Item Replacement Policy with Minimal Repair in Stepdown Warranty Model

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Abstract

This paper proposes age replacement policy in stepdown warranty policy. The replacement policy is considered in case of minimally repairable items. And renewal theory is used in analyzing warranty costs. The expected cost per unit time is presented in stepdown warranty policy, free replacement, prorata and hybrid policy. In this article it is assumed that item is replaced at the age of T but the any failure is minimally repaired before the age T. At this point, the expected cost per unit time is shown in customer's view point. And numerical example is explored in weibull time-to-failure distribution.

1. Introduction

Warranty policy has been introduced by the manufacturers for active Quality Assurance. Manufacturers take the full responsibility for products failures occurred during specified period after sales under warranty policy. In this paper, the procedures of optimum replacement period that is studied so far with repairable products under warranty policy are reviewed.

First, Amato and Anderson(1976) suggested estimation procedures of warranty future cost for repairable items. Karmarkar(1978) calculated warranty cost for repairable items, assumed as good as new repair when items failed.

Park and Yee(1984) calculated present value of warranty cost for minimal repair items, then minimal repair is as bad as old repair that unchanged item failure rate.

Namely minimal-repair are component repair like large system.

Park(1985) suggested replacement of new item in customer's view point and dicided econemic life model of item. And Kim(1988) suggested stepdown warranty policy model. He assumed repairable and irrepairable products and calculated manufacture and customer under warranty renewal policies.

This paper explains stepdown warranty policy and discusses it importance Replacement period is presented by proposing warranty renewal policy under three warranty policies.

2. Contents of the Research

Study in warranty policy depends on warranty rate and warranty renewal policy during the warranty period. Manufacturers should determine a liable rate for the products during the warranty period.

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Three warranty policies are commonly used according to warranty rate. In free replacement policy, manufactures take the full responsibility for all costs incurred during the warranty period. Prorate policy increases customer's liability as the product usage time goes on. Hybrid policy adopts free replacement policy within certain period of time and then uses prorata policy. In this paper, stepdown warranty policy which generalize these three policies is introduced.

Cost to manufacturer varies as the warranty contract renewed, Item replacement period is presented under such warranty policy as item is replaced at the age of T but the any failure is minimally repaired before the age T. The replacement period are also calculated, compared and analyzed under those three different warranty polices.

3. Assumption and Notation

Assumption

-Item failure rate is unchanged under minimal repair model and repair time is negligible.

-Warranty period is renewed as new item is replaced.

-Planning horizon of period is infinite.

-Failure rate function, h(t), is IFR (Increasing Failure Rate)

Notation

p : Replacement cost of new item

c : Minimal repair cost

h(t) : Failure rate function

H(T) : Accumulation failure rate function

f(t) : Item failure density function

F(t) : Item failure distribution function

T : Replacement period

T^* : Optimum replacement period

u(T) : Cost per unit time

R(T) : Cost of customer when a item fails at time T

W_i : Time interval in warranty period

4. Item Replacement Policy

As one item sold, item is replaced at the age of T but the any failure is minimally repaired before the age T.

At this point, in order to compute optimum replacement period T^*, define cost per unit time under the planning period. However the item have renewed every time T. From the renewal process theory

$$U(T) = \frac{\text{Cost per period}}{\text{Length of period}}$$

In this replacement policy, length of period is T and cost per period become to add new item replacement cost to repair cost by time T.

Therefore

$$U(T) = \frac{CH(T) - \Sigma C_i(H(W_i) - H(W_{i-1})) + p}{T} \qquad \cdots\cdots\cdots\cdots \quad (4.1)$$

R(t), cost of customer when a item fails at time t, is

$$R(t) = \begin{cases} c - c_i, & W_{i-1} < t \leq W_i \ , \ i=1,2,\ldots\ldots, k \\ c, & t > W_k \end{cases}$$

By the definition

$$CH(T) - \sum_{i=1}^{k} c_i(H(W_i) - H(W_{i-1})) = \int_0^T R(t)h(t)dt$$

Equations (4.1) finally

$$u(T) = \frac{p + \int_o^T R(t)h(t)dt}{T} \qquad \cdots\cdots\cdots\cdots \quad (4.2)$$

<u>Theorem 4.1</u>

T is optimal point as satisfies the next equation.

$$V(T) = R(T)h(T)T - \int_o^t R(t)h(t)dt = p \qquad \cdots\cdots\cdots\cdots \quad (4.3)$$

<u>Proof</u>

Differential equation of (4.2) is

$$u'(T) = \frac{R(T)h(T)T - (p + \int_o^t R(t)h(t)dt)}{T^2}$$

Consequently (4.3) equation is presented from u'(T) = o.

