Convergence Analysis of the Least Mean Fourth Adaptive Algorithm

최소평균사승 적응알고리즘의 수렴특성 분석

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ABSTRACT

The least mean fourth (LMF) adaptive algorithm is a stochastic gradient method that minimizes the error in the mean fourth sense. Despite its potential advantages, the algorithm is much less popular than the conventional least mean square (LMS) algorithm in practice. This seems partly because the analysis of the LMF algorithm is much more difficult than that of the LMS algorithm, and thus not much still has been known about the algorithm. In this paper, we explore the statistical convergence behavior of the LMF algorithm when the input to the adaptive filter is zero-mean, wide-sense stationary, and Gaussian. Under a system identification mode, a set of nonlinear evolution equations that characterizes the mean and mean-squared behavior of the algorithm is derived. A condition for the convergence is then found, and it turns out that the convergence of the LMF algorithm strongly depends on the choice of initial conditions. Performances of the LMF algorithm are compared with those of the LMS algorithm. It is observed that the mean convergence of the LMF algorithm is much faster than that of the LMS algorithm when the two algorithms are designed to achieve the same steady-state mean-squared estimation error.

요 약

최소평균사승 적응알고리즘은 추정오차의 평균사승값을 최소화하는 추정경도방법 가운데 하나이다. 알고리즘의 잠재적인 여러 장점에도 불구하고, 이 알고리즘은 현재 기존의 최소평균자승 알고리즘 보다 실제 적게 주목받고 있다. 그 이유는 최소평균사승 알고리즘의 수렴특성에 관한 통계적 분석이 최소평균자승 알고리즘에 비해 매우 어렵고, 따라서 아직 알고리즘에 대해 모르는 부분이 많기 때문으로 보인다. 본 논문에서는 적응필터의 입력신호가 평균이 영이고 시불변 가우시안 랜덤신호일 경우 최소평균사승 적응알고리즘의 통계적인 수렴특성에 대하여 연구하였다. 이를 위해, 시스템인지 모드에서 알고리즘의 평균 및 평균자승 특성을 나타내는 일련의 관계식을 유도하였다. 그리고 알고리즘의 평균특성이 수렴하기 위한 조건을 찾았는데, 여기서 최소평균사승 적응알고리즘의 수렴특성이 초기치의 선택에 크게 좌우됨을 알 수 있었다. 또한 최소평균사승 알고리즘의 성능을 기존의 최소평균자승 알고리즘과 실험적으로 비교하였고, 두개의 알고리즘이 정상상태에서 같은 값의 평균자승추정오차를 갖을 때 최소평균사승 알고리즘이 최소평균자승 알고리즘에 비해 매우 빠른 수렴속도를 갖을 수 있음을 확인하였다.

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I. Introduction

The adaptive LMS algorithm [1], [2] has received a great deal of attention during the last two decades and is now widely used in variety of applications due to its simplicity and relative ease of analysis. The algorithm tries to minimize the mean-squared estimation error at each iteration. There exist, however, only a limited number of researches for adaptive filtering algorithms that are based on higher order error conditions (or non-mean-square error criteria) [3]-[7]. Despite the potential advantages, these algorithms are less popular than the conventional LMS algorithm in practice. This seems partly because the analysis of the higher order error based algorithms is much more difficult, and thus not much still has been known about the algorithms.

The LMF adaptive algorithm is a special case for which the error function to be minimized is the mean of the estimation error to the fourth power. This error function is a perfect convex function of the filter coefficient vector, and therefore does not have local minima. Moreover, the LMF algorithm is intuitively appealing comparing to the LMS algorithm, since the estimation error during the early adaptation process is usually large so that the error to the higher power would improve the convergence speed, while the error during the steady-state is sufficiently small (surely, if designed well) so that the error to the higher power would increase the precision.

Walach and Widrow [7] presented a convergence analysis of the LMF algorithm, and compared its performances with the LMS algorithm under the "system identification mode". They evaluated the relaxation time constants for the weights, and showed that the time constants in the weight relaxation process for the LMF algorithm is proportional to those for the LMS algorithm. By evaluating the ratio between the misadjustment of the LMS algorithm and that of the LMF algorithm, they also showed that the LMF algorithm has substantially less noises in the filter coefficients than the conventional LMS algorithm for the same speed of convergence, except the case when the plant measurement noise of the unknown system has a Gaussian distribution. For the Gaussian plant noise, it was shown that the LMS algorithm outperforms the LMF algorithm. Conditions for the convergence of the mean and mean-squared behavior of the LMF algorithm were also derived. The results in [7] are, however, somewhat restrictive since the analysis is limited to the simple case in which the filter coefficients are already close to the optimal values. The assumption significantly simplifies the analysis, but any information about the transient behavior of the algorithm can not be obtained.

