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A Simplified Approach to the Analysis of the Ultimate Compressive Strength of Welded Stiffened Plates

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Abstract

In this paper, a method to calculate the ultimate compressive strength of welded one-sided stiffened plates simply supported along all edges is proposed. At first, initial imperfections such as distortions and residual stresses due to welding are predicted by using simplified methods. Then, the collapse modes of the stiffened plate are assumed and collapse load for each mode is calculated. Among these loads, the lower value is selected as the ultimate strength of the plate. Collapse modes are assumed as follows:

1. Overall buckling of the stiffened plate → Overall collapse due to stiffener bending
2. Local buckling of the plate → Local collapse of the plate → Overall collapse due to stiffener yielding
3. Local buckling of the plate → Overall collapse due to stiffener bending
4. Local buckling of the plate → Local collapse of the plate → Overall collapse due to stiffener tripping

The elastic large deformation analysis based on the Rayleigh-Ritz method is carried out, and plastic analysis assuming hinge lines is also carried out. Collapse load is defined as the crossing point of the two analysis curves. This method enables the ultimate strength to be calculated with small computing time and good accuracy. Using the present method, characteristics of the stiffener including bending and tripping can also be clarified.

1 Introduction

Ship structures are basically composed of welded stiffened plates and subjected to repeated sagging and hogging bending moments. Under hogging and sagging bending moment, compressive loads are acting on the stiffened plates of the decks and the bottom, which can cause collapse due to buckling and yielding.

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In order to design plate structures satisfying requirements of weight reduction and enhanced reliability, it is necessary to properly consider various factors which affect the strength of the plate structures. Initial distortions and residual stresses are inevitable in the welded plate structures, and these initial imperfections are known to be the important factors of strength degeneration[1]. Therefore, detailed analysis for the structural behaviors of welded stiffened plates subjected to compressive loads is required.

In stiffened plates under the compressive load, buckling occurs first, but unlike beams residual strength remains after buckling. And also, local buckling and global buckling occur sequentially or simultaneously due to the coupling effects of stiffeners and plates. Geometric nonlinearity also exists due to large deformation, and material nonlinearity exists due to progression of yielding. In order to analyze the strength of the stiffened plates considering geometric and material nonlinearity, computational methods such as the finite element method are necessary. Ueda et-al have reviewed in detail progress of collapse of the welded stiffened plate subjected to compressive load using the finite element method, which was adequate to nonlinear problems[2, 3]. They estimated initial imperfections due to welding, analysed coupling behaviors of the plates and the stiffeners, and proposed the minimum stiffness ratio which bounded the local collapse and the global one. However, the nonlinear finite element method have deficiencies such as large computing time. To overcome these deficiencies, Fujita et-al proposed a new method which could analyze the compressive strength of the welded stiffened plate[1]. In this method, elastic large deformation analysis and plastic analysis were carried out separately and the ultimate strength was selected to be the crossing point of the two curves. This method has usefulnesses such as good accuracy and efficiency, compared with the detailed finite element method. However the model used in formulation by Fujita et-al had symmetric stiffners around the plate, but this is not common in ship structures. Tanaka et-al[4] reviewed the behaviors of the stiffened plates under the compressive load using the method by Fujita et-al, but considered torsional rigidity of the stiffener.

In this paper, a simplified ultimate strength analysis method for the welded one-sided stiffened plates is proposed considering the effects due to welding, and the possibility for application of the method to real ship structures is also reviewed. The hinge line method proposed by Fujita et-al is used for economical and accurate analysis, but special effects are also considered such as eccentricity of the stiffener which was neglected in the study by Fujita et-al[1] and the torsional rigidity which was neglected in the study by Ueda et-al[2, 3]. The effects of the torsional rigidity on the local buckling of the stiffener can be shown and the sectional properties of the stiffener which bound the collapse due to bending of the stiffened plate and that due to tripping can also be found.

2 Estimation of welding initial imperfections

2.1 Estimation of residual stresses

Ueda et-al[3] proposed a method to predict welding residual stresses based on the thermal elasto-plastic analysis and experiments. According to the results by Ueda et-al, the width of the tensile residual stress b_w of the plate can be predicted as follows:

$$b_w = \frac{t_s}{2} + 0.619\frac{Q_{\max}}{(t_s + 2t)} \tag{1}$$

where

$$\begin{aligned} t_s &= \text{thickness of the stiffener(mm)} \\ t &= \text{thickness of the plate (mm)} \\ Q_{\max} &= \text{maximum heat input of the passes (J/mm)} \\ &= \eta\frac{IV}{v} \\ \eta &= \text{arc efficiency} \\ I &= \text{welding current (A)} \\ V &= \text{welding voltage (V)} \\ v &= \text{welding speed (mm/sec)} \end{aligned}$$

The magnitude of the tensile residual stress σ_{wl} can be estimated as follows:

$$\begin{aligned} \sigma_{wl}(SM41) &= \sigma_y(SM41) \\ \sigma_{wl}(SM50) &= \sigma_y(SM50) \\ \sigma_{wl}(SM53) &= \sigma_y(SM53) \\ \sigma_{wl}(SM58) &= 0.8\sigma_y(SM58) \\ \sigma_{wl}(HT80) &= 0.55\sigma_y(HT80) \end{aligned} \tag{2}$$

where, σ_y= yield stress

The width of the tensile residual stress b_s of the stiffeners can be predicted as follows:

$$b_s = 0.03h \tag{3}$$

2.2 Estimation of welding deformation

After fillet welding, angular distortions occur around the beads of joints. As the result, initial out-of plane deformations of the plates are formed, as shown in Fig.2. When expanding these initial deformations into Fourier series, many harmonic components appear, but the dominant component effective on the buckling strengh of the plate is the wave number which accords to the corresponding buckling mode. The effective initial deformation coefficient ζ proposed by Ueda et-al[3] can be calculated as follows:

$$\zeta = \frac{A_0}{W_0} \tag{4}$$

where,

A_0 = component of the initial deformation of the plate corresponding to the buckling mode
W_0 = maximum initial deformation of the plate

