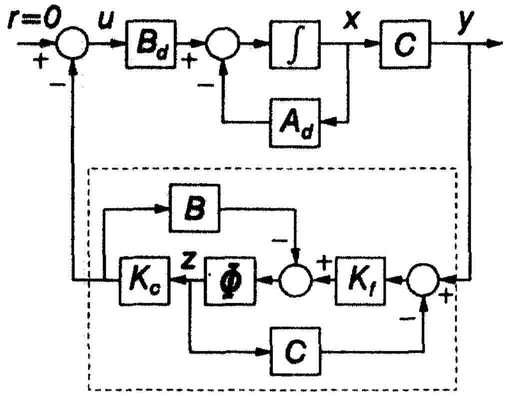

Fig. 2. Closed-loop system with output feedback controller

which guarantees that the output feedback (12) in the system (1)$'$ approaches those of the state feedback (6).

Theorem 2: Assume that $\Delta C = 0$. Then, for the uncertain system (1)$'$, the input breaking loop transfer function $K(s)G(s)$ in Fig. 2 asymtotically recovers the target loop transfer function $G_{LQ}(s)$ as $q \to \infty$, if the matrix $C\Phi_d B$ is minimum phase for all admissible uncertainties ΔA as described in (2). In this case, the closed-loop system is stable.

proof: Let $\Phi_c(s) = (sI - A + BK_c)^{-1}$. Then,

$$\begin{aligned} &K(s)G(s) \\ &= K_c(\Phi_c^{-1} + K_f C)^{-1} K_f C_d \Phi_d B_d \\ &= K_c[\Phi_c - \Phi_c K_f (I + C\Phi_c K_f)^{-1} C\Phi_c] \\ &\qquad \times K_f C_d \Phi_d B_d \\ &= K_c \Phi_c K_f (I + C\Phi_c K_f)^{-1} C_d \Phi_d B_d. \end{aligned}$$

Since $q^{-1} K_f \to BW$ as $q \to \infty$ [1], where

W is an orthonormal matrix, then as $q \to \infty$

$$
\begin{aligned}
&K(s)G(s) \\
&\to K_c \Phi_c B(C\Phi_c B)^{-1} C_d \Phi_d B_d \qquad (14) \\
&= K_c \Phi_d B[I+(D+K_c)\Phi_d B]^{-1} \\
&\quad \times \{C\Phi_d B[I+(D+K_c)\Phi_d B]^{-1}\}^{-1} \\
&\quad \times C_d \Phi_d B_d \qquad (15) \\
&= K_c \Phi_d B(C\Phi_d B)^{-1}(C+\Delta C) \\
&\quad \times \Phi_d B(I+E) \qquad (16) \\
&= G_{LQ}(s). \qquad (17)
\end{aligned}
$$

Here, $\Phi_c B = [\Phi_d^{-1} + B(D+K_c)]^{-1} B = \Phi_d B[I+(D+K_c)\Phi_d B]^{-1}$ is used to get (15) from (14). Since the inverse of the matrix $I+(D+K_c)\Phi_d B$ is cancelled in (15), $I+(D+K_c)\Phi_d B$ must be minimum phase. Note that the zeroes of $\det(I+(D+K_c)\Phi_d B)$ is the same as the poles of the state feedback system described by

$$
\begin{aligned}
\dot{x}(t) &= (A+\Delta A)x(t) + Bu(t), \\
u(t) &= -(D+K_c)x(t).
\end{aligned}
$$

By (2), the closed-loop system is given by

$$
\begin{aligned}
\dot{x}(t) &= (A+\Delta A - BD - BK_c)x(t) \\
&= (A - BK_c)x(t),
\end{aligned}
$$

which has nominally stable poles. Hence the matrix $I+(D+K_c)\Phi_d B$ is minimum phase and the cancellation in (15) is possible. In order to cancel the inverse of the matrix $C\Phi_d B$ in (16), $C\Phi_d B$ must be minimum phase. From (17), we see that

$$
\det(I+K(s)G(s)) \to \det(I+G_{LQ}(s)),
$$

as $q \to \infty$. Since the target feedback system is stable, the closed-loop system in Fig. 2 is stable. □

Remark 5: For matched uncertainties ΔA and ΔB illustrated in (2) and (3), ΔA is more important than ΔB. Because the condition for LTR depends on $C\Phi_d B$ being minimum phase for all admissible uncertainties of ΔA. Kharitonov's theorem [13] can be useful to check whether $C\Phi_d B$ is minimum phase for all addmissible uncertainties of ΔA.

Remark 6: Assume that $\Delta C \neq 0$, and $C\Phi_d B$ is minimum phase for all admissible uncertainties ΔA as described in (2). Then, the stability of the closed-loop system (1)′ with the control law in (10) and (12) can be guaranteed as $q \to \infty$ if

$$
\begin{aligned}
&\sigma_{\min}(I+G_{LQ}(j\omega)) \\
&> \sigma_{\max}(K_c \Phi_d(j\omega)B[C\Phi_d(j\omega)B]^{-1}\Delta C \\
&\qquad \times \Phi_d(j\omega)B_d), \qquad (18)
\end{aligned}
$$

for all $\omega \geq 0$, and for all admissible ΔA and ΔB. From (18), we can find the bound of ΔC which maintains the stability of the closed-loop system as $q \to \infty$ in LTR.

4. Conclusions

In this paper, an output feedback controller design method is proposed to stabilize uncertain linear systems, whose uncertainties are described in the time domain. It is assumed that the uncertainties satisfy the matching conditions and their bounds are known. A robust state feedback controller design method is used to get a target feedback loop for the LQG/LTR procedure. Then, LQG/LTR procedure is utilized. When $\Delta C=0$, it is shown that for the matched uncertainties ΔA and ΔB, the asymptotic recovery to the target state feedback control system for the output feedback control system is guaranteed if $C\Phi_d B$ is minimum phase for all admissible uncertainties of ΔA.

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