In this paper, we explore the statistical convergence behavior of the LMF algorithm when the input to the adaptive filter is zero-mean, wide-sense stationary, and Gaussian. Under a system identification mode, a set of nonlinear evolution equations that characterizes the mean and mean-squared behavior of the algorithm is derived. Price's theorem[9] as well as the decomposition property of the Gaussian higher order moments into multiplications of the second moments are used as the main tools for the analysis.

Now, consider the problem of the system identification as depicted in Figure 1, where $d(n)$ and $x(n)$ represent the primary and reference input signals, and $e_{min}(n)$ and $e(n)$ denote the measurement noise and the estimation error signals, respectively. The unknown plant to be identified is assumed to be linear and time invariant.

Let $H(n)$ denote the adaptive filter coefficient vector of size N. Define the reference input vector $X(n)$ as

$$X(n) = [x(n), x(n-1), \cdots, x(n-N+1)]^T, \qquad (1)$$

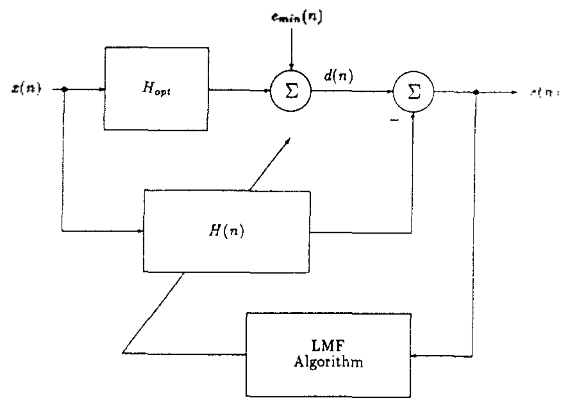

Figure 1. Adaptive system identification model.

where $[\cdot]^T$ denotes the transpose of $[\cdot]$. The LMF algorithm under consideration updates the coefficient vector $H(n)$ using

$$H(n+1)=H(n)+\mu X(n)e^3(n), \tag{2}$$

where μ is the adaptation step-size, and

$$e(n)=d(n)-X^T(n)H(n). \tag{3}$$

As can be seen is (2) and (3), the LMF algorithm requires additionally only two more multiplications for each iteration comparing to the LMS algorithm.

In the next section, under a set of assumptions and a set of mild approximations, the nonlinear difference equations that characterize the mean and mean-squared behavior of the filter coefficients and the mean-squared estimation error are derived. A condition for the mean convergence is also found.

II. Convergence Analysis

Before starting the analysis, let us define the following notations: Let H_{opt} denote the coefficient vector of the unknown system given by

$$H_{opt}=R_{XX}^{-1}\ R_{dX}, \tag{4}$$

where

$$R_{XX}=E\{X(n)X^T(n)\}, \tag{5}$$

$$R_{dX}=E\{d(n)X(n)\}, \tag{6}$$

and $E\{\cdot\}$ denotes the statistical expectation of $\{\cdot\}$. Also, define the coefficient misalignment vector $V(n)$ as

$$V(n)=H(n)-H_{opt}, \tag{7}$$

and its autocorrelation matrix $K(n)$ as

$$K(n)=E\{V(n)V^T(n)\}. \tag{8}$$

Using (7) in (2), we get

$$V(n+1)=V(n)+\mu X(n)e^3(n). \tag{9}$$

The optimal estimation error $e_{min}(n)$, that is the same as the measurement noise in the system identification mode, is given by

$$e_{min}(n)=d(n)-X^T(n)\ H_{opt}. \tag{10}$$

Combining (3), (7), and (10), it follows that

$$e(n)=e_{min}(n)-X^T(n)\ V(n). \tag{11}$$

Finally, let

$$\sigma_e^2(n)=E\{e^2(n)\} \tag{12}$$

and

$$\xi_{min}=E\{e_{min}^2(n)\} \tag{13}$$

denote the mean-squared estimation error and the minimum mean-squared estimation error (or the power of the measurement noise), respectively.

