Application of the Polar Parabolic Equation Method for Sound Propagation Over a Smooth Sea Mountain in the Ocean

해저구릉 위로의 음의 전파를 설명하기 위한 Polar PE의 적용

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ABSTRACT

The polar parabolic equation method (Polar PE) which introduces a series of "cascaded" boundary fitting coordinates into the parabolic equation method has been verified as a good numerical method for atmospheric sound propagation over a curved surface and hills. Polar PE is applied here to underwater sound propagation over a sea mountain assuming locally reacting boundary sea bottom and pressure release water surface for the boundary conditions. Calculations are presented for underwater propagation over a 450 m high sea mountain. Feasibility of Polar PE application for underwater sound propagation over a smooth mountain is discussed.

요 약

경계조건에 맞는 일련의 연속된 좌표계를 parabolic equation method에 적용한 polar parabolic equation method (Polar PE)는 하나의 곡면이나 언덕이 존재하는 경우 대기에서 음의 전파를 설명하는데 알맞은 수치 이론임이 입증되었다. 본 논문에서는 locally reacting 해저면과 pressure release 해수면의 경계조건을 사용하여 Polar PE 를 수중에서 해저구릉이 존재할 경우에 음의 전파를 설명하는데 적용하였다. 450 m 높이의 해저구릉이 존재할 경우, 음의 전파에 관하여 계산하고 그 결과를 살펴보았다. Polar PE 를 수중에 해저구릉이 있는 경우 음의 전파를 계산하는데 적용가능성을 논의하였다.

INTRODUCTION

Leontovich and Fock[1] introduced the parabolic equation method (PE) for electromagnetic wave propagation along the earth surface in 1946. Since then, the PE has been introduced to the underwater sound propagation by Tappert and Hardin[2] in 1973 and to the outdoor sound propagation by Gilbert and White [3] in 1989. The PE solves the acoustic wave propagation in range-dependent environment, using outgoing waves and neglecting backscattering waves.

For the atmospheric sound propagation, the PE solves the propagation with a realistic sound speed profile over a locally reacting ground surface. The PE treats the turbulence effects[4] in an upward refracting, sound propagation over ground with varying impedance with range[5], and diffraction of sound by a barrier[6]. Recently, the polar parabolic equation method (Polar PE) was developed to solve the sound propagation over hills and a curved surface[7-9]. The Polar PE uses a series of cascaded boundary-fitted coordinate systems with a wide angle PE developed by Gilbert and White. The new coordinate system consists of the distance along the ground surface and the height perpendicular to the ground surface, respectively, as the horizontal and vertical coordinates.

Up-to-date the effects of sea mountain for sound propagation have not received any attention even though there is an important physical phenomenon of sound diffraction over a sea mountain.

In this paper we will describe how to transform the coordinate system for the wave equation to solve the

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diffraction problem and how to apply the Polar PE to underwater sound proapagation over a smooth curved sea mountain. Using the finite sea floor ground impedance and pressure release water surface, the Polar PE calculation predicts the sound propagation with refracting sound speed profiles.

I. THEORY

A. Far field wave equation in standard cylindrical coordinates

The standard Helmholtz equation for wave propagation is

$$\nabla^2 P + k^2 P = 0. \tag{1}$$

where P is the acoustic pressure, $k = \omega/c$ is the wave number, ω is the angular frequency, and c is the speed of sound. The Helmholtz wave equation in the ocean and atomosphere can be approximated as wave propagation in an environment with azimuthal symmetry. That is, the sound field is assumed to propagate symmetrically about the z-axis. Considering this cylindrical symmetry along the z-axis, the wave equation can be written in cylindrical coordinates as

$$\frac{\partial^2 P}{\partial r^2} + \frac{1}{r}\frac{\partial P}{\partial r} + \frac{\partial^2 P}{\partial z^2} + k^2 P = 0. \tag{2}$$

