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Hydrodynamic Coefficients of an Oscillating Cylinder in Steady Horizontal Translation on the Free Surface

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Abstract

An integral equation associated with a mixed distribution of source and normal doublet on the wetted surface of a two dimensional body has been presented for calculating the hydrodynamic coefficients of an oscillating cylinder in steady horizontal translation on the free surface. This equation contains two dimensional equivalent of the line integral for a three dimensional surface-piercing body. It also contains an integral relation which eliminate the occurrence of irrelevant solution. The resulting overdetermined linear system is solved by the method of Householder. Hydrodynamic coefficients of a half immersed circular cylinder have been calculated for various Froude numbers up to 0.35 . The present numerical results for a very small Froude number agree well with those for zero Froude number. It seems that the present method yields reasonable numerical results for all frequencies without restriction on the magnitude of Froude number in the context of linear wave theory.

1 Introduction

The radiation problem for an oscillating body advancing on or below the free surface with a constant horizontal velocity has been studied by several researchers. Brard derived a Green function associated with a three dimensional pulsating Kelvin source advancing under the free surface[1]. Haskind derived a Green function in two dimensions[2]. The two dimensional hydrodynamic coefficients of a fully submerged cylinder have been calculated by making use of an integral equation whose unknown represents the density of source or of doublet distributed over the wetted surface[3][4][5].As for a surface-piercing body, no systematic comparative study on existing numerical methods based on a rigorous two or three dimensional formulation has yet been carried out[6][7][8][9].

In this paper, an integral equation with nonsymmetric kernel is derived from the method of source and doublet distribution on the wetted surface of a two dimensional surface-piercing body. Discretization of the equation yields an overdetermined linear system which can be solved numerically by Householder's method. Hydrodynamic coefficients of a half immersed circular cylinder have been calculated for various Froude numbers up to 0.35 .

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2 Formulation of the Problem

The fluid is assumed to occupy a space V bounded by the wetted surface S of a surface-piercing body at its mean position and by the free surface F of deep water under gravity. The body performs simple harmonic oscillations of small amplitude with circular frequency ω about its mean position which is moving with a steady horizontal velocity u. Cartesian coordinates(x, y) attached to the mean position of the body, are employed with the origin o in the waterplane W of the body at its mean position and the y axis vertically upwards. The waterplane lies in the plane zox and the plane xoy is perpendicular to the generatrices of the cylindrical body in order that the problem may be treated in two dimensions. With the usual assumptions of an incompressible fluid and irrotational flow without capillarity, the fluid velocity is given by the gradient of a velocity potential. The boundary condition is linearized assuming that both the magnitude of unsteady flow and the magnitude of steady flow due respectively to the oscillation and steady translation of the cylinder are small enough to neglect their products. Since the problem is linear, the potential can be decomposed into a steady potential due to the steady translation of the body and a unsteady potential $\Phi(x,\, y,\, t)$. The governing equations for $\Phi(x,\, y,\, t)$ are given as follows[4]:

$$\nabla^2\Phi = O \quad \text{in } V \tag{1}$$

$$\frac{\partial\Phi}{\partial n} = [\dot{a}_1\vec{e}_1 + (\dot{a}_2 - ua_3)\vec{e}_2 + \dot{a}_3\vec{e}_3 \times \vec{O_1M}] \cdot \vec{n} \quad \text{on } S \tag{2}$$

$$\frac{\partial^2\Phi}{\partial t^2} - 2u\frac{\partial^2\Phi}{\partial x\partial t} + u^2\frac{\partial^2\Phi}{\partial x^2} + g\frac{\partial\Phi}{\partial y} = 0 \quad \text{on } F \tag{3}$$

where $a_j(j = 1,2,3)$ denote respectively surge, heave and pitch motions, O_1 the center of rotation of the body, n the normal vector directed into the fluid region V from S and g the gravitational acceleration. The potential must also satisfy the radiation condition at infinity. Introducing complex amplitude defined as follows,

$$a_j = R_e\{(a_j^* + ia_j^{**})e^{-i\omega t}\} \tag{4}$$

and considering the body boundary condition (2), it can be found that Φ takes the following form:

$$\Phi = R_e\{-i\omega\sum_{j=1}^{3}[(a_j^* + ia_j^{**})\phi_j e^{-i\omega t} - u(a_3^* + ia_3^{**})\phi_2 e^{-i\omega t}\} \tag{5}$$