Convergence analysis of the LMF algorithm is much more complicated than that of the LMS algorithm due to existence of the higher order error signal in the coefficient update equation. We make the following assumptions to make the analysis mathematically more tractable :

Assumption 1 : $d(n)$ and $X(n)$ are zero-mean, wide-sense stationary, and jointly Gaussian random processes.

Assumption 2 : The input pair $\{d(n), X(n)\}$ at time n is independent of $\{d(k), X(k)\}$ at time k, if $n \neq k$.

Assumption 3 : The measurement noise $e_{min}(n)$ of the plant is also zero-mean, wide-sense stationary, and independent of $d(n)$ and $x(n)$.

A consequence of Assumption 1, which is of great importance for the analysis, is that the estimation error $e(n)$ given in (3) is also zero-mean and Gaussian when conditioned on the coefficient vector $H(n)$ (or equivalently, on $V(n)$). Assumption 2 is the commonly employed "independence assumption" [8] and is valid if μ is chosen to be sufficiently small. One direct consequence of Assumption 2 is that $H(n)$ is independent of the input pair $\{d(n), X(n)\}$ since $H(n)$ depends only on inputs at time $n-1$ and before. Note also that Assumption 2 does not restrict the nature of the matrix R_{XX}, and should not be confused with the white signal assumption.

Now, taking the statistical expectation on both sides of (9) gives

$$E\{V(n+1)\} = E\{V(n)\} + \mu\, E\{X(n)\, e^3(n)\}. \quad (14)$$

The last expectation of (14) can be simplified using the fact that for zero-mean and jointly Gaussian random variables x_1 and x_2,

$$E\{x_1\, x_2^3\} = E\{x_1 x_2\}\, E\{x_2^2\}. \quad (15)$$

Thus, using (15) in conjunction with Assumption 1, it follows that

$$\begin{aligned} E\{X(n)\, e^3(n)\} &= E\{E[X(n)\, e^3(n) \,|\, V(n)]\} \\ &= 3E\{E[e^2(n)|V(n)]\, E[X(n)e(n)|V(n)]\} \\ &= 3E\{\sigma^2_{e|V}(n)\, E[X(n)\, e(n)\,|\,V(n)]\}, \end{aligned} \quad (16)$$

where

$$\sigma^2_{e|V}(n) = E\{e^2(n) \,|\, V(n)\}, \quad (17)$$

and from (11)

$$\begin{aligned} E\{X(n)\, e(n) \,|\, V(n)\} &= E\{X(n)\,[\,e_{min}(n) - X^T(n)\, V(n)\,] \,|\, V(n)\} \\ &= -R_{XX}\, V(n). \end{aligned} \quad (18)$$

Note in (18) that we have made use of Assumption 2 as well as the orthogonality principle that is given by

$$E\{X(n)\, e_{min}(n)\} = 0_N, \quad (19)$$

where 0_N is the null vector of size N. Substituting (17) and (18) in (16) yields

$$\begin{aligned} E\{X(n)\, e^3(n)\} &= -3\, R_{XX}\, E\{\sigma^2_{e|V}(n)\, V(n)\} \\ &\approx -3\sigma_e^2(n)\, R_{XX}\, V(n). \end{aligned} \quad (20)$$

In (20), we have made use of an approximation as

$$E\{\sigma^2_{e|V}(n)\, V(n)\} \approx \sigma_e^2(n)\, E\{V(n)\}. \quad (21)$$

Therefore, using (20) in (14), we have the mean behavior for the coefficient misalignment vector of the LMF algorithm as

$$E\{V(n+1)\} \approx [\,I_N - 3\,\mu\, \sigma_e^2(n)\, R_{XX}\,]\, E\{V(n)\}, \quad (22)$$

where I_N denotes the $N \times N$ identity matrix. This expression can be equivalently rewritten using the coefficient vector as

$$E\{H(n+1)\} \approx [\,I_N - 3\mu\,\sigma_e^2(n)\, R_{XX}\,]\, E\{H(n)\} + 3\mu\,\sigma_e^2(n)\, R_{dX}. \quad (23)$$