$$\zeta = 1, \qquad \beta \leq 1.0$$
$$\zeta = 1 - 2/3(1 - 1/k)(\beta - 1), \qquad 1.0 \leq \beta \leq 2.5$$
$$\zeta = 1/k, \qquad \beta \geq 2.5$$
$$\beta = (b'/t)\sqrt{(\sigma_y/E)}$$

k = number of half wave of corresponding buckling mode

As out-of plane deformation W_0 is caused by shrinkage bending moments along the plate edges, it can be calculated by plate bending theory as follows:

$$W_o = \sum_{m=1}^{\infty}(-1)^{(m-1)/2}\frac{\frac{2b'}{\pi m}\delta_x + \frac{2\pi b'^4}{15a^3}m\delta_y}{\frac{\pi^4 b'^4}{60a^4}m^4 + \frac{\pi^2 b'^2}{3a^2}m^2 + 2} \tag{5}$$

where,

$$\delta_x = \{1 - (t_s + 2f)/b\}\delta_f$$
$$\delta_y = \{1 - (t_s + 2f)/a\}\delta_f$$

f = leg length
m = odd number

Angular distortion is also depenendent on heat input as shown in the following equations.

$$\delta_f = 1.42 \times 10^{-3}\frac{Q_{eq}}{t^2}, \quad 0 \leq Q_{eq}/t^2 \leq 20.95 J/mm^3 \tag{6}$$

$$\delta_f = -5.728 \times 10^{-4}\frac{Q_{eq}}{t^2} + 0.042, \quad 20.95 \leq Q_{eq}/t^2 \leq 73.32 J/mm^3 \tag{7}$$

where, $Q_{eq} = \frac{2t}{2t + t_s}2Q_{\max}$

After fillet welding, longitudinal deformation as well as angular distortion occurs, which contributes to the reduction of overall strength of the stiffened plate. Estimation of the longitudinal deformation can be possible by separating the effective breadth from the plate and using the following formula (for example 60t for steel)[11].

$$\frac{1}{r} = 0.366 \times 10^{-3}\frac{Q_{\max}e_s}{I_s} \tag{8}$$

where,

$\frac{1}{r}$ = radius of the curvature of the longitudinal deformation
e_s = distance of the weld line from the neutral axis of the beam including the effective breadth
I_s = moment of inertia of the beam section including the effective breadth

Deformation at the center of the stiffener can be calculated by

$$W_{s0} = \frac{1}{8r}a^2 \tag{9}$$

where, a = length of the stiffener

3 Classification of the collapse modes of the stiffened plate

The stiffened plate subjected to the compressive load shows various behaviors according to the stiffness of the stiffeners. Ueda et-al defined the stiffness ratio as follows[2];

$$\gamma = \frac{EI}{Db} \tag{10}$$

where,

E = Young's modulus
I = moment of inertia of the stiffener section
D = bending rigidity of the plate
$= \frac{Et^3}{12(1-\nu^2)}$

Ueda et-al also defined the stiffness ratio of the stiffener which could give the boundary between the local buckling and the overall one as γ^b_{min}, and the stiffness ratio of the stiffener which could give the boundary between the local collapse and the overall one, as γ^u_{min}. They defined the stiffness ratio considering only the bending rigidity of the stiffener, but actually torsional rigidity of the stiffener also affects collapse modes. In this paper, we classify collapse modes of the stiffened plates as follows:

1. Overall buckling of the stiffened plate → Overall collapse due to stiffener bending

2. Local buckling of the plate → Local collapse of the plate → Overall collapse due to stiffener yielding

3. Local buckling of the plate → Overall collapse due to stiffener bending

4. Local buckling of the plate → Local collapse of the plate → Overall collapse due to stiffener tripping

4 Ultimate strength analysis of welded stiffened plates according to collapse modes

4.1 Overall buckling → Overall collapse

When the stiffness of the stiffener is not large enough to cause local buckling, the stiffeners and the plate behave like one body and overall buckling and collapse occur. Each buckling load is calculated, and buckling corresponding to the lower value is assumed to occur first.

4.1.1 Elastic analysis

As shown in Fig.1, when the compressive load is applied to the stiffened plate in which initial imperfections such as residual stresses and deformations exist, the dominant shape function of the lateral deformation can be assumed as follows;

$$w = A \sin \frac{\pi}{a} x \ \sin \frac{\pi}{b} y \tag{11}$$

$$w_i = W_{s0} \sin \frac{\pi}{a} x \ \sin \frac{\pi}{b} y \tag{12}$$

where, w_i = initial deformation

The compatibility equation considering large deformation can be expressed as follows;

$$\frac{\partial^4 F}{\partial x^4} + 2\frac{\partial^4 F}{\partial x^2 \partial y^2} + \frac{\partial^4 F}{\partial y^4} = E\left[\left(\frac{\partial^2 w}{\partial x \partial y}\right)^2 - \frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2} + 2\frac{\partial^2 w_i}{\partial x \partial y}\frac{\partial^2 w}{\partial x \partial y} - \frac{\partial^2 w_i}{\partial x^2}\frac{\partial^2 w}{\partial y^2} - \frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w_i}{\partial y^2}\right] \tag{13}$$

where, F = Airy's stress function.

In-plane boundary conditions are

$$\frac{\partial^2 F}{\partial x \partial y} = 0 \quad \text{at} \quad \begin{aligned} x &= 0, \ a \\ y &= 0, \ b \end{aligned} \tag{14}$$

$$\begin{aligned} u(x=a) - u(x=0) &= \overline{u} \\ v(y=b) - v(y=0) &= constant \end{aligned} \tag{15}$$

where,

u = deformation in the x-direction

v = deformation in the y-direction

$$t\int_0^b \frac{\partial^2 F}{\partial y^2}dy = -\sigma bt \tag{16}$$

$$t\int_0^a \frac{\partial^2 F}{\partial x^2}dx = 0 \tag{17}$$

Substituting Eq. (11) and Eq. (12) into Eq. (13), the stress function can be calculated as

$$F = \frac{\sigma}{2}y^2 + \frac{E}{32}(A^2 + 2AW_{so})\left(\frac{a^2}{b^2}\cos\frac{2\pi}{a}x + \frac{b^2}{a^2}\cos\frac{2\pi}{b}y\right) + f(y) \tag{18}$$

where, $f(y)$ = component of the stress function which represents the residual stress.