<u>Theorem 4.2</u>

If T^* exists, T^* is unique as satisfies equation (4.3),
Also if T value doesnot exist, T value is infinite.

<u>Proof</u>

h(t) and R(t) are increasing function,then also g(t)=R(t)h(t) is increasing function.

Therefore

$$V' = g'(T)\,T + g(T) - g(T)$$
$$= g'(T)\,T \qquad T \geq 0$$

At this point V(T) is increasing function and if there exists T which satisfies equation (4.3), T is unique solution.
If there does not exist T which satisfies equation (4.3), V(T) - P is negative. U(T) becomes monoton decreasing function and T^* is infinite. If h(t) is monoton increasing ,
$\lim_{T \to 0} V(T)$ is infinite. The solution which satisfies equation (4.3) exists uniquely.

<u>Theorem 4.3</u>

i) Under free replacement policy T^* satisfies the following equation

$$h(T)T - (H(T) - H(W)) = p/c, \; T \geq W \qquad \cdots\cdots\cdots\cdots \qquad (4.4)$$

ii) Under prorata policy T^* satisfies the following equation

$$h(t)T^2 - \int_O^T H(t)dt - TH(T) = \frac{p}{c} W, \qquad \cdots\cdots\cdots\cdots \qquad (4.5)$$

and then

$$h(T)T - H(T) = \frac{p}{c} - \frac{1}{W}\int_O^W H(t)dt, \;\; T > W \qquad \cdots\cdots\cdots\cdots \qquad (4.6)$$

iii) Under hybrid policy T^* satisfies the following equation

$$h(T)T^2 + \int_{W_1}^T H(t)dt - W_1 h(T) - (T - W_1)H(T) = \frac{p}{c}(W - W_1), \; W_1 < T \leq W \qquad (4.7)$$

and then

$$h(T)T - H(T) = \frac{p}{c} - \frac{1}{W - W_1}\int_{W_1}^W H(t)dt, \;\; T > W \qquad \cdots\cdots\cdots\cdots \qquad (4.8)$$

<u>Proof</u>

i) Under free replacement policy R(t) is

$$R(t) = \begin{cases} c & , t > W \\ 0 & , t < W \end{cases}$$

ii) Under prorata policy R(t) is

$$R(t) = \begin{cases} \frac{c}{W} t & , t \le W_1 \\ c & , t > W \end{cases}$$

iii) Under hybrid policy R(t) is

$$R(t) = \begin{cases} o & , t \le W_i \\ \frac{c(t-W_i)}{W-W_1} & , W_1 < t \le W \\ c & , t > W \end{cases}$$

Therefore optimum replacement period T can be calculate by subtituting each customer's cost, R(t), in equation (4.3).

5. Numerical Example

The following example is analyzed through stepdown warranty poicy.

Example

As one item sold, manufacturer compensate customer for C1 = 30$ which is warranty cost of item failing during warranty period W1. The difference of cost in each step, C, is 10$. Replacement cost is p=10000. Expected costs per unit time are calculated in weibull failure distribution with scale parameter Lambda=1, shape parameter Beta.

The warranty period W = 3 years, free-replacement period W1 = 1 and the step of warranty period is 3.

Beta \ Policy	Stepdown	Free-Replacement	Prorata	Hybrid
1.5	10376.766	333.333	331.255	330.416
2.0	10570.000	333.333	330.333	329.000
3.0	11330.000	333.333	326.583	323.333
4.3	14212.262	333.333	312.084	301.554
5.0	18050.000	333.333	292.833	272.667

In table 5.1 customer's costs is increased under stepdown warranty policy as parameter beta is increased. But in free replacement policy customer's cost per unit time is constant value which is related with item replacement cost and minimal repair cost. As the value of shape parameter is increased, customer's cost per unit time is decreasing remarkably in prorata and hybrid policy.

6. Conclusion

In this paper, expected cost per unit time is calculated in stepdown warranty policy, free replacement, prorata and hybrid policy.

As shape parameter beta is 3, stepdown warranty policy has the most advantage one in other warranty policies in manufacturer's view point. And when customer's cost is compared with stepdown warranty policy with hybrid one, if parameter beta is increased cost per unit time is increased in the customer's points of view. But in hybrid and prorata policy, unit time cost is decreased with increasing parameter beta.

As a result of cost analysis, in manufacturer'sview point stepdown warranty policy is more available than other warranty policies in each parameter beta.

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