From (22), it is easy to show that the mean behavior of the coefficient misalignment vector $E\{V(n)\}$ converges to the zero vector (or equivalently, $E\{H(n)\}$ approaches H_{opt}) if the convergence parameter μ is selected to be

$$0<\mu<\frac{2}{3\lambda_{max}\,\sigma_e^2(n)}, \qquad \forall n, \tag{24}$$

where λ_{max} represents the maximum eigenvalue of the matrix R_{XX}. Notice, unfortunately, that there exists the time-varying function $\sigma_e^2(n)$ in the upper bound for μ. More restrictive but more sufficient condition for the convergence can be given by

$$0<\mu<\frac{2}{3\lambda_{max}\max\{\sigma_e^2(n)\}}, \tag{25}$$

where $\max\{\sigma_e^2(n)\}$ denotes the maximum value of $\sigma_e^2(n)$ for all n. Since $\sigma_e^2(n)$ is usually large at the beginning of adaptation processes, we can see that the convergence of the LMF algorithm strongly depends on the choice of initial conditions. It would be desirable to give the initial conditions such that the initial errors are kept as low as possible.

Recall that the condition for the mean convergence of the LMF algorithm sought by Walach and Widrow [7] is given by

$$0<\mu<\frac{2}{3\lambda_{max}\,\xi_{min}} \tag{26}$$

However, this condition was obtained under the very wild assumption that the convergence has already taken place. The upper bound in (26) is much looser than that in (25) particularry during the early adaptation processes ($\sigma_e^2(n)\geq\xi_{min}$ for all n), implying that for some initial values, the LMF algorithm may blow up. In fact, the condition in (26) is only a necessary condition for the mean convergence of the LMF algorithm.

We next derive an expression for the mean-squared estimation error $\sigma_e^2(n)$. Employing (11) in (12) yields

$$\sigma_e^2(n)=\xi_{min}+E\{V^T(n)\,X(n)\,X^T(n)\,V(n)\}$$
$$-2E\{V^T(n)\,X(n)\,e_{min}(n)\}. \tag{27}$$

Here, ξ_{min} is obtained by using (10) in (13) so that

$$\xi_{min}=E\{d^2(n)\}-H_{opt}^T R_{dX}, \tag{28}$$

and by the independence assumption,

$$E\{V^T(n)\,X(n)\,X^T(n)V(n)\}=tr\{K(n)\,R_{XX}\}, \tag{29}$$

where $K(n)$ is defined in (8), and $tr\{\cdot\}$ denotes the trace of $\{\cdot\}$. The last expectation of (27) becomes zero by the independence assumption (Assumption 2) as well as the orthogonality principle. It thus follows that

$$\sigma_e^2(n)=\xi_{min}+tr\{K(n)\,R_{XX}\}. \tag{30}$$

Finally, we derive an expression for $K(n)$ to complete the analysis. Substituting (9) in (8) leads to

$$K(n+1)=K(n)+\mu^2 E[X(n)\,X^T(n)\,e^6(n)]$$
$$+\mu E[V(n)\,X^T(n)\,e^3(n)]+\mu E[X(n)\,V^T(n)\,e^3(n)]. \tag{31}$$

Here, using Assumptions 1 and 2, the second expectation on the right-hand side (RHS) of (31) becomes

$$E[V(n)\,X^T(n)\,e^3(n)]$$
$$=E\{V(n)\,E[X^T(n)\,e^3(n)\,|V(n)]\}$$
$$=3E\{V(n)\,E[X^T(n)\,e(n)\,|V(n)]\,E[e^2(n)\,|V(n)]\}$$
$$=-3E\{\sigma_{e|V}^2(n)\,V(n)\,V^T(n)\,R_{XX}\}$$
$$\approx -3\sigma_e^2(n)\,K(n)\,R_{XX}. \tag{32}$$

In (32), we have made use of the result in (20) and the approximation as

$$E\{\sigma_{e|V}^2(n)\,V(n)\,V^T(n)\}\approx\sigma_e^2(n)\,K(n). \tag{33}$$

In a similar way, the last expectation on the RHS of (31) leads to

$$E[X(n)V^T(n)e^3(n)] \approx -3\sigma_e^2(n) R_{XX} K(n). \quad (34)$$

In order to simplify the first expectation on the RHS of (31) any further, the following result will be used :

- Let x_1, x_2, and x_3 be zero-mean, jointly Gaussian random variables with covariance matrix

$$R = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{12} & r_{22} & r_{23} \\ r_{13} & r_{23} & r_{33} \end{bmatrix}.$$