In-plane strain energy can be calculated by

$$U_I = \frac{t}{2E}\int_0^a\int_0^b\left[\left(\frac{\partial^2 F}{\partial x^2}\right)^2 + \left(\frac{\partial^2 F}{\partial y^2}\right)^2 - 2\nu\frac{\partial^2 F}{\partial x^2}\frac{\partial^2 F}{\partial y^2} + 2(1+\nu)\left(\frac{\partial^2 F}{\partial x \partial y}\right)^2\right]dydx \tag{19}$$

where, ν = Poisson's ratio

Plate bending strain energy can be calculated by

$$U_B = \frac{D}{2}\int_0^a\int_0^b\left[\left(\frac{\partial^2 w}{\partial x^2}\right)^2 + \left(\frac{\partial^2 w}{\partial y^2}\right)^2 + 2\nu\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2} + 2(1-\nu)\left(\frac{\partial^2 w}{\partial x \partial y}\right)^2\right]dydx \tag{20}$$

Deformation along the x-direction can be calculated by

$$u = \frac{1}{b}\int_0^a\int_0^b\left[\frac{1}{E}\left(\frac{\partial^2 F}{\partial y^2} - \nu\frac{\partial^2 F}{\partial x^2}\right) - \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2 - \frac{\partial w}{\partial x}\frac{\partial w_i}{\partial x}\right]dydx \tag{21}$$

To calculate the total potential energy of the stiffened plate, the strain energy of the stiffeners must be found. The stresses of the stiffeners can be calculated by the condition that the stresses of the stiffeners and the plates are the same at the connecting point. That is to say, the stress can be expressed by

$$\begin{aligned}\sigma_{si} &= \left[\frac{\partial^2 F}{\partial y^2} - Ez\frac{\partial^2 w}{\partial x^2}\right]_{y=y_i} + g_s(s)\\ &= -\frac{EA^2\pi^2}{8a^2}\cos\frac{2\pi}{b}y_i - \sigma + E(s+c)A\frac{\pi^2}{a^2}\sin\frac{\pi}{a}x\sin\frac{\pi}{b}y_i + g_s(s)\end{aligned} \tag{22}$$

where,
$g_s(s)$ =function for representing the residual stress of the stiffener
c = distance from the neutral axis of the plate to the neutral axis of the stiffener itself

y_i = position of the stiffener

The strain energy of the stiffeners can be calculated by the following equation,

$$U_S = \sum_i^n \frac{t_s}{2E} \int_0^a \int_{-h/2}^{h/2} \sigma_{si}^2 ds dx \tag{23}$$

The distribution of the residual stress is shown in Fig.1, but as the effective component on the compressive strength is the compressive residual stress, it is assumed that only the compressive residual stress is uniformly distributed along the plate and the stiffeners. That is to say,

$$g(y) = g = \frac{\sigma_{c1}(b' - 2b_w)}{b'} \tag{24}$$

$$g_s(y) = g_s = \frac{\sigma_{s1}(h - b_s)}{h} \tag{25}$$

In case that the stiffeners are attached at one side of the plate, additional moment due to eccentricity must be considered. Thus external work done can be expressed as

$$U_W = -\left[(\sigma + g)bt + (\sigma + g_s)nht_s\right] u + \sigma d(bt + nht_s)\frac{1}{b}\int_0^b \left(\frac{\partial w}{\partial x}\right)_{x=0} dy \tag{26}$$

where, d = distance from the neutral axis of the stiffener to the loading point.

The total potential energy can be given by

$$\pi = U_I + U_B + U_S - U_W \tag{27}$$

By using Rayleigh-Ritz method, equilibrium condition can be obtained.

$$\frac{\partial \pi}{\partial A} = 0 \tag{28}$$

Thus the equilibrium equation can be deduced as

$$\begin{aligned}
&\frac{(W_1^2 - W_{s01}^2)W_1}{8}\frac{h}{b}\frac{t_s}{t}\left(\frac{b}{a}\right)^2 \sum_i^n cos^2\frac{2\pi y_i}{b} \\
&-\frac{3}{\pi}(W_1^2 - W_{s01}^2)\frac{c}{t}\frac{h}{b}\frac{t_s}{t}\left(\frac{b}{a}\right)^2 \sum_i^n sin\frac{\pi y_i}{b} cos\frac{2\pi y_i}{b} \\
&+(W_1 - W_{s01})\left[\frac{1}{6}\left(\frac{h}{b}\right)^3\left(\frac{b}{t}\right)^2\frac{t_s}{t}\right. \\
&\left.+2\left(\frac{c}{t}\right)^2\frac{h}{b}\frac{t_s}{t}\right]\left(\frac{b}{a}\right)^2 \sum_i^n sin^2\frac{\pi y_i}{b} \\
&+\frac{(W_1^2 - W_{s01}^2)W_1}{16}\left[\left(\frac{b}{a}\right)^2 + \left(\frac{a}{b}\right)^2\right]
\end{aligned}$$

$$
\begin{aligned}
&+\frac{(W_1 - W_{s01})}{12(1-\nu^2)}(\frac{b}{a}+\frac{a}{b})^2 \\
&+\frac{W_1}{\pi^2}\frac{(\sigma+g_s)}{\sigma_y}\frac{\sigma_y}{E}(\frac{b}{t})^2\frac{t_s}{t}\frac{h}{b}\sum_i^n cos\frac{2\pi y_i}{b} \\
&-\frac{8}{\pi^3}\frac{(\sigma+g_s)}{\sigma_y}\frac{\sigma_y}{E}(\frac{b}{t})^2\frac{t_s}{t}\frac{h}{b}\frac{c}{t}\sum_i^n sin\frac{\pi y_i}{b} \\
&-\frac{W_1}{\pi^2}\frac{\sigma_y}{E}(\frac{b}{t})^2[\frac{(\sigma+g)}{\sigma_y}+n\frac{(\sigma+g_s)}{\sigma_y}\frac{h}{b}\frac{t_s}{t}] \\
&-\frac{16}{\pi^4}\frac{\sigma}{\sigma_y}\frac{\sigma_y}{E}(\frac{b}{t})^2\frac{d}{t}(1+n\frac{h}{b}\frac{t_s}{t}) \\
&=0
\end{aligned}
\tag{29}
$$

where,

$$
\begin{aligned}
W_1 &= (A + W_{s0})/t \\
W_{s01} &= W_{s0}/t
\end{aligned}
$$

4.1.2 Plastic analysis

Assuming that plastic hinge lines are formed and additional moment due to loading eccentricity is acting, the following equation can be obtained by virtual work principle.