where

$$\phi_j = \phi_j^* + i\phi_j^{**}, \quad j = 1,2,3 \tag{6}$$

where $\phi_j(j = 1,2,3)$ denote complex valued elementary potentials. Taking into account of (2) and (5), the following body boundary conditions for ϕ_j can be found:

$$\frac{\partial\phi_j}{\partial n} = \vec{e}_j \cdot \vec{n} \quad \text{on } S \quad \text{for } j = 1,2 \tag{7}$$

$$\frac{\partial \phi_3}{\partial n} = (\vec{e}_3 \times O_1\vec{M}) \cdot \vec{n} \quad \text{on} \quad S \tag{8}$$

Other boundary conditions for ϕ_j are identical to those for ϕ.

3 Integral Equation

The explicit form of the Green function in the complex plane $z = x + iy$ has already been introduced in the reference[3]. Its final expression is as follows:

$$G(z, z'; t) = \frac{1}{2\pi}(\cos\omega t \log\frac{z - z'}{z - \overline{z}'} + I_1 e^{-i\omega t} + I_2 e^{i\omega t}) \tag{9}$$

$$I_1 = \frac{1}{\sqrt{1 + 4\gamma}}\{e^{\zeta_2}[\epsilon_1(\zeta_2) + 2i\pi] - e^{\zeta_1}[\epsilon_1(\zeta_1) + 2i\pi]\}, \ \forall\ \gamma \tag{10}$$

$$I_2 = \begin{cases} \frac{1}{\sqrt{1-4\gamma}}\{e^{\zeta_4}[\epsilon_1(\zeta_4) + 2i\pi] - e^{\zeta_3}\epsilon_1(\zeta_3)\}, & \gamma < \frac{1}{2}, \quad \gamma \neq \frac{1}{4} \\ \frac{1}{\sqrt{1-4\gamma}}[e^{\zeta_4}E_1(\zeta_4) - e^{\zeta_3}E_1(\zeta_3)], & \gamma \geq \frac{1}{2} \end{cases} \tag{11}$$

where $\gamma = \omega \cdot u/g$ denotes the Brard number, E_1 the complex exponential integral and ϵ_1 the modified complex exponential integral[4]. Applying Green's theorem to an elementary potential and the Green function in the fluid region V, the following integral relation can be found:

$$\int_S \left(\phi\frac{\partial G}{\partial n} - G\frac{\partial \phi}{\partial n}\right) dl \ + \ \frac{2i\omega u}{g}\int_F \left(\phi\frac{\partial G}{\partial x} - G\frac{\partial \phi}{\partial x}\right) dl$$
$$+ \ \frac{u^2}{g}\int_F \frac{\partial^2}{\partial x^2}(\phi G - G\phi) dl = 0 \tag{12}$$

Applying Stokes' theorem to the second integral in (12) and assuming that

$$\int_W \frac{\partial \phi}{\partial x} G dl = 0 \tag{13}$$

the integral reduces to a simple expression as follows:

$$\phi_C G_C - \phi_D G_D \tag{14}$$

In (13), the subscript W denotes the waterplane and in (14), subscripts C and D denote respectively left and right intersecting points of the free surface F and the body surface S as shown in Figure 1. In case of a three dimensional floating body, the expression (14) shall be replaced by the following line integral around the waterline WL:

$$\int_{WL} \phi G dz \tag{15}$$

Simple application of Stokes' theorem to the third integral in (12) yields the following expression:

$$\left(\phi\frac{\partial G}{\partial x}\right)_C - \left(\phi\frac{\partial G}{\partial x}\right)_D - \left[\left(G\frac{\partial \phi}{\partial x}\right)_C - \left(G\frac{\partial \phi}{\partial x}\right)_D\right] \tag{16}$$

Substitution of (14) and (16) into (12) yields

$$\begin{aligned} &\int_S \left(\phi\frac{\partial G}{\partial n} - G\frac{\partial \phi}{\partial n}\right) dl + \frac{2i\omega u}{g}[\phi_C G_C - \phi_D G_D] \\ + &\frac{u^2}{g}\left[\left(\phi\frac{\partial G}{\partial x}\right)_C - \left(\phi\frac{\partial G}{\partial x}\right)_D\right] - \left[\left(G\frac{\partial \phi}{\partial x}\right)_C - \left(G\frac{\partial \phi}{\partial x}\right)_D\right] = 0 \end{aligned} \tag{17}$$