Then, by Price's theorem [9] as well as the decomposition property of the Gaussian higher order moments into multiplications of the second moments, it follows

$$E\{x_1 x_2 x_3^6\} = 15 r_{33}^2 [r_{12} r_{33} + 6 r_{13} r_{23}] \quad (35)$$

Now, since

$$E\{X(n)X^T(n)e^6(n)\} = E\{E[X(n)X^T(n)e^6(n)|V(n)]\}, \quad (36)$$

using (35) and (18) in (36), it is not difficult to get

$$E\{X(n)X^T(n)e^6(n)\}$$
$$= 15E[\sigma_{e|V}^4(n)[\sigma_{e|V}^2(n)R_{XX} + 6R_{XX}V(n)V^T(n)R_{XX}]]$$
$$\approx 15\sigma_e^4(n)[\sigma_e^2(n)R_{XX} + 6R_{XX}K(n)R_{XX}]. \quad (37)$$

In (37), we have made use of another two approximations that

$$E\{\sigma_{e|V}^6(n)\} \approx \sigma_e^6(n), \quad (38)$$

and

$$E\{\sigma_{e|V}^4(n)R_{XX}V(n)V^T(n)R_{XX}\} \approx \sigma_e^4(n)R_{XX}K(n)R_{XX}. \quad (39)$$

Therefore, combining (32), (34), and (37) with (31), we obtain an expression for the second order behavior of the coefficients of the LMF algorithm as

$$K(n+1) \approx K(n) - 3\mu\sigma_e^2(n)[K(n)R_{XX} + R_{XX}K(n)]$$
$$+ 15\mu^2\sigma_e^4(n)[\sigma_e^2(n)I_N + 6R_{XX}R_{XX}K(n)]R_{XX}. \quad (40)$$

Ⅲ. Experimental Results

Here, we present experimental results for which the LMF algorithm is used in identifying a linear and time-invariant system to demonstrate the validity of our analysis. The reference input $x(n)$ to the adaptive filter is modeled as a pseudorandom white Gaussian process with zero-mean and unit variance. The corresponding primary input $d(n)$ is generated by processing $x(n)$ through the linear and time-invariant FIR system with seven coefficients given by

$$H_{opt} = [0.1, 0.3, 0.5, 0.7, 0.5, 0.3, 0.1]^T, \quad (41)$$

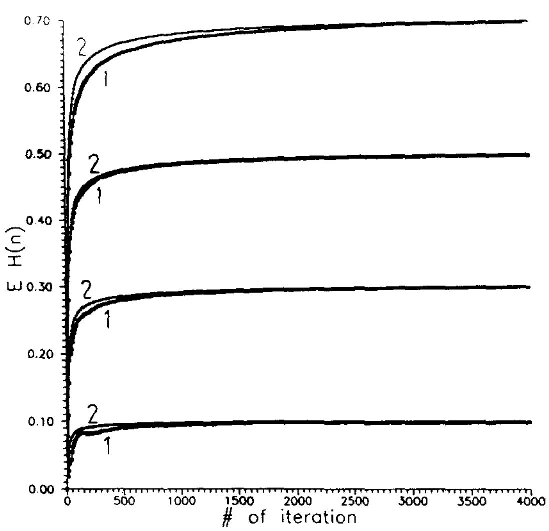

Figure 2. Mean behavior of the coefficients, $E\{H(n)\}$ (Only curves for the first four coefficients are displayed) : 1 = simulation result, 2 = theoretical result.

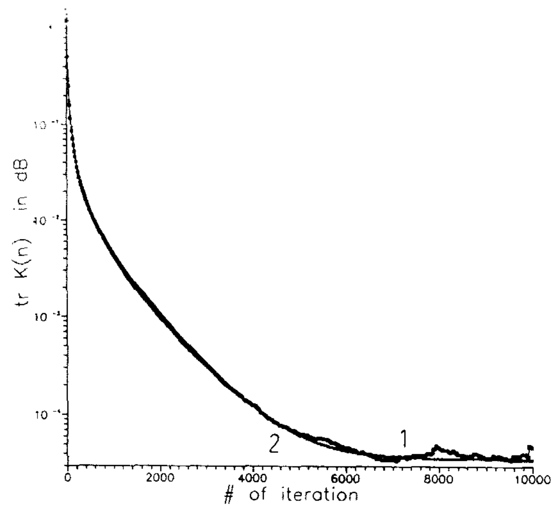

Figure 3. Mean-squared behavior of the coefficients, $tr\{K(n)\}$ in dB : 1 = simulation result, 2 = theoretical result.