$$
\begin{aligned}
&\sigma(bt+nht_s)\dot{u}+\sigma(bt+nht_s)\dot{\theta} \\
&= \sum_m \int_{l_m}(M_{(m)}+W_{(m)}N_{(m)})\dot{\Theta}_{(m)}dl_m - \sum_m \int_{l_m} N_{(m)}\dot{U}_{(m)}dl_m
\end{aligned}
\tag{30}
$$

where,

$\theta =$ rotational displacement of the stiffened plate at x=0
$u =$ uniform displacement at the loading point
$M_{(m)} =$ resultant moment at the hinge line
$N_{(m)} =$ resultant stress normal to the hinge line
$\theta_{(m)} =$ rotational displacement of the hinge line
$U =$ in-plane displacement of the hinge line
$l_m =$ hinge line
designates rate of change of displacement

When overall collapse occurs, as stiffeners collapse altogether, plastic hinge lines are formed as shown in Fig 3. The plastic collapse moment of the stiffeners must be considered. Using Eq. (30), equations for finding plastic deformation can be deduced as follows;

When $a/b \leq 1$,

$$W_1 = \frac{1}{\alpha(4\frac{b}{a} - 2\tan\phi)}\left[\frac{\xi(\phi)}{\cos\phi\sin\phi} + \xi(90°)(\frac{b}{a} - \tan\phi) + 2(\frac{h}{b})^2\frac{b}{a}\frac{b}{t}\frac{t_s}{t}n\right] - 4\frac{(\frac{b}{a} + n\frac{h}{b}\frac{b}{a}\frac{t_s}{t})}{(4\frac{b}{a} - 2\tan\phi)}\frac{d}{t} \quad (31)$$

When $a/b > 1$,

$$W_1 = \frac{1}{2\alpha}\left[\frac{\xi(\phi)}{\sin^2\phi} + \xi(0°)(\frac{b}{a} - \cot\phi)\cot\phi + 2(\frac{h}{b})^2\frac{b}{t}\frac{t_s}{t}n\right] - 2(1 + n\frac{h}{b}\frac{t_s}{t})\frac{d}{t} \quad (32)$$

Plastic bending moment of the hinge lines can be expressed by Von Mises's theory as follows:

$$\xi(\phi) = \frac{M}{M_P} = \frac{2(1-\alpha^2)}{\sqrt{4 - 3\alpha^2(\cos^2\phi - \sin^2 2\phi)}} \quad (33)$$

where,

$$M = \text{plastic bending moment}$$
$$M_P = \frac{1}{4}\sigma_y t^2$$
$$\alpha = \sigma/\sigma_y$$

4.1.3 Ultimate strength analysis

Ultimate strength can be defined as the crossing point of the plastic analysis curve and the elastic analysis one. Angle of the plastic hinge line is taken to be 32 ° using the results of the reference[10].

4.2 Local buckling → Local collapse → Yielding of stiffener

4.2.1 Elastic analysis

When the bending rigidity of the stiffener is large and the torsional rigidity is small, local buckling of the plate occurs first. The stiffeners and the plate deform separately. The deformation of the plate can be assumed as follows;

$$w = A\sin\frac{\pi}{a'}x\,\sin\frac{\pi}{b'}y \quad (34)$$

$$w_i = A_0\sin\frac{\pi}{a'}x\,\sin\frac{\pi}{b'}y \quad (35)$$

where,

$$\begin{aligned} a' &= a/K \\ K &= int(a/b')\text{: when remainder does not remain} \\ K &= int(a/b')+1\text{: when remainder remains} \\ int(x) &= \text{integer part of x} \end{aligned}$$

In this case, h, d and c of Eq. (29) are all zeros, and a, b and W_{s01} are replaced by a', b' and W_{01} , respectively.

4.2.2 Plastic analysis

In plastic analysis, h, d and c of Eq. (31) and (32) are also all zeros, and a and b are replaced by a' and b' , respectively.

4.2.3 Ultimate strength analysis

Ultimate strength of the plate can be found by the method introduced in Sec 4.1.3. As the stiffeners collapse due to full plastic yielding, the collapse load of the stiffened plate is calculated by the following equation.

$$\frac{\sigma}{\sigma'_y} = \frac{(bt\sigma_{1c} + nht_s\sigma_s y)}{(\sigma_y bt + \sigma_{sy} nht_s)} \tag{36}$$

where,

$$\begin{aligned} \sigma_{col} &= \text{collapse stress of the stiffened plate} \\ \sigma_{1c} &= \text{collapse stress of the plate} \\ \sigma_{sy} &= \text{yield stress of the stiffener} \\ \sigma_y &= \text{yield stress of the plate} \\ \sigma'_y &= \frac{\sigma_y bt + \sigma_{sy} nht_s}{bt + nht_s} \end{aligned}$$

4.3 Local buckling - Overall collapse due to stiffener bending

As the stiffness of the stiffeners increases enough to prevent overall buckling, local buckling of the plate occurs first. But if the bending stiffness of the stiffeners is not large enough to cause only local collapse, overall collapse due to bending of the stiffeners occurs. Which buckling mode occurs first is determined by the magnitude of the buckling loads of the two modes. That is to say, local buckling strength of the plate and overall buckling strength of the stiffened plate are calculated and buckling mode corresponding to the smaller value are assumed to occur first. The simplest method to calculate the overall buckling strength is to distribute stiffness of the stiffeners uniformly along the plate and to use the simple formulas. Local buckling strength is calculated by

$$\sigma_{lc} = K_c D \pi^2 \left(\frac{t}{b'}\right)^2 \tag{37}$$

Overall buckling strength is calculated by

$$\sigma_{oc} = K_c D_e \pi^2 \left(\frac{t_e}{b}\right)^2 \tag{38}$$

where,

$$\begin{aligned} t_e &= t + \frac{nht_s}{b} \\ D &= \frac{Et^3}{12(1-\nu^2)} \\ D_e &= \frac{Et^3}{12(1-\nu^2)} + \frac{nEt_s h^3}{12b} \\ K_c &= \text{buckling stress coefficient according to aspect ratio} \end{aligned}$$

When local buckling occurs, the plate contribute to the stiffness of the whole stiffened plate by effective width. Effective width is defined as

$$b_{ep} = \frac{\sigma b'}{\sigma_{\max}} \tag{39}$$

At collapse, as maximum stress amounts to the yield stress, it can be assumed to be yield stress. That is to say,

$$\sigma_{\max} = \sigma_y \tag{40}$$

where, $\sigma_y =$ yield stress.

