Simulation of the Flow around and Estimation of The Force Exerted to a Cylindrical Body By the Discrete Vortex Method

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Abstract

Vortex shedding from a circular cylinder is simulated by means of the discrete vortex method. The shear layer emanating from the separation point is approximated as a sheet vortex which is in turn represented by a sequence of discrete vortices. The strength of these vortices is calculated from the vorticity shedding rate and introduced at a small distance off the side ($\theta = \pm\pi/2$) of the cylinder surface in regular time step. Sheet vortex cutting, rediscretization and replacement of vortex by vortex segment are put to use to enhance stability of the sheet vortex evolution. The simulated vortex distribution pattern very well reproduces structure like the Karman vortex street. However, as for the force coefficients, the qualitative properties are correctly predicted but some more improvements are needed for the quantitative accuracy.

1 Introduction

The flow past a long cylindrical body is characterized by the alternating shedding of vorticity from the both sides of the body for a considerable range of Reynolds number, the most conspicuous manifestation being what is commonly called the Karman vortex street. Examples of structural elements apt to expose to this situation are abundant and are specifically commonplace in offshore structures. This is highly undesirable phenomenon which usually accompanies noise, vibration and sometimes disastrous destructive forces. It is therefore essential for a designer to have a tool by which the alternating frequency and the magnitude of forces exerted to the structural elements can be estimated.

The discrete vortex method(DVM) is a promising candidate as the tool since it is intrinsically more suitable to deal with the vortex-dominated problems than other numerical techniques based on the velocity. This method models viscous rotational flow by inviscid one with embedded vortices, and as such produces realistic flow patterns through only moderate computing efforts. The implemented principles of DVM to cope with the viscosity-related processes and effects may make one as much say as the legitimacy of the inviscid flow

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concept as the substitute for the real viscous flow has been considerably strengthened by the appearence of the method.

Since Gerrard[1] introduced the method to investigate vortex shedding from a circular cylinder, it has been further developed and refined by many investigators, notably, Sarpkaya, Fink and Soh, Stansby and Dixon among others. Within the context of the method one can treat the vortex shedding from a cylindrical body by mainly two different ways. In the first, one pays attention only to the vortex shedding rate from the body into the flow field and the shear layer where the vorticity is concentrated. This way of taking the problem results in the sheet vortex model since the shed vorticity is assumed to form a sheet vortex leaving the whole flow field irrotational except this sheet. In the other, one tries to represent the rotational flow, that is the flow within the boudary layer and the wake, by randomly located vortices all of which were created at one time or another in the vicinity of the body surface and convected to their current position. This way of treating the problem gives birth to the vortex cloud model.

The present investigation follows the sheet vortex model and introduces techniques to overcome difficulties intrinsically confronted with therin. This model was taken up by Sarpkaya[2,3] in his series of investigations. Fink and Soh[4] made a notable contribution to improve the workability of the method. It is a well known fact that numerical treatment of evolution of sheet vortex is highly likely to become unstable, the reason coming from the two facts that a sheet vortex itself shows intrinsic instability and that the numerical error due to the discretization can bring in over-sensitive consequences.

2 Description of the flow field

As the viscosity is assumed to be ineffective and the vorticity prevalent in the region is replaced by a number of discrete lumped vortices, the flow field shown in Fig.1 can be represented by the complex velocity potential. The boundary condition of zero normal component of relative velocity at the cylinder surface is accurately satisfied by a doublet at the cylinder center for the uniform oncoming stream and an image vortex at the inverse point for each point vortex outside the cylinder. The image vortex of a vortex located at z_k with strength Γ_k is given by

$$\begin{aligned} &\text{strength}: \quad \Gamma_{Ik} = -\Gamma_k \\ &\text{position}: \quad z_{Ik} = a^2/\bar{z}_k. \end{aligned} \tag{1}$$

Then the complex potential describing the flow field is expressed, when there are M_t vortices outside the cylinder, by

$$\varphi(z) = U\left(z + \frac{a^2}{z}\right) - \frac{i}{2\pi}\sum_{k=1}^{M_t}\Gamma_k\left[\log(z - z_k) - \log(z - z_{Ik})\right].$$

If every variable is nondimensionalized on the basis of U and a, and is denoted by just the plain symbol, this equation becomes

$$\varphi(z) = z + \frac{1}{z} - \frac{i}{2\pi}\sum_{k=1}^{M_t}\Gamma_k\left[\log(z - z_k) - \log(z - z_{Ik})\right] \tag{2}$$