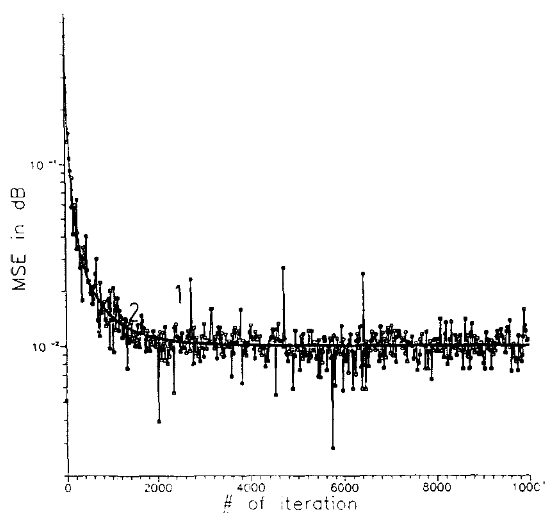

Figure 4. Mean-squared estimation error, $\sigma_e^2(n)$ in dB : 1 = simulation result, 2 = theoretical result.

and then corrupting the system output using a zero-mean and white random sequence with variance 0.01.

Figure 2 illustrates the empirical and theoretical results for the mean behavior $E\{H(n)\}$ of the first four adaptive filter coefficients, and Figures 3 and 4 show those for the mean-squared behavior of the coefficients and for the mean-squared estimation error, respectively. Displayed in Figure 3 are only the curves for the sum of all the diagonal elements of the matrix $K(n)$ (i.e., $tr\{K(n)\}$), for convenience. In Figures 2-4, curves 1 and 2 represent the empirical and theoretical results, respectively. The empirical results are obtained by averaging over 50 independent runs using 10,000 samples each, and the convergence parameter μ is chosen to be 0.02. It can be seen that the theoretical results show a fairly good agreement with the empirical results.

Next, we empirically compare the convergence speed of the LMF algorithm with that of the conventional LMS algorithm. The results are once again obtained by averaging over 50 independent runs with 10,000 samples, and the convergence parameter μ_{LMF} for the LMF algorithm is chosen to be 0.02. In order to perform a fair comparison, we select the convergence parameter μ_{LMS} for the LMS algorithm 0.0012 in such a way that the two

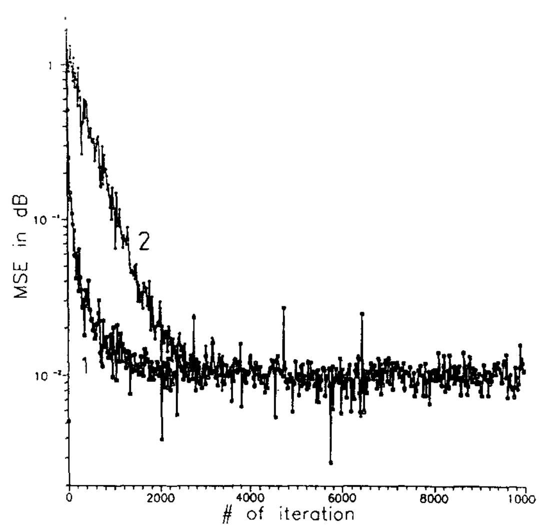

Figure 5. Mean-squared estimation error, $\sigma_e^2(n)$ in dB : 1 = LMF algorithm (μ_{LMF} = 0.02), 2 = LMS algorithm (μ_{LMS} = 0.0012).