The stiffness of the plate except effective width is assumed to be zero and the total potential energy is calculated again and the equilibrium equation is deduced as follows.

$$\begin{aligned} & \frac{(W_1^2 - W_{s01}^2)W_1}{8} \frac{h}{b}\frac{t_s}{t} \left(\frac{b}{a}\right)^2 \sum_i^n \cos^2 \frac{2\pi y_i}{b} \\ - & \frac{3}{\pi}(W_1^2 - W_{s01}^2)\frac{c}{t}\frac{h}{b}\frac{t_s}{t} \left(\frac{b}{a}\right)^2 \sum_i^n \sin\frac{\pi y_i}{b} \cos\frac{2\pi y_i}{b} \\ + & (W_1 - W_{s01}) \left[\frac{1}{6}\left(\frac{h}{b}\right)^3 \left(\frac{b}{t}\right)^2 \frac{t_s}{t} + 2\left(\frac{c}{t}\right)^2 \frac{h}{b}\frac{t_s}{t}\right] \left(\frac{b}{a}\right)^2 \sum_i^n \sin^2\frac{\pi y_i}{b} \\ + & \frac{(W_1^2 - W_{s01}^2)W_1}{16} \left[\frac{b_{c1} b}{a^2} + \left(\frac{a}{b}\right)^2\right] \\ + & \frac{(W_1 - W_{s01})}{12(1-\nu^2)} \left(\frac{bb_{s2}}{a^2} + \frac{a^2}{b^2}\frac{b_{s2}}{b} + 2\frac{b_{c2}}{b} + 2\nu\frac{b_{s2}}{b} - 2\nu\frac{b_{c2}}{b}\right) \end{aligned}$$

$$
\begin{aligned}
&+ \frac{W_1}{\pi^2}\frac{(\sigma+g_s)}{\sigma_y}\frac{\sigma_y}{E}\left(\frac{b}{t}\right)^2\frac{t_s}{t}\frac{h}{b}\sum_i^n \cos\frac{2\pi y_i}{b} \\
&- \frac{8}{\pi^3}\frac{(\sigma+g_s)}{\sigma_y}\frac{\sigma_y}{E}\left(\frac{b}{t}\right)^2\frac{t_s}{t}\frac{h}{b}\frac{c}{t}\sum_i^n \sin\frac{\pi y_i}{b} \\
&- \frac{W_1}{\pi^2}\frac{\sigma_y}{E}\left(\frac{b}{t}\right)^2[\frac{(\sigma+g)}{\sigma_y} + n\frac{(\sigma+g_s)}{\sigma_y}\frac{h}{b}\frac{t_s}{t}] \\
&- \frac{16}{\pi^4}\frac{\sigma}{\sigma_y}\frac{\sigma_y}{E}\left(\frac{b}{t}\right)^2\frac{d}{t}\left(1+n\frac{h}{b}\frac{t_s}{t}\right) \\
&= 0
\end{aligned}
\qquad (41)
$$

where,

$$
\begin{aligned}
b_{c1} &= b_{ep}(n+1) + \frac{b}{4\pi}\sum_i^n\left[\sin\frac{4\pi}{b}(ib'-b_{ep}) - \sin\frac{4\pi}{b}\{(i-1)b'-b_{ep}\}\right] \\
b_{c2} &= b_{ep}(n+1) + \frac{b}{2\pi}\sum_i^n\left[\sin\frac{2\pi}{b}(ib'-b_{ep}) - \sin\frac{2\pi}{b}\{(i-1)b'-b_{ep}\}\right] \\
b_{s2} &= b_{ep}(n+1) - \frac{b}{2\pi}\sum_i^n\left[\sin\frac{2\pi}{b}(ib'-b_{ep}) - \sin\frac{2\pi}{b}\{(i-1)b'-b_{ep}\}\right]
\end{aligned}
$$

Ultimate strength is calculated by the same procedure shown in sec.4.1.

4.4 Local buckling → Overall collapse due to stiffener tripping

Until now, it was assumed that in ultimate strength analysis, torsional rigidity of the stiffener was neglected and tripping of the stiffener did not occur. But, actually the stiffener has finite torsional rigidity, so local buckling of the plate and tripping of the stiffener occur simultaneously due to coupling effect. The stiffener attains to ultimate state through out-of plane deformation.

4.4.1 Elastic analysis

To consider rotational deformation of the stiffener, the deformations of the plate and the stiffener are expressed as follows, referring to Fig.5.

$$w = A\sin\frac{\pi}{a'}x\sin\frac{\pi}{b'}y \qquad (42)$$

$$w_i = A_0\sin\frac{\pi}{a'}x\sin\frac{\pi}{b'}y \qquad (43)$$

$$v = -\frac{\pi}{b'}Az\sin\frac{\pi}{a'}x \qquad (44)$$

$$v_i = -\frac{\pi}{b'}A_0z\sin\frac{\pi}{a'}x \qquad (45)$$

By substituting the above functions into the compatibility equation of elastic large deformation, the stress function is obtained by

$$\begin{aligned} F_s &= \frac{Ea'^2}{32b'^2}(A^2+2A^2A_0^2)\cos\frac{2\pi}{a'}x + \frac{E\pi^4 z^4}{48a'^2b'^2}(A^2+2A^2A_0^2) - \frac{\sigma}{2}z^2 \\ &+ f(z) - \frac{E\pi^2 z^2}{16a'^2}(A^2+2A^2A_0^2) \end{aligned} \quad (46)$$