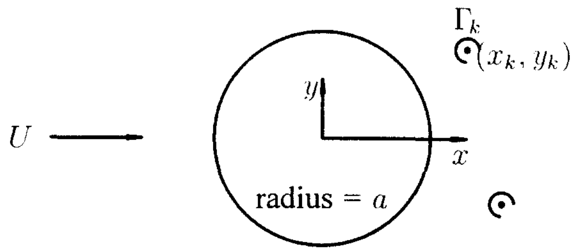

Figure 1: Flow around a circular cylinder

$$\text{where } z_{Ik} = 1/\overline{z}_k. \tag{3}$$

The expression for the velocity is given in conjugate form by the derivative of this equation,

$$\begin{aligned}\overline{w}(z) &= \frac{d\varphi}{dz} \\ &= u - iv = \left(1 - \frac{1}{z^2}\right) - \frac{i}{2\pi}\sum_{k=1}^{M_t}\Gamma_k\left(\frac{1}{z - z_k} - \frac{1}{z - z_{Ik}}\right).\end{aligned} \tag{4}$$

3 Evolution of the shear layer

The shear layer, its thickness being ignored, is supposed to form a sheet vortex starting from the separation point. This sheet vortex is of course to be represented by discrete vortices all of which set out sequentially at regular time interval ideally from the separation point but actually, for brevity, from a prescribed fixed point.

3.1 Nascent vortices

The vortex to be newly introduced at each time step is called the nascent vortex. Its strength is determined from the vorticity shedding rate as shown in the following equation[5]

$$\gamma = \frac{1}{2}u_t^2\,\Delta t \qquad \text{where } \Delta t : \text{time step} \tag{5}$$

in which u_t denotes, in the real situation, the velocity at the edge of the boundary layer at the separation point. In the present inviscid flow model, the boundary layer thickness has to be taken as none but zero and, as for the separation points, the control points set at $\theta = \pm\pi/2$ on the both sides of cylinder surface are chosen instead to estimate the shedding rate. Then, to endow the nascent vortices with appropriate sign, it is better to write the expression for the nascent vortex strengths as

$$\gamma_j = \frac{1}{2}u_{\theta j}\,|u_{\theta j}|\,\Delta t, \qquad \text{where } j = 1, 2. \tag{6}$$

The circumferential component of velocity u_θ, the positive direction being counterclockwise, is calculated from

$$u_{\theta j} = -\Im(e^{i\theta_{cj}}\,\overline{w}_{cj}) \quad (7)$$

where $\overline{w}_{cj} = \overline{w}(z_{cj})$
$\theta_{cj} = \arg z_{cj}$: angular position of the j-th control point
$j = 1, 2.$

The initial starting position of this nascent vortices is set to a point a small distance δ off the cylinder surface to make them convect easily. If these positions are denoted by ζ_1 and ζ_2, they are expressed as

$$\zeta_j = (1+\delta)\, z_{cj}, \quad j = 1, 2. \quad (8)$$

Once created, these nascent vortices are enrolled to the sequence of vortices previously shed into the flow field.

3.2 Convection of the vortices

As a two-dimensional point vortex does not induce velocity at its own position, the expression for the velocity eq.(4) should be accordingly adjusted when the velocity at a vortex position is calculated. If the j-th vortex is considered, for instance,

$$\overline{w}(z_j) = 1 - \frac{1}{z_j^2} - \frac{i}{2\pi}\left[\sum_{\substack{k=1\\k\neq j}}^{M_t}\Gamma_k\left(\frac{1}{z_j - z_k} - \frac{1}{z_j - z_{Ik}}\right) - \Gamma_j\frac{1}{z_j - z_{Ij}}\right] \quad (9)$$

and the j-th vortex convects with this velocity so that its position at the next time step is

$$(z_j)_{\text{new}} = z_j + w(z_j)\,\Delta t. \quad (10)$$

4 Schemes to enhance stability

The convection scheme shown as eq.(10) is liable to develop instability originating from discretization if no remedy is provided to keep the sheet vortex evolution an orderly process.

4.1 Sheet vortex cutting

The vortex sequence emerging from a nascent vortex position exhibits an inflection point at some stage and the sheet curls up with advance of the time steps thereafter around this nucleus to form a double-layered spiral. If this double-layered spiral is formed, the distance between vortices lying on the neighbouring sheet can be smaller than that between neighbouring vortices along the sheet so that the induced velocities are unduly great as to destroy the spiral structure. Before this happens therefore, it is desirable to cut the sheet at around the inflection point when certain cutting conditions are satisfied and let the separated group of vortices go by themselves. The actual cutting is done through two operations; one removing a few vortices from the tail of the group to be detached and the other performing rediscretization separately within each group of vortices. A reasonable set of cutting conditions might be:

(1) The detached vortices should be greater than a certain specified number, typically any between fifteen to twenty;

(2) An inflection point should appear;

(3) The curvature in front or after the inflection point should be greater than a specified value.