Here, in order to alleviate the manipulation to construct an integral equation without modifying the nature of the problem, it is assumed that the free surface and the wetted surface intersect at a right angle. Then, following the convention used in the singularity distribution method, the formula (17) becomes

$$\begin{aligned} &\frac{\mu}{2} + \int_S \mu\frac{\partial G}{\partial n} dl + \frac{2i\omega u}{g}[\mu_C G_C - \mu_D G_D] - \frac{u^2}{g}\left[\left(\mu\frac{\partial G}{\partial n}\right)_C - \left(\mu\frac{\partial G}{\partial n}\right)_D\right] \\ = &-\int_s \sigma G dl + \frac{u^2}{g}[\sigma_C G_C + \sigma_D G_D] \end{aligned} \tag{18}$$

where

$$\mu = -\phi \quad \text{on } S \tag{19}$$

$$\sigma = \frac{\partial \phi}{\partial n} \quad \text{on } S \tag{20}$$

The equation (19) assumes that the mixed distribution of μ and σ on S generates a null scalar field outside the fluid region V. However, the solution of (18) may not satisfy the assumptions given by (13) and (19). In order to prevent the occurrence of an irrelevant solution, the following condition should be added to (18):

$$\int_S \left(\sigma G - \mu\frac{\partial G}{\partial n}\right) dl = 0 \quad \text{on } W \tag{21}$$

or

$$\int_S \mu G_n dl = -\int_S \sigma G dl \quad \text{on } W \tag{22}$$

The above integral relation signifies that the potential induced by the source and doublet distribution over S vanishes on W, the portion of the free surface inside the body. Since $\phi = 0$ on W , its derivatives also vanish on W and so does the integral of (13). It also assures that the potential in the domain bounded by S and W is identically null. It is very useful characteristic of a mixed surface distribution of sources and normal doublets to be able to impose a null scalar field outside the domain under consideration. Simultaneous resolution of (18) and (22) can be carried out by the method of Householder after discretization. The pressure on S can then be obtained by making use of the Bernoulli equation:

$$p = -\rho \left(\frac{\partial \Phi}{\partial t} - u \frac{\partial \Phi}{\partial x} \right) \tag{23}$$

The hydrodynamic pressure forces and moment due to the unsteady potential can be obtained as

$$F_1 \vec{e}_1 + F_2 \vec{e}_2 = - \int_S p \vec{n} dS \quad \text{on} \quad S \tag{24}$$

$$F_3 \vec{e}_3 = - \int_S p \left(\frac{\vec{O_1 M}}{L} \times \vec{n} \right) dS, \quad M \in S \tag{25}$$

In (25), L denotes the characteristic length of the body. Introduction of non-dimensional added-mass and wave-damping coefficients M_{ij} and D_{ij} into (24) and (25) leads to the following expression for F_i :

$$F_i = -\rho g L \sum_{j=1}^{3} [M_{ij} \ddot{a}_j + \omega D_{ij} \dot{a}_j], \quad i = 1, 2, 3 \tag{26}$$

4 Numerical Results and Discussion

The hydrodynamic coefficients of a circular cylinder half immersed as shown in Figure 2 are computed for various Froude numbers, $F_n = u/\sqrt{gR}$. They are all plotted as functions of $\omega\sqrt{R/g}$. Here, it should be noted that ω is actually the circular frequency of encounter given by the following formula :

$$\omega = \omega_o (1 + u \, \omega_o / g) \tag{27}$$

where ω_o denotes the circular frequency of incident waves whose direction of propagation is opposed to that of the steady translation of the body. So, the expression (27) holds in case of head sea. In case of following sea, the relation between ω and ω_o is given as follows :

