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Effects of Air Compressibility on the Hydrodynamic Forces of a Bag

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Abstract

The hydrodynamic problem when the pressurized bag submerges partially into water and oscillates was formulated by Lee(1992), and the solution method was given. In his formulation, however, the compressilbility of air was neglected and the pressure inside the bag was assumed to be constant.

In this paper, the formulation was done including the air compressibility and the wall to block fling around phenomenon. The compression process was assumed to be a isothermal process for a static problem, isentropic process for a dynamic problem. And the stability was analyzed for the static problem. Through the various numerical calculations, the forces and the shape of the bag were compared with those of a rigid body case, constant pressure case, and variable pressure case.

1 Introduction

The stern bag of SES plays an important role in preventing the air from leakage out of the cushion chamber, and this is very effective because of its flexibility. The bag also has an effect on the motion of the craft, especially on the pitch motion.

Ozawa[1] studied the dynamic characteristics of the seal system of SES by theoretical method and experiments, however in his study the hydrodynamics is not included. The hydrodynamics of the bag submerged partially into water was formulated and calculated by Lee[2], but he assumed a constant pressure in the bag, so the compressibility of air was ignored.

The hydrodynamics of a bag has peculiar characteristics: the boundary condition on the bag is represented globally not point-wisely and it has a very complex form because the pressure change in certain portion of a bag affects the shape of the bag on the whole. And it is a moving boundary like a free surface boundary so the treatment of it is difficult.

The motion of a bag can be divided into two modes by its mechanism, one mode is due to the pressure change in the bag, and another due to the movement of structure to which the bag is attached. In the later case, the pressure in the bag can be constant or

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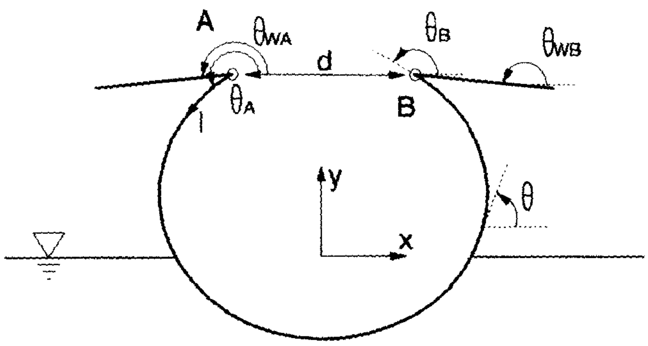

Figure 1: Shape of a bag and coordinate system

variable, it is almost constant when the air of the bag is fed by a huge reservoir and the opening area to the bag is large, and when the opening of the bag is closed the pressure inside is to be varied in order to satisfy the state equation of air. There are two processes in compression-expansion, isothermal process and isentropic process. For static problem, it is assumed to be isothermal because there is enough time to flow out the resulting heat, and for dynamic problem isentropic because there is too short time to flow out the resulting heat totally.

In this paper, the static and dynamic problem were formulated including the compressibility of air. Because most bags are made of fiber, the mass of the bag and girth-wise elongation were ignored. And the hydrodynamic problem is treated by a potential theory. The wall to block the 'fling around' is included, and the static stability was analyzed more thoroughly. Through a number of numerical calculations, the comparison was done with rigid body, compressible air, constant pressure cases.

2 Static Problem

In this chapter, the shape and the static problem were studied for a pressurized bag submerged partially into water. The pressure in the bag may remain constant, or may be varied because of the compressibility of air. The isothermal process is assumed. In this study, the mass of a bag and elongation in girth-wise length are ignored like Lee[2].

2.1 The shape of a Bag

Suppose that there is no tangential force on the surface of a bag, so the tension is constant along the perimeter. The shape of a bag and coordinate system is shown in Fig.1.

The end points of a bag are attached to the structure, these points are denoted by point A, B and their positions are (x_A, y_A), (x_B, y_B). The angles between the tangential direction of the bag and x-axis are θ_A, θ_B at end points, and $\theta(l)$ between them. Here l is the girth-wise length, and total length is L. If the bag is overlapped with the wall, the angles of end points are θ_{WA}, θ_{WB}, and the overlapping lengths are l_A, l_B respectively.

The relationship of the pressure, tension, curvature of the bag is given by the following Laplace formula.[3]

$$P_b - P = \frac{T}{R}, \tag{1}$$

where P_b is the pressure in the bag, P out of the bag, T tensile force, R radius of curvature. The definition of radius of curvature is the reciprocal of the derivative of tangential angle with respect to arc length, so the tangential angle can be obtained from it.

$$\begin{aligned} \theta(l) &= \int_0^l \frac{1}{R} du + \theta_A \\ &= \int_0^l \frac{P_b - P(u)}{T} du + \theta_A, \end{aligned} \tag{2}$$

and when the bag is overlapped with the wall, it is

$$\theta(l) = \begin{cases} \theta_{WA} & l < l_A \\ \theta_{WA} + T^{-1} \int_{l_A}^{l} (P_b - P(u)) du & l_A < l < L - l_B \\ \theta_{WA} + T^{-1} \int_{l_A}^{L-l_B} (P_b - P(u)) du & l > L - l_B. \end{cases} \tag{3}$$