Introducing a new variable, $U = \sqrt{r}\,P$, the wave equation becomes

$$\frac{\partial^2 U}{\partial r^2} + \frac{\partial^2 U}{\partial z^2} + \left(\frac{1}{4r^2} + k^2\right)U = 0. \tag{3}$$

Using the far field assumption, $kr \gg 1$, the far field wave equation can be derived as

$$\frac{\partial^2 U}{\partial r^2} + \frac{\partial^2 U}{\partial z^2} + k^2 U = 0. \tag{4}$$

Equation (4) is useful for propagation over flat sea floor and is transformed to solve the propagation over a sea hill.

B. Far field wave equation over a sea mountain

We treat sound propagation over a sea mountain shown in Fig.1. The solution in standard cylindrical coordinates is used for flat surfaces of regions I and V. Assuming that the far fields propagate over all the regions, the far field wave equation is applied to all the regions but with different meanings for the coordinates and fields depending on the area. To do so, a new polar coordinate system and a boundary fitted coordinate system are introduced in succession to the far field wave equation of Eq.(4) for propagation over regions II, III and IV.

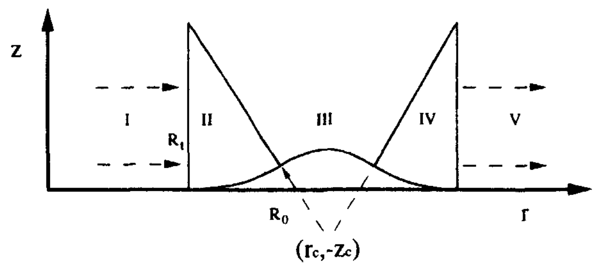

Fig.1 Schematic diagram showing the propagation regions over a sea mountain. R_t is the radius of the region II and IV and R_0 is the radius of the hill-top.

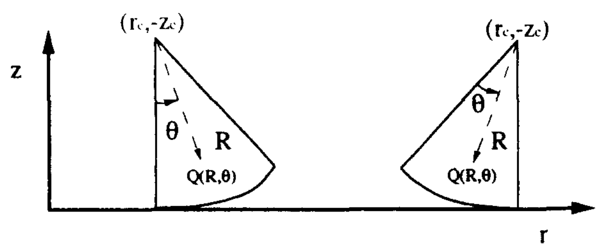

Fig.2 Polar coordinates of the region I (left) and region III (right).

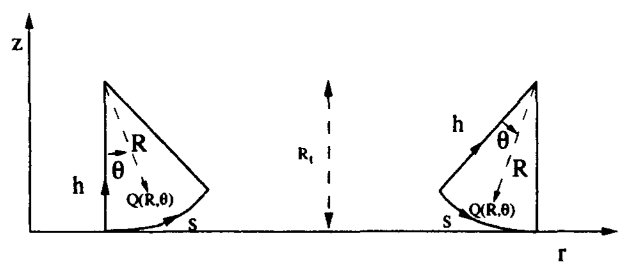

Fig.3 New coordinates (s, h) of the region I (left) and region III (right).

Far field wave equation for regions II and IV

For regions II and IV, a polar coordinate system (R, θ) is introduced as shown in Fig.2. The centers of regions II and IV are (r_c, z_c) and (r_a, z_a), respectively. A point Q is located at (R, θ) in polar coordinates and (r, z) in cylindrical coordinates. The relations between polar and cylindrical coordinates from Fig.2 are

$$R^2 = (r - r_c)^2 + (z_c - z)^2 \text{ and}$$

$$\theta = \tan^{-1}\frac{r - r_c}{z_c - z} \quad \text{for region II}, \tag{5}$$

and $R^2 = (r_a - r)^2 + (z_a - z)^2$ and

$$\theta = \tan^{-1}\frac{r_a - r}{z_a - z} \quad \text{for region IV}. \tag{6}$$