$$\omega = \omega_o (1 - u \, \omega_o / g) \tag{28}$$

In this paper, ω is considered as that of the head sea. The contour of semicircle is discretized in eighty line segment of equal length and $\overline{CD}$ in forty. In figures 3 through 6, the well-known added mass and wave-damping coefficients due to surge and heave motions of the present numerical model with zero Froude number are presented by solid lines. The hydrodynamic coefficients calculated by the present method for three nonzero Froude numbers are shown by other lines. It is shown that the present results for a very small Froude number of 0.005 agree well with those for zero Froude number. It is also shown that the present numerical results are almost speed independent at low Froude numbers up to 0.1 for the present numerical model. It should be noted that the computed values of hydrodynamic coefficients fluctuate slightly as the Froude number grows. It is a numerical problem caused by the discretization of the integral equation. The fluctuation will be diminished by increasing the size of linear system. In figures 7 through 10, the hydrodynamic coefficients for five Froude numbers from 0.15 to 0.35 are presented. The coefficient M_{22} is still shown to be almost speed independent. Other coefficients are significantly reduced from $F_n = 0.15$ to $F_n = 0.35$ for $\omega < \omega_c$ where $\omega_c = \gamma_c \cdot g/u$. $\gamma_c = 0.25$ is the critical Brard number where the Green function fails to exist as shown in (11). It is interesting to note that M_{11} , D_{11} and D_{22} take their local minima at about $\omega = \omega_c$. For $\omega > \omega_c$, D_{11} and D_{22} show significant increase from $F_n = 0.15$ to $F_n = 0.35$ and their peak values become clear as the Froude number grows while M_{11} is slightly increased. It should be noted that the local minima of D_{11} and D_{22} of the present numerical model become negative in the neighborhood of ω_c for $F_n \geq 3.0$. It probably is due, in part, to the local resonance at ω_c as explained in the reference [4]. The nonlinear effect might have another part in this problem. Since it is physically impossible for the damping coefficient to be negative, the present method for the present numerical model might be invalid for $F_n \geq 3.0$.

5 Conclusions

An improved integral equation is presented to solve the radiation problem for a floating body advancing on the free surface. From the solution of the integral equation, the hydrodynamic coefficients of an oscillating cylinder in steady horizontal translation on the free surface can be found for all frequencies. In principle, there is no restriction on the magnitude of Froude number under the assumption of linearization. But the upper bound of F_n should be taken where the computed wave-damping coefficient becomes negative. It might be closely related to the wave making characteristic of the cylinder. The present numerical results are open to experimental results as well as to other numerical results for comparison.

References

[1] Brard, R., "Introduction à l'étude théorique du tangage en marche," Bulletin de l'ATMA, vol. 47, Paris, 1948.

[2] Haskind, M. D., "On Wave Motion of a Heavy Fluid," Applied Mathematics and Mechanics, Vol. 18, 1954.

[3] Hong, D. C. , " Calcul des coefficients hydrodynamiques d'un flotteur cylindrique en mouvement de translation uniforme et soumis à des oscillations forcées."Rapport de D.E.A., l'ENSM de Nantes, France, 1978.

[4] Hong, D. C., "Unsteady Wave Generation by an Oscillating Cylinder Advancing under the Free Surface,"Journal of the SNAK, vol. 25, No. 2, 1988.

[5] Park, S. H. , "Oscillation of Cylinders near the Free Surface of a Deep Fluid in Horizontal Currents," Dissertation, Ocean Eng. Dept., Stevens Inst. of Tech., May 1983.

[6] Lee, S. C., "Current Effect on the Motion and Drift Force of Cylinders Floating in Waves," Dissertation, Ocean Eng. Dept., Stevens Inst. of Tech., May 1986.

[7] Chang, M. S., "Computations of Three-Dimensional Ship-Motions with Forward Speed," Proc. of the 2nd Int. Conference on Numerical Hydrodynamics, Berkeley, Sept. 1977.

[8] Bougis, J., "Etude de la diffraction radiation dans le cas d'un flotteur indéformable animé d'une vitesse moyenne constante et sollicité par une houle sinusoïdale de faible amplitude," Thése de Docteur-Ingénieur, l'ENSM de Nantes, France, juillet 1980.

[9] Inglis, R. B. and Price, W. G., "A Three Dimensional Ship Motion Theory ; Comparison between Theoretical Predictions and Experimental Data of the Hydrodynamic Coefficients with Forward Speed," Transaction RINA, vol. 124, 1981.