$\theta_{WA} = \theta_A$ when $l_A = 0$. With this angle, the shape of the bag can be represented by

$$\begin{aligned} x(l) &= \int_0^l \cos(\theta(u)) du + x_A, \\ y(l) &= \int_0^l \sin(\theta(u)) du + y_A. \end{aligned} \tag{4}$$

Once the two points A, B and length L and pressure difference are given, the shape can be obtained. The unknowns are T, θ_A for $l_A = 0$, or T, l_A for $l_A > 0$. For the mathematical convenience, introduce the generalized θ_A.

$$\theta_A = \begin{cases} \theta_A & \text{if } l_A = 0 \\ l_A & \text{if } l_A > 0. \end{cases} \tag{5}$$

The following state equation of air is to be used when the compressibility of air is included.

$$P_b V = \text{const}.$$

Above equation is for the isothermal process. In the static problem, there is enough time to flow out the resulting heat totally, so the assumption of isothermal process is reasonable. Including the compressibility, the pressure inside P_b is also an unknown to be sought for, thus three unknowns can be found from three equations below.

$$\begin{aligned} f_1 &= x(L) - x_B = 0 \\ f_2 &= y(L) - y_B = 0 \\ f_3 &= P_b V - P_{b0} V_0 = 0, \end{aligned} \tag{6}$$

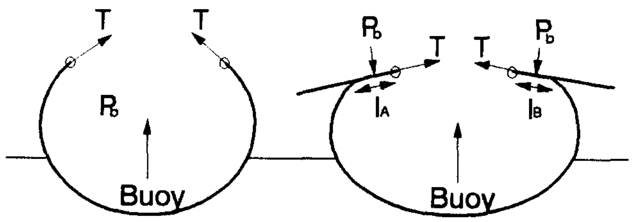

Figure 2: Forces and shape of a bag

where the volume inside is

$$V = \int_0^L y\frac{dx}{dl}dl + \frac{1}{2}(y_A + y(L))(x_A - x(L)).$$

The pressure out of the bag has a value $P(l) = -\rho g y(l)$ only in the portion below the free surface $y = 0$. Eq.(6) are non-linear equations, the solution is obtained using Newton-Raphson method.

$$\left\{ \begin{array}{c} T^{k+1} \\ \theta_A^{k+1} \\ P_b^{k+1} \end{array} \right\} = \left\{ \begin{array}{c} T^{k} \\ \theta_A^{k} \\ P_b^{k} \end{array} \right\} - \left\{ \begin{array}{c} \Delta T^{k} \\ \Delta\theta_A^{k} \\ \Delta P_b^{k} \end{array} \right\},$$

$$\left\{ \begin{array}{c} \Delta T^{k} \\ \Delta\theta_A^{k} \\ \Delta P_b^{k} \end{array} \right\} = \left[\begin{array}{ccc} \frac{\partial f_1}{\partial T} & \frac{\partial f_1}{\partial \theta_A} & \frac{\partial f_1}{\partial P_b} \\ \frac{\partial f_2}{\partial T} & \frac{\partial f_2}{\partial \theta_A} & \frac{\partial f_2}{\partial P_b} \\ \frac{\partial f_3}{\partial T} & \frac{\partial f_3}{\partial \theta_A} & \frac{\partial f_3}{\partial P_b} \end{array} \right]^{-1} \left\{ \begin{array}{c} f_1^k \\ f_2^k \\ f_3^k \end{array} \right\}. \quad (7)$$

The derivatives used in above equations are derived in Appendix A. The initial guesses are given as the same manner as in the work of Lee[2].

2.2 Static Stability

The stability of the static equilibrium state of a bag is analyzed here.

Firstly, suppose the case that the bag is not overlapped with the wall. The external force on the bag is

$$\begin{aligned} f_x &= -T\cos\theta_A + T\cos\theta_B, \quad (8) \\ f_y &= -P_b d + Buoy - T\sin\theta_A + T\sin\theta_B. \end{aligned}$$

If the shape is symmetric in y-axis, $\theta_A = -\theta_B$, so the force in the x-direction vanishes. When an infinitesimal force in the x-direction exists, the angles are changed by an amount of $\Delta\theta_A, \Delta\theta_B$ respectively, and these changes are positive. The change in f_x is

$$\Delta f_x = T\sin\theta_A\Delta\theta_A - T\sin\theta_B\Delta\theta_B \quad (9)$$

For the case that $\theta_B < \pi$, Δf_x is negative and its direction is opposed to the direction of movement, so the positive restoring results out. The static instability takes place when $\theta_B > \pi$, and a neutral state when $\theta_B = \pi$.