The above relations yield the far field wave equation for region II and IV in the polar coordinate system (R, θ) :

$$\frac{\partial^2 U}{\partial R^2} + \frac{1}{R}\frac{\partial U}{\partial R} + \frac{1}{R^2}\frac{\partial^2 U}{\partial \theta^2} + k^2 U = 0. \tag{7}$$

By introducing an auxiliary variable $\psi = \sqrt{R}\,U$, Eq. (7) becomes

$$\frac{\partial^2 \psi}{\partial R^2} + \frac{1}{R^2}\frac{\partial^2 \psi}{\partial \theta^2} + \frac{1}{4R^2}\psi + k^2\psi = 0. \tag{8}$$

In all regions of physical importance, we have this condition

$$\frac{1}{4R^2} \ll k^2. \tag{9}$$

Near the top boundary, Eq.(9) does not hold but the pressure release water surface is implemented far before the top boundary in our applications.

$$\frac{\partial^2 \psi}{\partial R^2} + \frac{1}{R^2}\frac{\partial^2 \psi}{\partial \theta^2} + k^2\psi = 0 \tag{10}$$

Here a boundary fitted coordinate system (s, h) shown in Fig.3 is introduced for computations where s is the coordinate along the ground surface and h is perpendicular to the s and directed radially inward. R_t is radius of a wedge. From Fig.3,

$$s = R_t \times \theta, \quad h = R_t - R. \tag{11}$$

Using the above relations the transformed far field wave equation in the new coordinate systems is

$$\frac{\partial^2 \psi}{\partial h^2} + \frac{1}{(1 - \frac{h}{R_t})}\frac{\partial^2 \psi}{\partial s^2} + k^2\psi = 0 \tag{12}$$

Far field wave equation for region III

The polar coordinate system (R, θ) for region III is shown in Fig.4. R_0 is the radius of curvature which represents as the curved sea floor. The origin of polar coordinate system is chosen as $(r_c, -z_c)$ at the center of curvature. Considering the point Q in the two coordinate systems, the relation between R and r is

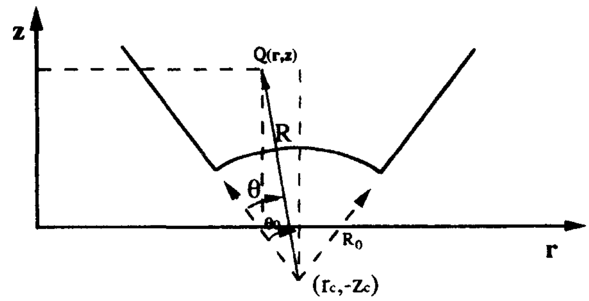

Fig.4 Polar coorninates (R, θ) in region II.

$$R^2 = (r_c - r)^2 + (z + z_c)^2 \text{ and } \theta_0 - \theta = \tan^{-1}\frac{r_c - r}{z + z_c}. \tag{13}$$

Therefore, the farfield wave equation Eq.(4) can be transformed from (r, z) to (R, θ) as

$$\frac{\partial^2 U}{\partial R^2} + \frac{1}{R}\frac{\partial U}{\partial R} + \frac{1}{R^2}\frac{\partial^2 U}{\partial \theta^2} + k^2 U = 0 \tag{14}$$

By introducing an auxiliary variable $\phi = \sqrt{R}\,U$, Eq. (14) becomes

$$\frac{\partial^2 \phi}{\partial R^2} + \frac{1}{R^2}\frac{\partial^2 \phi}{\partial \theta^2} + \frac{1}{4R^2}\phi + k^2\phi = 0 \tag{15}$$

The sound wave propagates over a curved surface with radius of curvature, R_0. The third term can be generally neglected because of $1/4R^2 \ll k^2$.

$$\frac{\partial^2 \phi}{\partial R^2} + \frac{1}{R^2}\frac{\partial^2 \phi}{\partial \theta^2} + k^2\phi = 0 \tag{16}$$