In plane strain energy U_{IS} , torsional strain energy U_{TS} , and external work done of the stiffener U_{WS} are calculated following the previous procedure. Using Rayleigh-Ritz method, equilibrium equation can be obtained.

$$\pi = U_I + U_B - U_W + U_{IS} + U_{TS} - U_{WS} \quad (47)$$

$$\frac{\partial \pi}{\partial A} = 0 \quad (48)$$

The resulting equation is given by

$$\begin{aligned} &\frac{1}{16}\frac{h}{b}\frac{t_s}{t}\left(\frac{a'}{b'}\right)^2(W_1^2-W_{01}^2)W_1 + \frac{\pi^4}{10}\left(\frac{h}{b'}\right)^5\frac{t_s}{t}\left(\frac{b'}{a'}\right)^2(W_1^2-W_{01}^2)W_1 \\ &- \frac{\pi^2}{12}\left(\frac{h}{b'}\right)^3\left(\frac{b'}{a'}\right)^2\frac{t_s}{t}(W_1^2-W_{01}^2)W_1 + \frac{\pi^2}{18(1-\nu^2)}\left(\frac{t_s}{t}\right)^3\left(\frac{h}{b'}\right)^3\left(\frac{b'}{a'}\right)^2(W_1-W_{01}) \\ &+ \frac{1}{3(1+\nu)}\frac{h}{b'}\left(\frac{t_s}{t}\right)^3(W_1-W_{01}) + \frac{1}{16}\left[\left(\frac{a'}{b'}\right)^2+\left(\frac{b'}{a'}\right)^2\right](W_1^2-W_{01}^2)W_1 \\ &+ \frac{1}{12(1-\nu^2)}\left(\frac{b'}{a'}+\frac{a'}{b'}\right)^2(W_1-W_{01}) - \frac{2}{3}\left(\frac{\sigma+g_s}{\sigma_y}\right)\frac{\sigma_y}{E}\left(\frac{h}{b'}\right)^3\left(\frac{b'}{t}\right)^2\frac{t_s}{t}W_1 \\ &- \frac{1}{\pi^2}\left(\frac{\sigma+g}{\sigma_y}\right)\frac{\sigma_y}{E}\left(\frac{b'}{t}\right)^2 W_1 = 0 \end{aligned} \quad (49)$$

where,

$$\begin{aligned} W_1 &= (A+A_0)/t \\ W_{01} &= A_0/t \end{aligned}$$

4.4.2 Plastic analysis

Plastic analysis is carried out following the same procedure presented in sec 4.2.2, and the plastic analysis of the stiffner is carried out by assuming the hinge lines shown in Fig.6.

$$W_{s1} = \frac{1}{2\alpha(4h/a' - \tan\phi_s)} \frac{\xi(\phi_s)}{\sin\phi_s \cos\phi_s} \tag{50}$$

$$\tan\phi_s = \frac{a'}{2h} \tag{51}$$

$$W_s = W_1 \frac{ht}{b_2 t_s} \tag{52}$$

$$b_2 = \frac{a\tan\phi_s}{2} \tag{53}$$

$$W_{s1} = \frac{W_s}{t}$$

4.4.3 Ultimate strength analysis

The ultimate strength of the stiffened plate considering tripping of the stiffener can be calculated as the sum of that of the plate and that of the stiffener. The ultimate strength of the plate is calculated by the method presented in sec.4.2.3 and that of the stiffener is calculated by substituting deformation of the plate W_1 into Eq.(50). Ultimate strength of the stiffened plate is calculated using the concept suggested in Eq.(36).

5 Calculated results and discussion

A procedure to calculate the ultimate strength of the welded stiffened plate is shown in Fig.7. To confirm the validity of the procedure, the ultimate strength of the plate alone is calculated neglecting the torsional rigidity of the stiffener and compared with the results by Yao[2, 7]. The results are shown in Fig.8, where the distribution of the residual stresses and the initial deformation is assumed and the aspect ratio a'/b' is 1. The calculated ultimate strengths are plotted as the parameter $\beta = (b/t)\sqrt{(\sigma_y/E)}$. The utimate strengths taking into account the bending rigidity of the stiffener are shown in Fig.9 and compared with the results by Yao, where, $\sigma_y = \sigma_{sy} = 34.68kg/mm^2$, $W_{01} = 0.01$, $W_{S01} = 0.01$ and $b_w = 8t$. The collapse loads due to tripping of the stiffener are calculated and compared with the experimental results as shown in Table 1. The differences between the calculated results and experimental ones are within 7 % . The model for calculation presented in Table 1 have h/t greater than 15, which results from the purpose that overall collapse should be prevented and only the local collapse mode be allowed. Yao did not consider the tripping of the stiffener in the analysis of the local collapse and the torsional rigidity, but in this paper the effect of torsional rigidity of the stiffener on the ultimate strength is considered using the procedure shown in Fig.7. In the calculation by Yao, the torsional rigidity of the stiffener is assumed to be negligible and only the bending rigidity is assumed to be effective. According to this assumption, when the stiffness ratio as to bending becomes a certain limit, ultimate state due to the collapse of the plate is considered to be attained. For the stiffener with more than the minimum stiffness ratio as to bending, the local collapse loads are constant as shown in Fig.10, but when torsional rigidity of the stiffener is considered, local collapse loads increase as the torsional rigidity increases, which is ommited in Yao's calculations. The effect of the torsional rigidity of the stiffener is revealed in Fig.10, of which quantity is the difference between the dotted line and the solid one. The effect of the torsional rigidity becomes more apparent as height-to-thickness ratio decreases. The ultimate strength of the stiffened plate is the sum of the collapse load of the plate and that of the stiffener. Ueda et-al[2] have introduced the concept of the minimum stiffness ratio as to bending of the stiffener but the concept must be altered when torsional rigidity is included. Fig.11 shows variation of the ultimate strengths of the stiffened plates considering tripping and bending after local buckling according to height-thickness ratio. Fig.11 indicates that a limit height-to-thickness ratio exists which gives a bound to prevent tripping of the stiffener. In design of stiffener, it must be remembered that the thickness of the stiffener should be increased to prevent tripping as the height of the stiffener increases.