Secondly, suppose the case that the bag is overlapped with the wall. Similarly, the external force on the bag is

$$\begin{aligned} f_x &= -T\cos\theta_{WA} + T\cos\theta_{WB} \\ &\quad -P_b l_A \sin\theta_{WA} - P_b l_B \sin\theta_{WB}, \\ f_y &= -P_b d + Buoy - T\sin\theta_{WA} + T\sin\theta_{WB} \\ &\quad +P_b l_A \cos\theta_{WA} - P_b l_B \cos\theta_{WB}. \end{aligned} \tag{10}$$

If the shape is symmetric, $\theta_{WA} = \theta_{WB}$. When an infinitesimal force in the x-direction exists, the overlapped length are changed by an amount of $\Delta l_A, \Delta l_B$, where Δl_A is negative, Δl_B positive. The change in f_x is

$$\Delta f_x = -P_b \Delta l_A \sin\theta_{WA} - P_b \Delta l_B \sin\theta_{WB}. \tag{11}$$

From the above equation, it can be said that it is stable when $\theta_{WB} < \pi$, unstable when $\theta_{WB} > \pi$, neutral when $\theta_{WB} = \pi$. This result is the same as the case of no wall.

3 Dynamic Problem

The hydrodynamic problem when the bag moves is treated here. The motion of a bag can be divided into two modes by its mechanism, one mode is due to the pressure change in the bag, and another due to the movement of structure to which the bag is attached. In the later case, the pressure in the bag can be constant or variable, including the compressibility of air it is to be varied in order to satisfy the state equation of air, and an isentropic process is assumed.

3.1 Shape Change due to Pressure Change

In order to obtain the boundary condition on the bag, the shape change due to the pressure change in and out has to be known before.

Firstly, suppose the case that the pressure in the bag changes. The change of P_b results in the changes of T, θ_A, and the shape. Denote the change of P_b by dP_b, and the shape by dX_1, dY_1. Even though P_b changes, the end points of the bag remain at the same points, that is, the first two equations in Eq.(6) must hold. The differentials of them are

$$\begin{aligned} \frac{\partial x(L)}{\partial P_b} dP_b + \frac{\partial x(L)}{\partial T} dT + \frac{\partial x(L)}{\partial \theta_A} d\theta_A &= 0, \\ \frac{\partial y(L)}{\partial P_b} dP_b + \frac{\partial y(L)}{\partial T} dT + \frac{\partial y(L)}{\partial \theta_A} d\theta_A &= 0. \end{aligned} \tag{12}$$

$dT, d\theta_A$ can be obtained from the above equations.

$$\begin{Bmatrix} dT \\ d\theta_A \end{Bmatrix} = -\begin{bmatrix} \frac{\partial x(L)}{\partial T} & \frac{\partial x(L)}{\partial \theta_A} \\ \frac{\partial y(L)}{\partial T} & \frac{\partial y(L)}{\partial \theta_A} \end{bmatrix}^{-1} \begin{Bmatrix} \frac{\partial x(L)}{\partial P_b} \\ \frac{\partial y(L)}{\partial P_b} \end{Bmatrix} dP_b. \tag{13}$$

The change of the shape is

$$dX_1(l) = \frac{\partial x(l)}{\partial T}dT + \frac{\partial x(l)}{\partial \theta_A}d\theta_A + \frac{\partial x(l)}{\partial P_b}dP_b, \tag{14}$$

$$dY_1(l) = \frac{\partial y(l)}{\partial T}dT + \frac{\partial y(l)}{\partial \theta_A}d\theta_A + \frac{\partial y(l)}{\partial P_b}dP_b. \tag{15}$$

Substituting Eq.(13) into above equations, $dX_1(l), dY_1(l)$ can be represented by only one variable P_b. The derivatives used in above equations are derived in Appendix A.

Secondly, let us seek the shape change due to variations of the outside pressure. Hereafter the equations are derived for the case of compressible air, if the constant pressure is wanted it is achieved by setting $dP_b = 0, V =$const. In the isentropic process, the state equation of air is

$$P_b^{1/\gamma}V - P_{b0}^{1/\gamma}V_0 = 0, \tag{16}$$

where γ is the specific heat of air, its value is 1.4 for dry air. The differentials of the first two equations in Eq.(6) and the above equation are as follows.

$$\begin{aligned} &\frac{\partial x(L)}{\partial P}dP + \frac{\partial x(L)}{\partial T}dT + \frac{\partial x(L)}{\partial \theta_A}d\theta_A + \frac{\partial x(L)}{\partial P_b}dP_b = 0, \\ &\frac{\partial y(L)}{\partial P}dP + \frac{\partial y(L)}{\partial T}dT + \frac{\partial y(L)}{\partial \theta_A}d\theta_A + \frac{\partial y(L)}{\partial P_b}dP_b = 0, \\ &\frac{\partial V}{\partial P}dP + \frac{\partial V}{\partial T}dT + \frac{\partial V}{\partial \theta_A}d\theta_A + \left(\frac{V}{\gamma P_b} + \frac{\partial V}{\partial P_b}\right)dP_b = 0. \end{aligned} \tag{17}$$