It is convenient to introduce a new beoundary fitted coordinate system (f, g) shown in Fig.5, where f is the coordinate along the curved surface and g is the vertical coordinate perpendicular to f. From Fig.5,

$$f = R_0 \times \theta, \quad g = R - R_0. \tag{17}$$

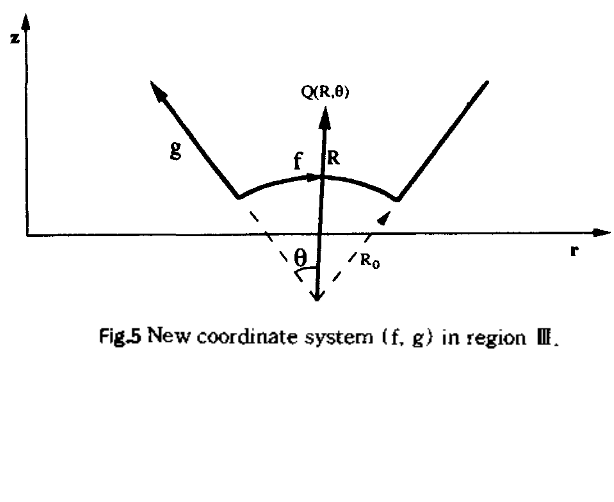

Fig.5 New coordinate system (f, g) in region III.

For the boundary fitted coordinate system (f, g), the following far field wave equation is found :

$$\frac{1}{(1+g/R_0)^2}\frac{\partial^2\psi}{\partial f^2}+\frac{\partial^2\psi}{\partial g^2}+k^2\psi=0. \qquad (18)$$

Unified far field wave equation for all regions

Three different differential wave equations were derived depending on the flat surfaces of Eq.4, the regions II and IV of Eq.12, and the regions III of Eq. 18. Although the fields have different geometries and the coordinate variables are defined differently, the unified far field wave equation for all regions can be written as

$$I(y)\frac{\partial^2 U}{\partial x^2}+\frac{\partial^2 U}{\partial y^2}+k^2U=0 \qquad (19)$$

where for the flat surface, $I(y)=1$, $U=\sqrt{r}\,P$, $x=r$ and $y=z$, for the regions II and IV, $I(y)=1/(1-h/R_t)^2$, $U=\sqrt{r(R_t-h)}\,P$, $x=s$ and $y=h$ and for the region III, $I(y)=1/(1+g/R_0)^2$, $U=\sqrt{r(R_0+g)}\,P$, $x=f$ and $y=g$.

D. Solution of the wave equation

The main difference of the far field wave equation is the factor I(y) depending on the flat surface, regions II and IV, and region III. Both the Polar PE and the conventional PE are followed the finite element method and can be represented as a linear equation :

$$\bar{\bar{T}}\,\bar{E}\,(x+\triangle x)=\bar{\bar{R}}\,\bar{E}\,(x) \qquad (20)$$

where $\bar{\bar{T}}$ and $\bar{\bar{R}}$ are tridiagonal matrices[2]. The vector $\bar{E}$ is $e^{-ik_0x}\,\bar{U}$, where k_0 is the reference wavenumber and the matrix element U_j is proportional to the acoustic pressure P at the height y_j perpendicular to the ground surface. Note that the only difference between the Polar PE and the conventional PE is the tridiagonal matrix elements due to the differences of the factor I(y) depending on the regions. As in the conventional PE, the Gaussian starting field is given at the source and is marched in range to solve the field at the next range step. This procedure is then repeated using the calculated field as a source field. To solve linear equation in Eq. 20, $\bar{\bar{T}}$ is represented by the product of two matrices which are lower triangular and upper triangular. Since they are tridiagonal matrix, the procedure of matrix decomposition for N×N matrix takes only N-operations for each forward and backward substitution[10].