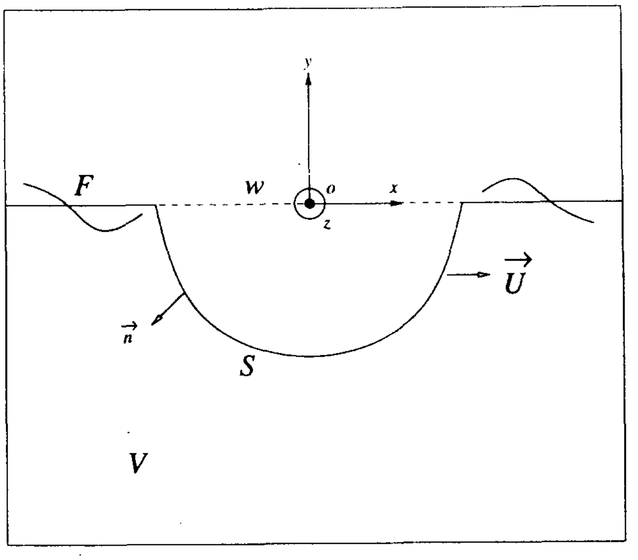

Figure 1: Coordinate systems

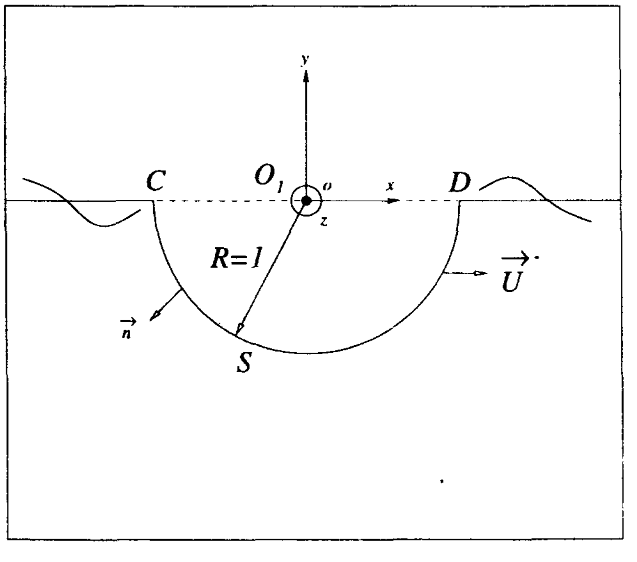

Figure 2: Numerical model

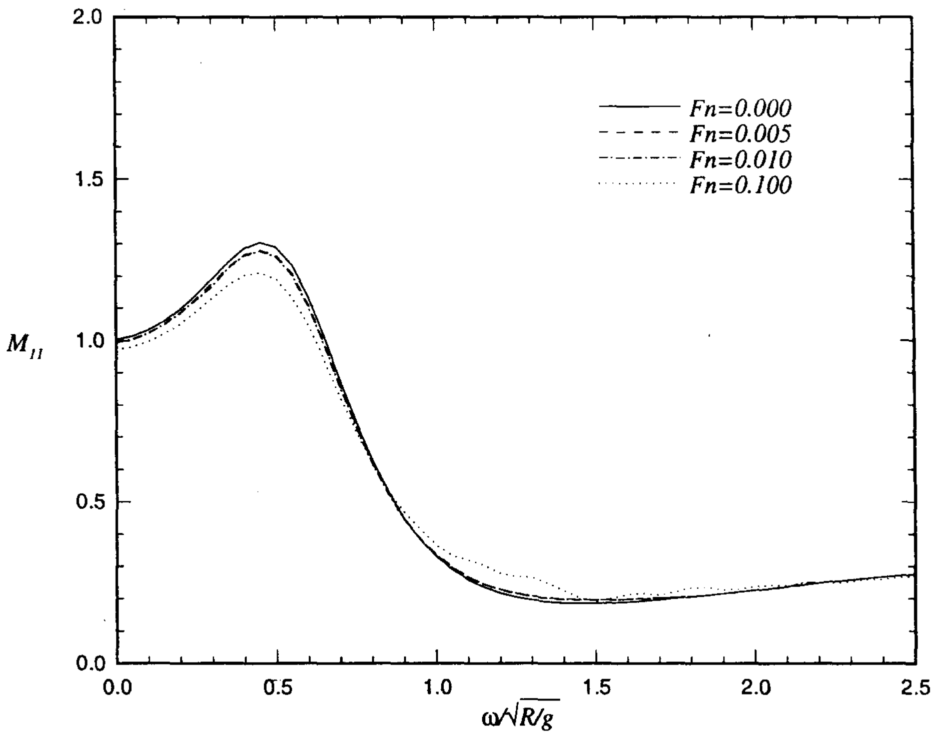

Figure 3: Surge induced surge added-mass coefficients