$dT, d\theta_A, dP_b$ can be obtained from the above equations as follows.

$$\begin{Bmatrix} dT \\ d\theta_A \\ dP_b \end{Bmatrix} = -\begin{bmatrix} \frac{\partial x(L)}{\partial T} & \frac{\partial x(L)}{\partial \theta_A} & \frac{\partial x(L)}{\partial P_b} \\ \frac{\partial y(L)}{\partial T} & \frac{\partial y(L)}{\partial \theta_A} & \frac{\partial y(L)}{\partial P_b} \\ \frac{\partial V}{\partial T} & \frac{\partial V}{\partial \theta_A} & \frac{V}{\gamma P_b} + \frac{\partial V}{\partial P_b} \end{bmatrix}^{-1} \begin{Bmatrix} \frac{\partial x(L)}{\partial P}dP \\ \frac{\partial y(L)}{\partial P}dP \\ \frac{\partial V}{\partial P}dP \end{Bmatrix}. \tag{18}$$

The derivatives used in above equations are derived in Appendix A. and B. Because $dP(l)$ is a function, derivative with respect to $dP(l)$ is somewhat different from that with respect to a scalar, and it has to be obtained in the distribution sense. Thus derivatives with respect to a function $dP(l)$ is linear operators not values, Appendix B. contains more details. The shape change of the bag can be obtained as follows.

$$dx(l) = \frac{\partial x(l)}{\partial T}dT + \frac{\partial x(l)}{\partial \theta_A}d\theta_A + \frac{\partial x(l)}{\partial P_b}dP_b + \frac{\partial x(l)}{\partial P}dP, \tag{19}$$

$$dy(l) = \frac{\partial y(l)}{\partial T}dT + \frac{\partial y(l)}{\partial \theta_A}d\theta_A + \frac{\partial y(l)}{\partial P_b}dP_b + \frac{\partial y(l)}{\partial P}dP.$$

Using Eq.(18), the above equations are rewritten as below.

$$
\begin{aligned}
dx(l) &= \frac{\partial x(l)}{\partial P} dP + \left\{ \begin{array}{c} \frac{\partial x(l)}{\partial T} \\ \frac{\partial x(l)}{\partial \theta_A} \\ \frac{\partial x(l)}{\partial P_b} \end{array} \right\}^T \left\{ \begin{array}{c} dT \\ d\theta_A \\ dP_b \end{array} \right\} \\
&= L_x \cdot dP, \qquad (20)
\end{aligned}
$$

$$
\begin{aligned}
dy(l) &= \frac{\partial y(l)}{\partial P} dP + \left\{ \begin{array}{c} \frac{\partial y(l)}{\partial T} \\ \frac{\partial y(l)}{\partial \theta_A} \\ \frac{\partial y(l)}{\partial P_b} \end{array} \right\}^T \left\{ \begin{array}{c} dT \\ d\theta_A \\ dP_b \end{array} \right\} \\
&= L_y \cdot dP. \qquad (21)
\end{aligned}
$$

where L_x, L_y are linear operators.

3.2 Boundary Condition

The boundary condition on the surface of a rigid body is well known and simple, but on the surface of a flexible body, like a pressurized bag, the boundary condition has a complicated nature.[2] In the previous section, the shape change due to the change of pressure is given, but practically in a fluid the bag can not make such shape change because of the static and dynamic pressure in the fluid.

$$P = -\rho g y - \rho \phi_t \qquad \text{for } y < 0,$$

where ϕ is a velocity potential. The motion excluding this dynamic condition, that is, the movement due to the change of the pressure in the bag and due to the movement of the end points is represented by

$$
\begin{aligned}
dX_E(l) &= dX_1(l) + dX_2, \qquad (22) \\
dY_E(l) &= dY_1(l) + dY_2,
\end{aligned}
$$

where dX_1, dY_1 are the shape changes due to the pressure inside, and dX_2, dY_2 are the motion displacements due to the movement of its end points. If the compressibility of air is included, dX_1, dY_1 need not to be considered. The actual displacement of the surface of a bag is as follows, using the dynamic condition.

$$
\begin{aligned}
dy &= L_y \cdot dP + dY_E \\
&= -\rho g L_y^* \cdot dy - \rho L_y^* \cdot d\phi_t + dY_E,
\end{aligned}
$$

$$dy = [I + \rho g L_y^*]^{-1} [-\rho L_y^* \cdot d\phi_t + dY_E], \qquad (23)$$

in which I is an identity operator and L_y^* is an operator which is reduced from L_y by ignoring the part of $y > 0$. Similarly dx can be obtained as follows,

$$\begin{aligned} dx &= L_x \cdot dP + dX_E \\ &= -\rho L_x^*[I + \rho g L_y^*]^{-1} d\phi_t \\ &\quad -\rho g L_x^*[I + \rho g L_y^*]^{-1} dY_E + dX_E. \end{aligned} \tag{24}$$