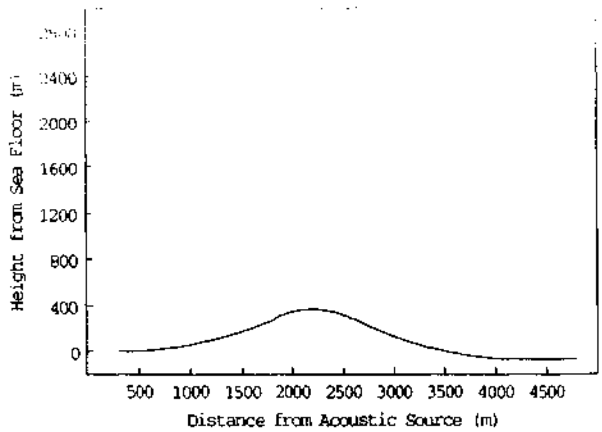

Fig.6 The geometry of sea mountain for calculation. The water surface is placed at 1500 m from the sea floor.

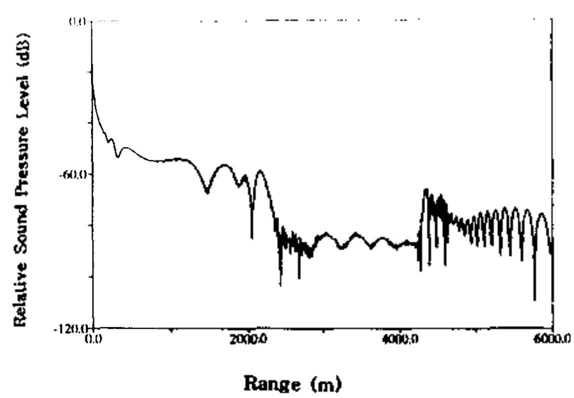

Fig.7 Relative sound pressure level versus distance at 200 Hz when the acoustic source and receiver are located at 1450 m from the water surface.

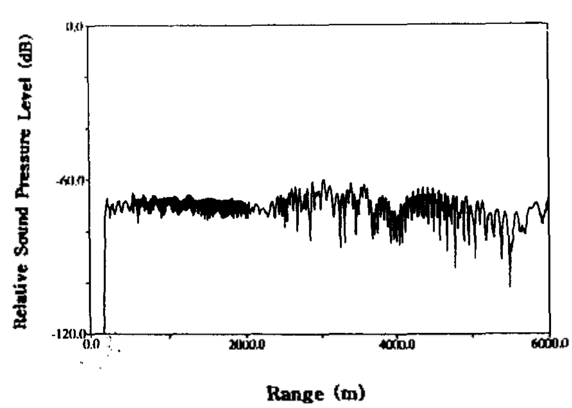

Fig.8 Relative sound pressure level versus distance at 200 Hz when the acoustic source and receiver are located at 1450 m and 500 m from the water surface, respectively.

Equation (20) is solved successively by advancing the field over flat surfaces and over ground surfaces of regions Ⅱ, Ⅲ, and Ⅳ. Since I(y) of Eq.(19) has different values depending on the region, the matrixes $\overline{\overline{T}}$ and $\overline{\overline{R}}$ have different elements at each region.

Ⅱ. SOUND PROPAGATION OVER A SMOOTH 450 m HIGH SEA MOUNTAIN

In general most outdoor ground surfaces can be approximated by locally reacting. The locally reacting ground assumption requires that the specific acoustic ground impedance be independent of the angle of incidence. In the atmosphere, the upper boundary should be treated as a surface that does not reflect the sound wave. Therefore, the radiation boundary condition is used so that the outgoing wave never see the top of the numerical grid by gradually damping wave. As a preliminary calculation for the sound propagation over a sea mountain, we treat the sea floor as a locally reacting surface and the water surface as a pressure release surface.