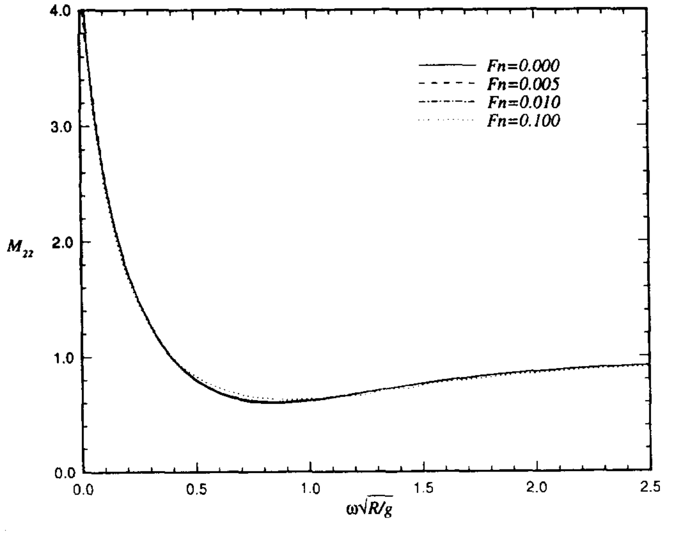

Figure 4: Heave induced heave added-mass coefficients

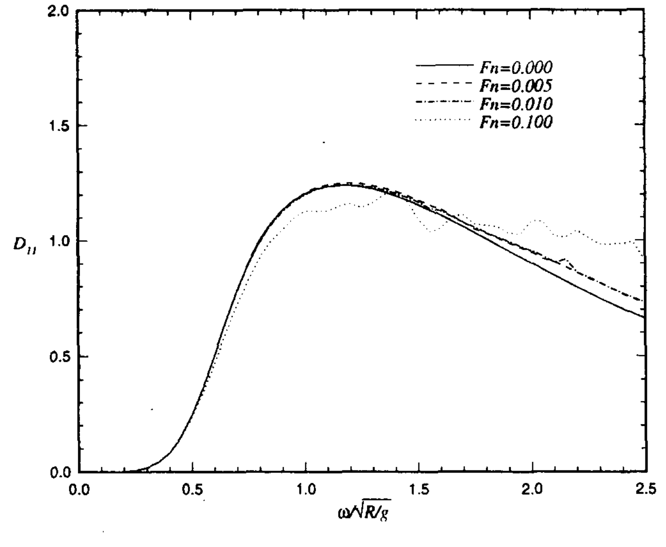

Figure 5: Surge induced surge wave-damping coefficients

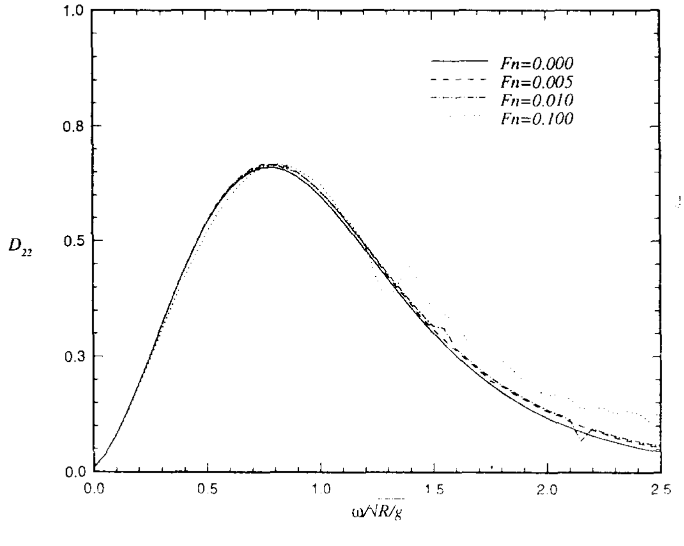