Only the portion $y < 0$ is required to solve the boundary value problem. Examining the above two equations closely, we know that the portion $y > 0$ has no effect on the portion $y < 0$ because the pressure remains constant over the portion $y > 0$. Thus we can rewrite the equations only for $y < 0$.

$$\begin{aligned} dx^* &= -\rho K_x[I + \rho g K_y]^{-1} d\phi_t \\ &\quad -\rho g K_x[I + \rho g K_y]^{-1} dY_E^* + dX_E^*, \\ dy^* &= [I + \rho g K_y]^{-1}[-\rho K_y d\phi_t + dY_E^*], \end{aligned} \tag{25}$$

where K_x, K_y are the linear operators which are reduced from L_x^*, L_y^* by ignoring the part of $y > 0$, and defined only on the portion $y < 0$.

The kinematic condition is

$$\begin{aligned} \phi_n &= n_x x_t^* + n_y y_t^* \\ &= n_x X_{Et}^* + C_y Y_{Et}^* - C_\phi \phi_{tt}, \end{aligned} \tag{26}$$

where

$$\begin{aligned} C_y &= [n_y - n_x \rho g K_x][I + \rho g K_y]^{-1}, \\ C_\phi &= [n_x \rho K_x [I + \rho g K_y]^{-1} + n_y \rho [I + \rho g K_y]^{-1} K_y]. \end{aligned}$$

The boundary condition can be represented globally as shown above. Above boundary condition seems to be similar to that of free surface, but it is global while that of free surface is point-wise.

3.3 Boundary Value Problem

In the framework of potential theory, the governing equation is Laplace equation, and there must exist boundary condition on the whole boundary. This boundary value problem is summarized as follows.

$$\begin{aligned} \nabla^2 \phi &= 0 \qquad \text{in fluid domain}, \\ \phi_y + 1/g\ \phi_{tt} &= 0 \qquad \text{on } y = 0, \\ \phi_n + C_\phi \phi_{tt} &= n_x X_{Et}^* + C_y Y_{Et}^* \\ &\qquad \text{on the Surface of the bag}, \end{aligned} \tag{27}$$

and appropriate radiation condition. The solution of this boundary value problem can be obtained by Green's identity

$$\phi(P) = \int_S \{G_{nQ}(P,Q)\phi(Q) - G(P,Q)\phi_n(Q)\} dS(Q), \tag{28}$$

where P is the field point and Q source point, and $G(P,Q)$ is the fundamental solution of Laplace equation which satisfies the free surface boundary condition and the radiation condition.[4] Discretizing the surface submerged, assuming that the values of ϕ, ϕ_n are constant over each segment and equal to the values at midpoint, and performing integration over each segment analytically, the above equation becomes the matrix equation below.

$$\{\phi\} = [G_n]\{\phi\} - [G]\{\phi_n\}. \tag{29}$$

Suppose the case of time harmonic motion. Substituting the boundary condition into the above equation results in

$$[I - G_n + \omega^2 G C_\phi]\{\phi\} = -[G]\{n_x X^*_{Et} + C_y Y^*_{Et}\}. \tag{30}$$

After discretization, the operator C_ϕ, C_y turn into matrices and n_x, n_y diagonal matrices. Once the solution of the above equation is found, the pressure on the surface of a bag can be calculated by

$$\begin{aligned} dP &= -\rho g dy - \rho d\phi_t \\ &= -\rho[I + \rho g K_y]^{-1}\{g dY_E + d\phi_t\} \quad \text{for } y < 0. \end{aligned} \tag{31}$$

The first term of the above equation is static pressure and the second term is hydrodynamic pressure. The force acting on the bag is calculated by integrating dP on the surface of it. Substituting the above equation into Eq.(20),(21), we can obtain the shape change of the bag, and into Eq.(18), the tension, θ_A, P_b.

4 Numerical Resutls

All calculations were carried out with single precision on the i386 based PC. The total length of the perimeter of a bag was divided into 100 elements. And non-dimensionalization is as follows: perimeter length L/d , submerged depth $depth/d$, volume inside $V' = V/d^2$, submerged area $A' = A/d^2$, pressure inside $p' = p_b/\rho g d$, tension $T' = T/pd$, frequency $\omega/\sqrt{g/d}$, added mass $a/\rho d^2$, damping coefficient $b/\rho d^2\sqrt{g/d}$.

4.1 Static Problem

All integrations used for the static problem were performed by the trapezoidal method. Lee[2] used the modified Newton method to solve the non-linear equations, however in this paper, the conventional Newton-Raphson method makes no problem in solving them because the wall to block 'fling around' makes the solution scheme more stable. But the special numerical treatment is required when the bag starts to be overlapped with the wall.