6 Conclusion

Until now we presented a method to calculate the ultimate compressive strength of the welded stiffened plate, and compared the results with the detailed analysis results and experimental ones and proved the validity of the method. It was revealed that the ultimate strength of the stiffened plate could be found by calculation of collapse load of each failure mode, which was governed by the dominant component of deformation. Unlike the results by other researchers which neglected the torsional rigidity of the stiffener, it was found that the effect of the torsional rigidity of the stiffener becomes important as the thickness of the stiffener increases. And also the calculated results by the present method show that there exists height-to-thickness ratio which gives the bound between the collapse due to tripping and the collapse due to bending. It is thought that by using this method, the effect of welding on the ultimate strength of the stiffened plate can be clarified economically and accurately.

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Table 1: Particulars of test models and their ultimate strength($mm, kg/mm^2$)

Details of Plate					Details of Stiffener			σ_{col}/σ_y	
b'	t	σ_y	W_0	σ_w	h	t_s	σ_{sy}	exp.[4]	cal.
360	6.15	23.9	0.101	-2.1	110	9.77	29.3	0.931	0.997
360	5.95	25.9	0.119	-3.47	118.5	7.98	29.0	0.990	0.995
300	4.38	45.1	0.515	-3.80	65	4.38	45.1	0.547	0.55
300	4.38	45.1	0.503	-2.55	90	4.38	45.1	0.527	0.535
360	4.38	45.1	0.523	-3.85	65	4.38	45.1	0.510	0.502

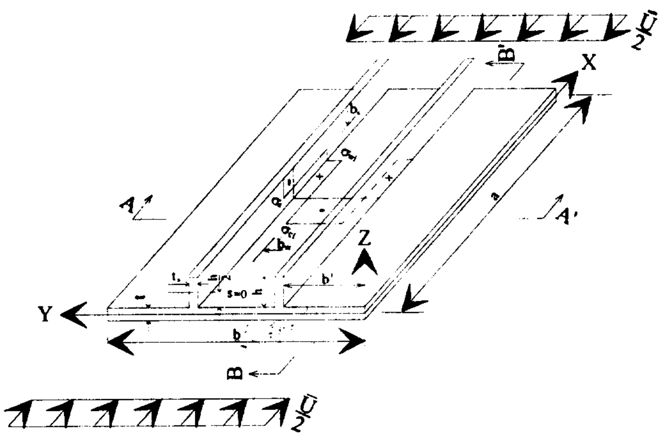
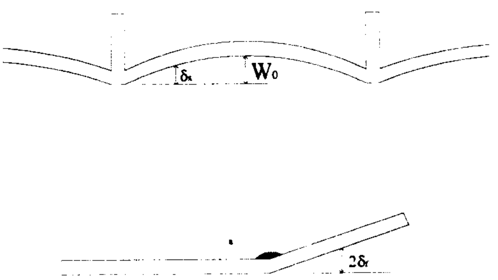
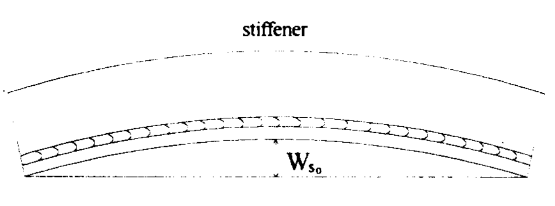


Figure 1: Coordinate system of the welded stiffened plate

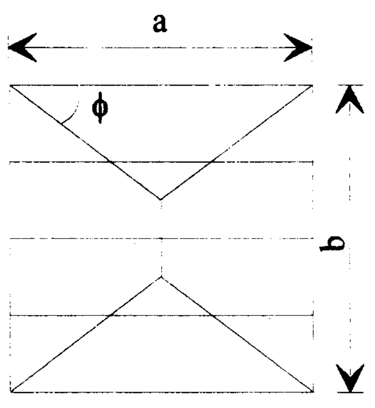


(a) Angular distortion and out-of plane distortion of plate

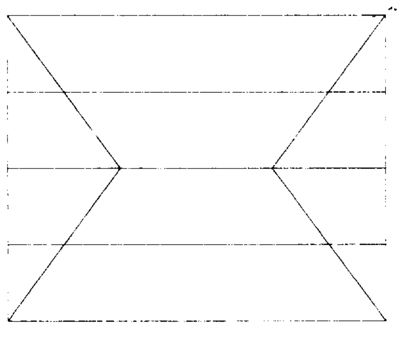


(b) Longitudinal distortion of stiffeners

Figure 2: Welding distortions



(a) a / b ≤ 1



(b) a / b > 1

Figure 3: Hinge lines of the stiffened plate

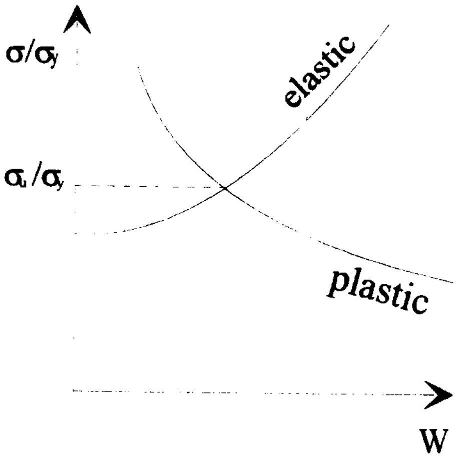
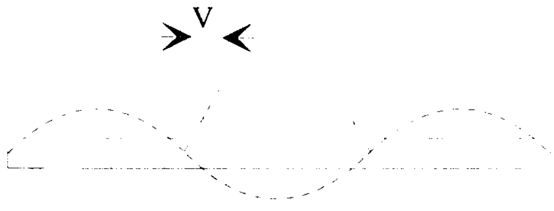
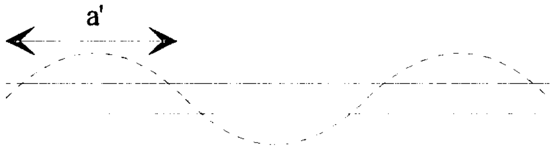