Figure 6: Heave induced heave wave-damping coefficients

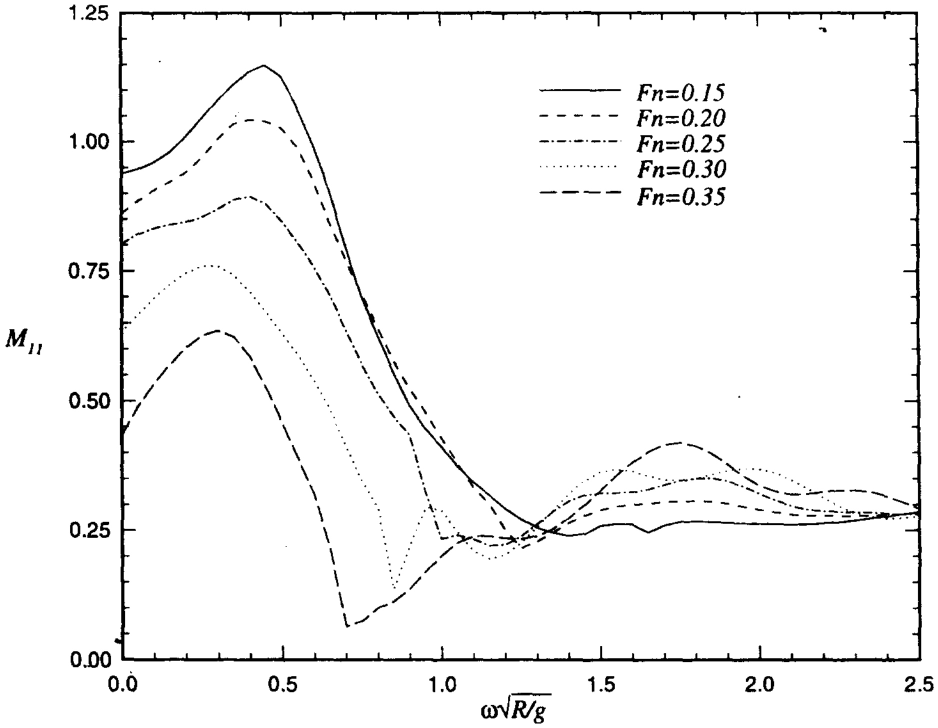

Figure 7: Surge induced surge added-mass coefficients

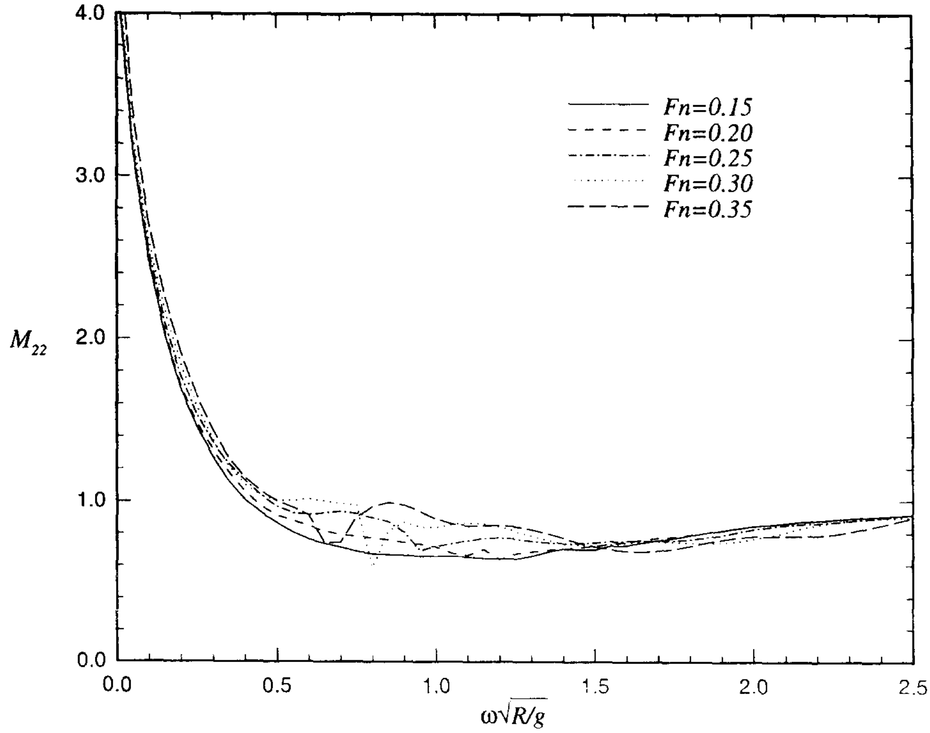