The shapes of a bag with various submerged depths are shown in Fig.3 and Fig.4. The results for constant pressure are in Fig.3, and for compressible air in Fig.4. Comparing two cases, it is known that the shapes are similar for small depths, but as the depth increases the shape change is smaller and the bottom position is lower in the case of compressible air. This is due to the fact that the inside pressure grows as the depth increases.

In Fig.5, the inside volume, submerged area, tension, inside pressure are drawn for the various submerged depth. Including the compressibility of air, it can be said that all values are larger than those for the constant pressure case. The buoyancy force can be obtained by multiplying the submerged area by ρg, and the restoring force by differentiating this with respect to depth, so the restoring force is proportional to the slope of the submerged area. For the constant pressure, the slope is nearly constant while the length of water line increases, but for the compressible air the slope grows large as the depth increases, and therefore larger restoring force than that of constant pressure case.

4.2 Dynamic Problem

The heave added mass and damping of a bag are shown in Fig.6 and Fig.7. In low frequency, the added mass and damping have similar behavior to those of rigid body while their values are small. As the pressure increases, the added mass and damping become close to that of rigid body. The effect of compressibility is not shown significantly. In the medium frequency ranged from 1.5 to 2.7 for the Fig.6 and Fig.7, the added mass and damping behave quite strangely, this phenomenon seems to be a resonance of the bag. The bag itself is a spring-mass-damper system because the bag above the free surface has a similar role of spring. Therefore the bag has a natural frequency, and as the pressure inside grows large, tension increases, and does the equivalent spring constant. As the result, the resonant frequency of high pressure is higher than that of low pressure. The possibility of irregular frequency has been considered, but this is not the case because the submerged shape is nearly the same so the irregular frequency must remain near a certain frequency, and the added mass and damping of the rigid body of same shape in the range under considering have a smooth behavior. Thus this phenomenon can not be considered as that of irregular frequency. Therefore author concluded that this is the resonance of the bag, but a more careful study on this phenomenon must be done.

5 Conclusions

In this paper, the static and dynamic problem was analyzed including the compressibility of air when a bag filled with pressurized air submerges into water. This problem has some distinct nature: both the kinematic and dynamic conditions are required for the boundary condition on the surface of a bag because the surface of a bag can be deformed easily by the pressure acting on it, and the boundary condition is represented not locally but globally.

In this paper, the formulation was done in the framework of potential theory including the compressibility of air, and the wall to block 'fling around' is included. Following conclusions are drawn:

1. The effect of air compressibility was shown in static problem especially in large depth, but in dynamic problem the effect is small.

2. The added mass and damping behaves strangely near a certain frequency, this behavior seems to be a resonance of the bag not an irregular frequency phenomenon.

Author hopes advances in the resonance of a bag, and the application to the stern bag of SES.

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Appendix A. Derivatives w.r.t. Scalar

The definition of the tangential angle $\theta(l)$ is given in Eq.(3). Derivative of $\theta(l)$ with respect to a generalized θ_A is, for $l_A = 0$,

$$\frac{\partial\theta(l)}{\partial\theta_A} = 1, \tag{A.1}$$

for $l_A > 0$,

$$\frac{\partial\theta(l)}{\partial\theta_A} = \begin{cases} 0 & l < l_A \\ -\{P_b - P(l_A)\}/T & l_A < l < L - l_B \\ -\{P_b - P(l_A)\}/T & l > L - l_B. \end{cases} \tag{A.2}$$

Derivative of $\theta(l)$ with respect to T, P_b.

$$\frac{\partial\theta(l)}{\partial T} = \begin{cases} 0 & l < l_A \\ -T^{-2}\int_{l_A}^{l}\{P_b - P(u)\}du & l_A < l < L - l_B \\ -T^{-2}\int_{l_A}^{L-l_B}\{P_b - P(u)\}du & l > L - l_B, \end{cases} \tag{A.3}$$

$$\frac{\partial\theta(l)}{\partial P_b} = \begin{cases} 0 & l < l_A \\ (l - l_A)/T & l_A < l < L - l_B \\ (L - l_B - l_A)/T & l > L - l_B. \end{cases} \tag{A.4}$$

Derivatives of x, y

$$z(l) = x(l) + iy(l) = \int_0^l e^{i\theta(v)} dv + x_A + iy_A, \tag{A.5}$$

$$\frac{\partial z(l)}{\partial T} = \int_0^l e^{i\theta(v)} i \frac{\partial \theta(v)}{\partial T} dv, \tag{A.6}$$

$$\frac{\partial z(l)}{\partial \theta_A} = \int_0^l e^{i\theta(v)} i \frac{\partial \theta(v)}{\partial \theta_A} dv, \tag{A.7}$$

$$\frac{\partial z(l)}{\partial P_b} = \int_0^l e^{i\theta(v)} i \frac{\partial \theta(v)}{\partial P_b} dv. \tag{A.8}$$