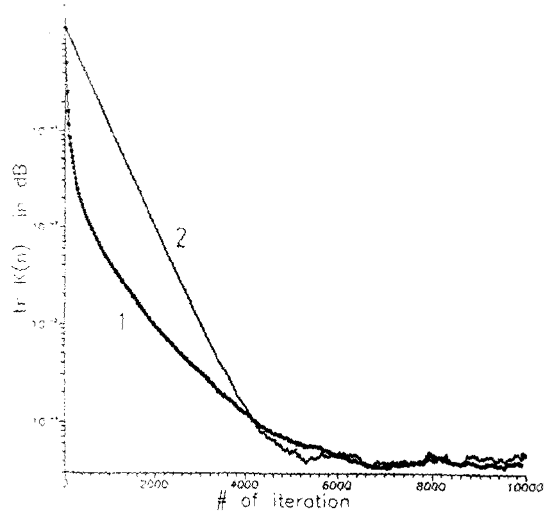

Figure 6. Mean-squared behavior of the coefficients, $tr\{K(n)\}$ in dB : 1 = LMF algorithm (μ_{LMF} = 0.02), 2 = LMS algorithm (μ_{LMS} = 0.0012).

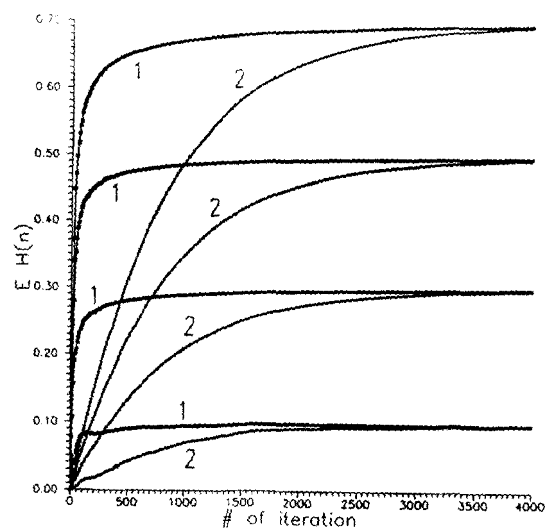

Figure 7. Mean behavior of the coefficients, $E\{H(n)\}$ (Only curves for the first four coefficients are displayed) : 1 = LMF algorithm (μ_{LMF} = 0.02), 2 = LMS algorithm (μ_{LMS} = 0.0012).

algorithms produce the same mean-squared estimation error in the steady state. Illustrated in Figures 5 and 6 are simulation curves of $\sigma_e^2(n)$ and $tr\{K(n)\}$ for both the LMF and the LMS algorithm, respectively. We can see that the mean-squared behavior of the coefficients and the mean-squared estimation error of the two algorithms indeed approach the same value in the steady-state when the convergence parameters μ_{LMF} and μ_{LMS} are selected to be 0.02 and 0.0012, respectively. However, the LMF algorithm shows much faster convergence than the LMS algorithm in the mean-squared sense. Now, Figure 7 displays the corresponding mean behavior of the two algorithms. It is observed that the mean convergence of the LMF algorithm is also much faster than that of the LMS algorithm when the two algorithms are designed to achieve the same steady-state mean-square error.

Ⅳ. Conclusion

In this paper, the statistical convergence analysis of the LMF algorithm is presented in the system identification mode. The wild assumption employed in the previous work [7] is relaxed. In particular, the transient behavior of the LMF algorithm is investigated when the input signals are zero-mean, wide-sense stationary, and Gaussian. Price's theorem [9] as well as the decomposition property of the Gaussian higher order moments into multiplications of the second moments are used as the main tools for the analysis. A condition for the mean convergence is also found, and it turns out that the convergence of the LMF algorithm strongly depends on the choice of initial conditions. Computer simulations show that our theoretical results agree with simulation ones fairly well. Also observed is that the mean convergence of the LMF algorithm is much faster than that of the LMS algorithm when the two algorithms are designed to achieve the same steady-state mean-squared error. Moreover, through extensive computer simulations, we have found that there are many other cases in which the LMF algorithm outperforms the conventional LMS algorithm from the viewpoints of the convergence speed as well as the precision.

We have, however, had difficulties in deriving exact expressions for the mean and mean-squared convergence of the LMF algorithm. As an alternative, we have made use of the four approximations as were in (21), (33), (38), and (39). Surely these approximations misleads our analytical results to some extent. We are currently working on obtaining better results by removing the approximations. As a future work, we will work on deriving expressions for the steady-state responses and a condition on μ for the mean-squared convergence of the LMF algorithm, and also on making quantitative comparisons with the LMS algorithm.

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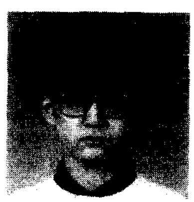

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