Figure 8: Heave induced heave added-mass coefficients

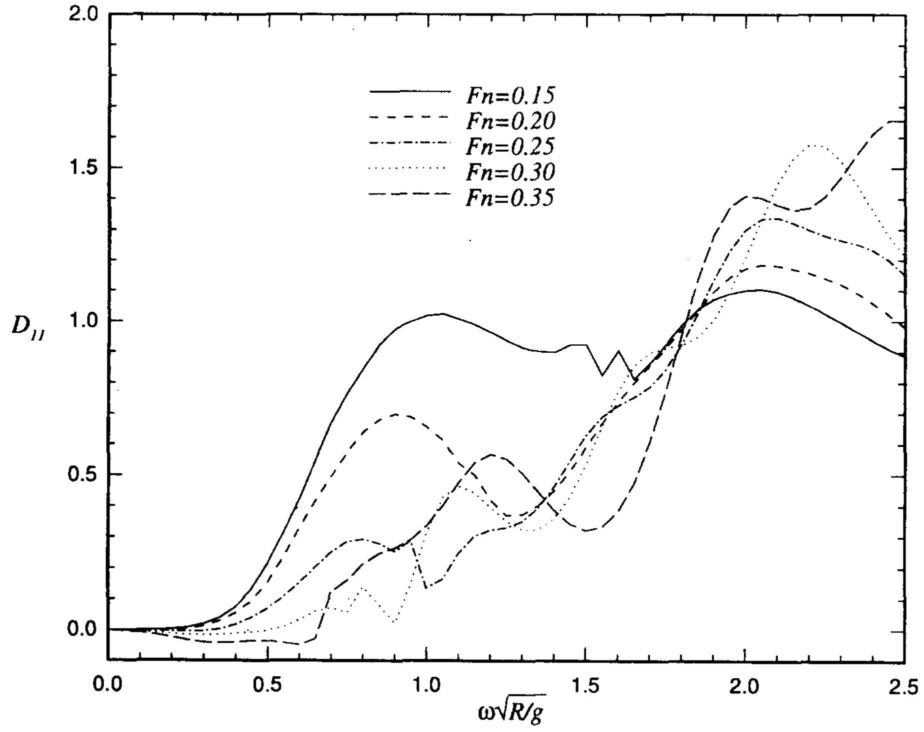

Figure 9: Surge induced surge wave-damping coefficients

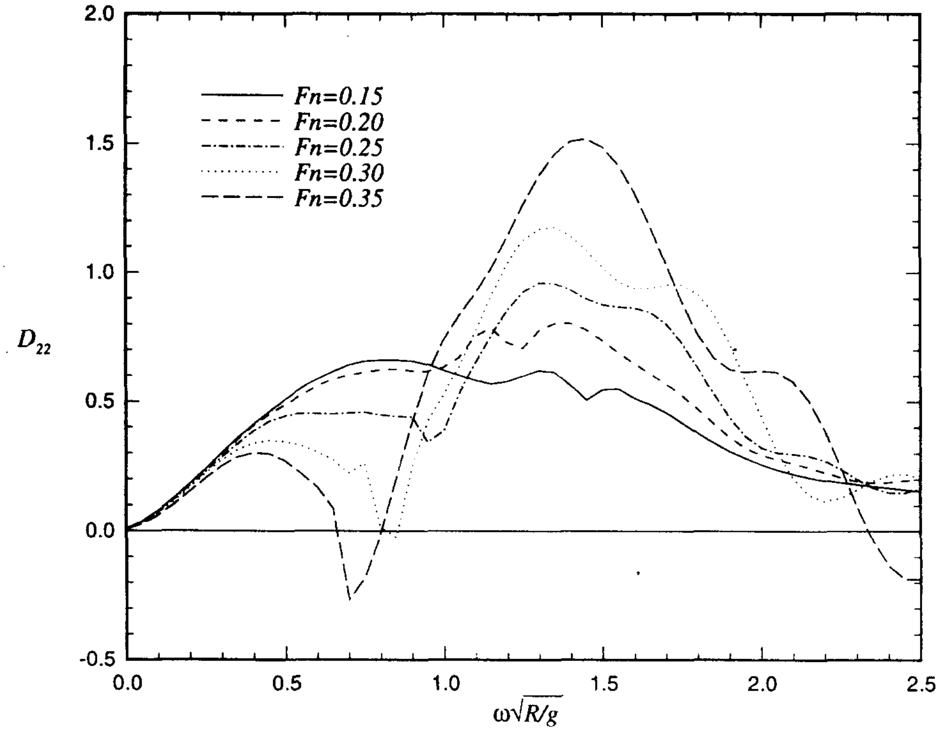

Figure 10: Heave induced heave wave-damping coefficients