Derivative of Volume V

$$V = \int_0^L y \frac{dx}{dl} dl + \frac{1}{2}(y_A + y(L))(x_A - x(L)), \tag{A.9}$$

$$\frac{\partial V}{\partial T} = \int_0^L \left\{ \frac{\partial y(l)}{\partial T} \frac{dx(l)}{dl} + y(l) \frac{\partial}{\partial T} \frac{dx(l)}{dl} \right\} dl \tag{A.10}$$
$$+ \frac{1}{2} \left\{ \frac{\partial y(L)}{\partial T}(x_A - x(L)) - (y_A + y(L)) \frac{\partial x(L)}{\partial T} \right\},$$

$$\frac{\partial V}{\partial \theta_A} = \int_0^L \left\{ \frac{\partial y(l)}{\partial \theta_A} \frac{dx(l)}{dl} + y(l) \frac{\partial}{\partial \theta_A} \frac{dx(l)}{dl} \right\} dl \tag{A.11}$$
$$+ \frac{1}{2} \left\{ \frac{\partial y(L)}{\partial \theta_A}(x_A - x(L)) - (y_A + y(L)) \frac{\partial x(L)}{\partial \theta_A} \right\},$$

$$\frac{\partial V}{\partial P_b} = \int_0^L \left\{ \frac{\partial y(l)}{\partial P_b} \frac{dx(l)}{dl} + y(l) \frac{\partial}{\partial P_b} \frac{dx(l)}{dl} \right\} dl \tag{A.12}$$
$$+ \frac{1}{2} \left\{ \frac{\partial y(L)}{\partial P_b}(x_A - x(L)) - (y_A + y(L)) \frac{\partial x(L)}{\partial P_b} \right\},$$

where
$$\frac{dx(l)}{dl} = \cos(\theta(l)),$$
$$\frac{\partial}{\partial T} \frac{dx(l)}{dl} = -\sin(\theta(l)) \frac{\partial \theta(l)}{\partial T},$$
$$\frac{\partial}{\partial \theta_A} \frac{dx(l)}{dl} = -\sin(\theta(l)) \frac{\partial \theta(l)}{\partial \theta_A},$$
$$\frac{\partial}{\partial P_b} \frac{dx(l)}{dl} = -\sin(\theta(l)) \frac{\partial \theta(l)}{\partial P_b}.$$

Appendix B. Derivatives w.r.t. a Function

When a function f is a function of $P(l)$, the change of f due to a change of P , $dP(l)$ can be obtained in the following manner. The derivative of f w.r.t. P when P is varied at one point $l = s$ by an amount of $\Delta P \cdot \delta(l - s)$ is

$$\left(\frac{\partial f}{\partial P}\right)' = \lim_{\Delta P \to 0} \frac{f(P(l) + \Delta P \cdot \delta(l - s)) - f(P(l))}{\Delta P}. \tag{B.1}$$

This has a similar meaning with an impulse-reponse function. Therefore the change of f due to $dP(l)$ can be obtained as follows.

$$\frac{\partial f}{\partial P} dP = \int_0^L \left(\frac{\partial f}{\partial P}\right)' (l - s) \cdot dP(l) ds. \tag{B.2}$$

The derivative of θ w.r.t P is

$$\left(\frac{\partial \theta}{\partial p}\right)' (l - s) = -\frac{1}{T} H(l - s). \tag{B.3}$$

With this result, the change due to dP can be obtained.

$$\begin{aligned} \frac{\partial z(l)}{\partial P} dP &= \int_0^L \left\{ \int_0^l e^{i\theta(v)} i \left(\frac{\partial \theta}{\partial P}\right)' (v - s) dv \right\} \cdot dP(s) ds \\ &= \int_0^l \left\{ \int_s^l e^{i\theta(v)} \frac{-i}{T} dv \right\} \cdot dP(s) ds, \end{aligned} \tag{B.4}$$

$$\begin{aligned} \frac{\partial V}{\partial P} dP = \int_0^L \Bigg[\int_0^L \left\{ \left(\frac{\partial y}{\partial P}\right)' \frac{dx(l)}{dl} + y(l) \left(\frac{\partial}{\partial P} \frac{dx(l)}{dl}\right)' \right\} dl \\ + \frac{1}{2} \left\{ \left(\frac{\partial y(L)}{\partial P}\right)' (x_A - x(L)) - (y_A + y(L)) \left(\frac{\partial x(L)}{\partial P}\right)' \right\} \Bigg] dP(s) ds. \end{aligned} \tag{B.5}$$