Figure 4: Definition of ultimate strength

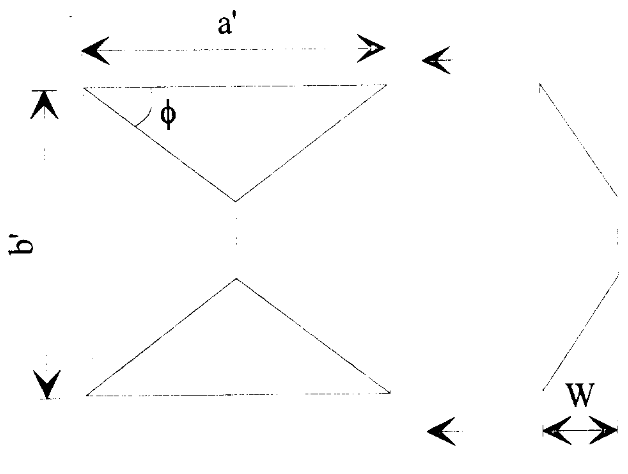


(a) Section A-A'

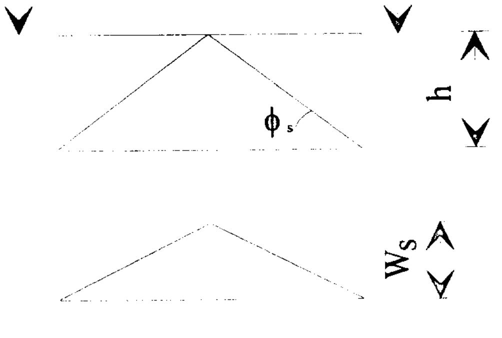


(b) Section B-B'

Figure 5: Deflected shape of the plate and stiffener



(a) plate



(b) stiffener

Figure 6: Hinge lines of the plate and stiffener at tripping collapse

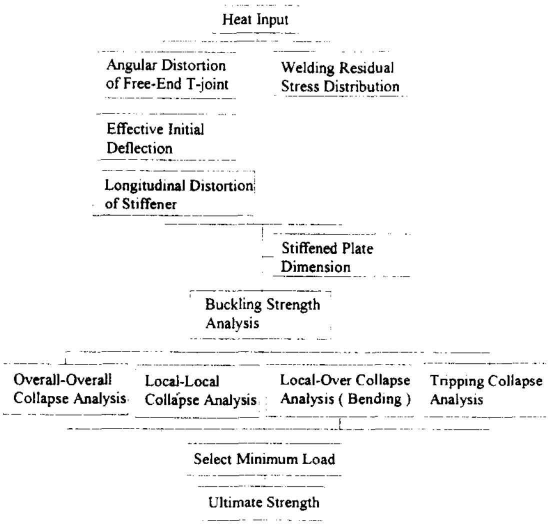


Figure 7: Flow for the calculation of ultimate strength of welded stiffened plates

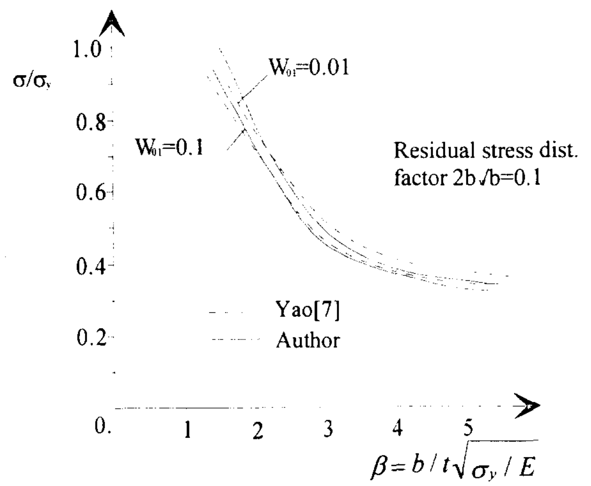


Figure 8: Collapse loads of local plates

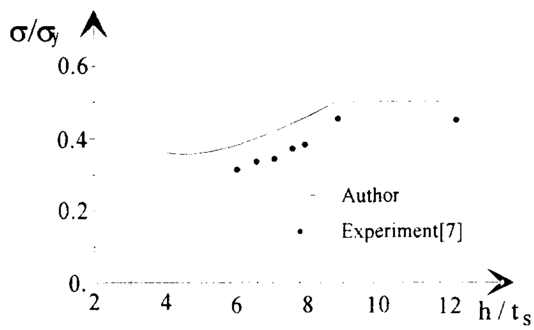


Figure 9: Ultimate strength of the welded stiffened plate (neglecting torsional rigidity of the stiffener)

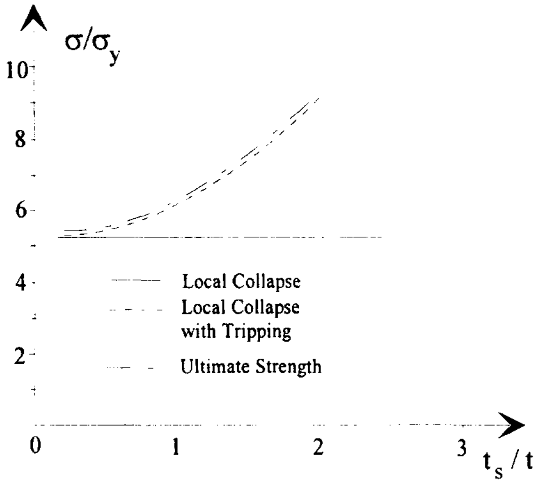


Figure 10: Effect of torsional rigidity of stiffeners

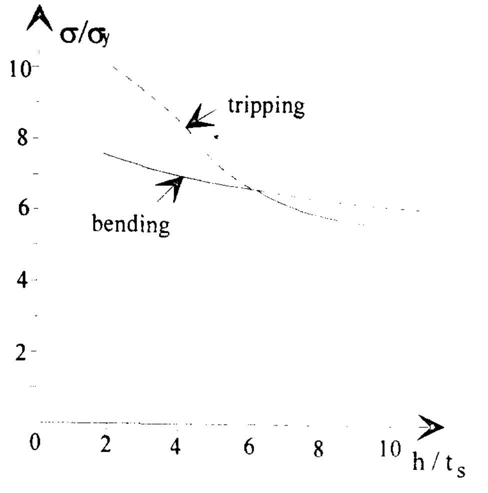


Figure 11: Change of ultimate strength according to variation of stiffener dimension