The operating acoustic frequency, water depth, and mountain height are 200 Hz, 1500 m, and 450 m, respectively. For the calculation, the normalized acoustic impedance of sediment is assumed as (1.48, 0.03) for 200 Hz. The speed of sound for the calculation is

$$c(z) = 1530 - 0.026 \times z \quad \text{m/s for } z \le 1000 \text{ m}$$

$$c(z) = 1486 + 0.016 \times (z - 1000) \quad \text{m/s for } 1000 \text{ m} \le z \le 1500 \text{ m}$$

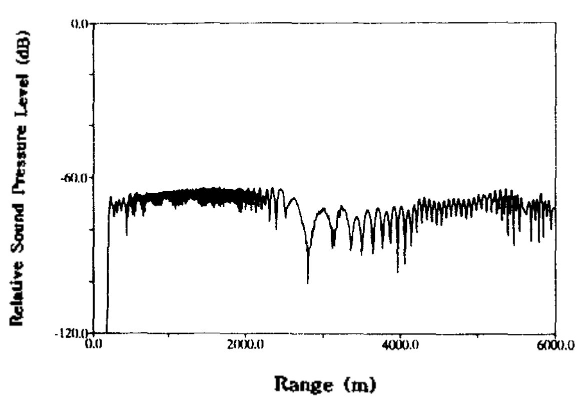

Fig.9 Relative sound pressure level versus distance at 200 Hz when the acoustic source and receiver are located at 500 m and 1450 m from the water surface, respectively.

where z is the depth from the water surface. The above parameters of frequency, acoustic impedance of sediment, sound speed profile, and the depth of source and receiver are used as input data to calculate the linear equation of Eq. 20. Four difference cases for depths of source and receiver are considered for calculations : case 1 – source 1450 m and receiver 1450 m, case 2 – source 1450 m and receiver 500 m, case 3 – source 500 m, receiver 1450 m, and case 4 – source 500 m and receiver 500 m. The geometry of sea mountain for the calculation is shown in Fig.6. Figure 7 gives the result of case 1 which implies non-line of sight sound propagation. Figures 8 and 9 show the calculations for case 2 and case 3 which either source or receiver is placed near to the water surface. Figure 10 is the results of case 4 which both the source and receiver are located at 500 m from the water surface. Those figures tell the differences of diffraction effects depending on the heigh of acoustic source and receiver. In the relative sound pressure level, the reference sound level is the source level calculated at 1 m from the source. The sea mountain ranges 300 m to 4300 m from the acoustic source.

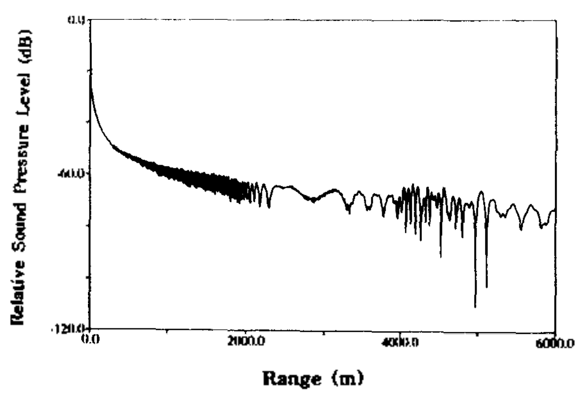

Fig.10 Relative sound pressure level versus distance at 200 Hz when both of the acoustic source and receiver are located at 500 m from the water surface.

Ⅲ. CONCLUSIONS

The boundary fitted coordinates of Polar PE were introduced to solve the sound propagation over a sea mountain. The Polar PE implementing locally reacting sea floor and pressure release water surface calculated 4 different cases depending on the depth of acoustic source and receiver. The purpose of this paper is to show the feasibility solving the sound propagation over a sea mountain wherever the acoustic source and receiver are located in the existence of sea mountain. However the sediment parameters should be incorporated into the present trial case for more realistic sound propagation prediction with sea mountains. The calculation of case 1 for non-line of sight sound propagation shows the significant decrease of about 20 dB in the area between the top and base of sea mountain compared with other cases.

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