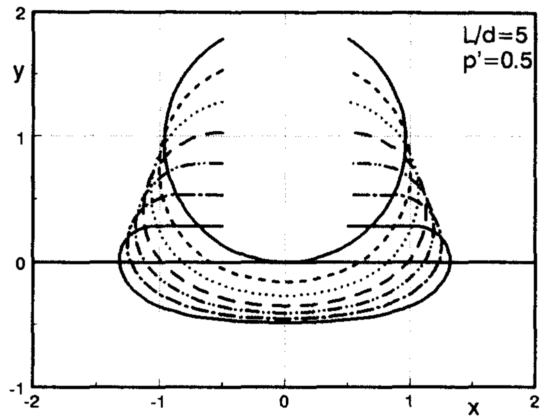

Figure 3: Shape of a bag with various depths for constant pressure case

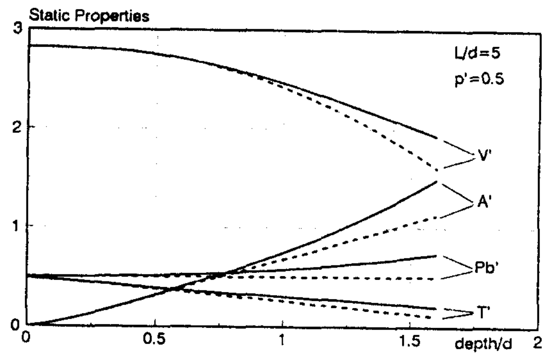

Figure 5: Static properties of a bag with changing depths for compressible air (solid line) and constant pressure (dashed line) cases

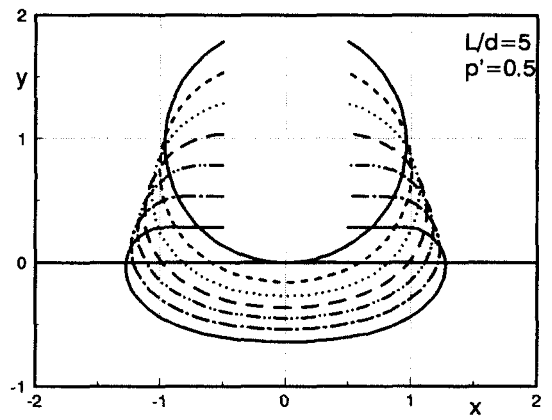

Figure 4: Shape of bag with various depths including air compressibility effect

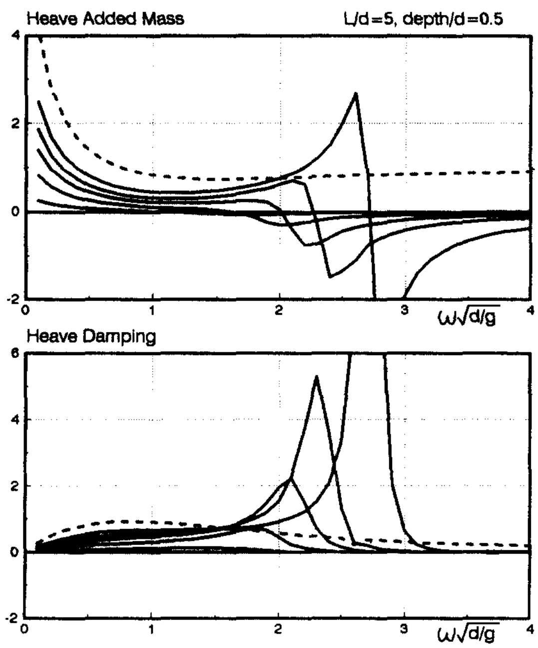

Figure 6: Heave added mass & damping for constant pressure. Non-dimensioned pressures are 0.25, 0.5, 0.75, 1.0, 1.5. Dashed lines are for the rigid body of the same shape.($\rho' = 1$)

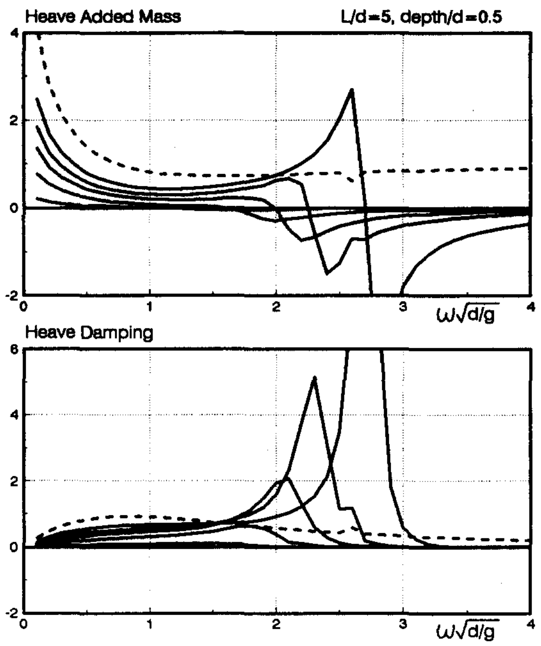

Figure 7: Heave added mass & damping for compressible air. Non-dimensioned pressures are 0.25, 0.5, 0.75, 1.0, 1.5. Dashed lines are for the rigid body of the same shape.($\rho' = 1$)