The group of vortices so separated is called a vortex cluster and there would be a number of vortex clusters emerging from each side of the cylinder.

4.2 Rediscretization of sheet vortex

Suppose that a straight sheet vortex segment of constant strength density exists in the field by itself. Obviously this segment cannot translate, although can rotate around its midpoint, by its own induced velocity. This fact suggests that the representative point of a vortex segment, the so-called control point, has to be chosen at its midpoint to prevent occurrence of the inadvertent translation.

When the shed vortices are convected according to the convection scheme eq.(10) distances between the neighbouring vortices along a vortex cluster at any time step are generally uneven. It is then desirable to relocate the vortices so that each of them lies at the mid-point of the segment that it assumes to represent. The simplest way of doing this is to put the vortices even-spaced along the curve of the vortex cluster. Fink and Soh[4] called this operation rediscretization and demonstrated that it resulted in a considerable delay of onset of the numerical instability. The principles to be observed in the rediscretization are:

(1) The vortex cluster curve should be unchanged;

(2) The strength density along the curve should be kept unchanged.

4.3 Restriction on the induced velocity magnitude

Notwithstanding these operations, a vortex can still haphazardly come close to another so that the mutual induced velocities are excessively great. As this kind of trouble is due to discretization, it can be avoided by replacing the concerned vortices by vortex segments in calculating the induced velocity. It is necessary to apply this rectification whenever any two vortices approach closer than their either side segment length.

5 Administration of the shed vortices

A systematic way of handling the shed vortices may be the use of arrays to store number of clusters and number of vortices in each cluster such as shown below.

$$\begin{array}{lll} n_c(j) & : & \text{number of clusters originating from the } j\text{-th side} \\ n_{vc}(k_j, j) & : & \text{number of vortices in the } (k_j, j) \text{ cluster} \\ & & \text{where } k_j = 1, 2, \ldots, n_c(j) \\ & & \qquad\quad j = 1, 2. \end{array}$$

The number of vortices shed from the j-th side and the total number of vortices are then calculated respectively from

$$m_t(j) = \sum_{k=1}^{n_c(j)} n_{vc}(k, j) \tag{11}$$

$$M_t = m_t(1) + m_t(2). \tag{12}$$

It is to be noted that enrolling the nascent vortices is acomplished by

$$[m_t(j)]_{\text{new}} = m_t(j) + 1 \tag{13}$$

$$[n_{vc}(n_c(j), j)]_{\text{new}} = n_{vc}(n_c(j), j) + 1 \tag{14}$$

$$z(m_t(j), j) = \zeta_j \tag{15}$$

$$\Gamma(m_t(j), j) = \gamma_j \tag{16}$$

where $j = 1, 2$

and whenever the cluster cutting is performed $n_c(j)$, $n_{vc}(n_c(j) - 1, j)$ and $n_{vc}(n_c(j), j)$ are to be renewed.

6 Stimulation of asymmetry

Imposing some artificial disturbance to trigger asymmetry of flow pattern is a necessity in the DVM simulation. Sudden shift of vortices shed from one side of the cylinder at a suitable time step works well and is employed in the present investigation, as shown below.

$$[z(k, 2)]_{\text{shifted}} = z(k, 2) + \xi, \qquad k = 1, 2, \ldots, m_{t_s}(2) \tag{17}$$

where t_s : shifting time step.

7 Force calculation

Although the force exterted to the cylinder can be calculated in principle by the pressure integration over the surface, it is more convenient to use the following force formulae[6] obtained from the extended Blasius theorem[7]

$$C_L + iC_D = \sum_{k=1}^{M_t} \Gamma_k(\overline{w}_k - \overline{w}_{Ik}) + \left(1 - \frac{1}{1+\delta}\right) \sum_{k=1}^{2} \frac{d\gamma_k}{dt} e^{-i\theta_k}. \tag{18}$$

The velocity of the image vortices and the growth rate of the nascent vortices appearing in this equation are dealt with in the following.

* velocity of image vortices

Since the position of an image vortex is specified by eq.(3), its velocity is expressed by the time derivative of that equation. Hence,

$$\begin{aligned} \overline{w}_{Ik} &= u_{Ik} - iv_{Ik} \\ &= -\frac{w_k}{z_k^2}. \end{aligned} \tag{19}$$

* growth rate of the nascent vortices

Since the strength of the nascent vortices are determined from eq.(6)

$$\begin{aligned} \frac{d\gamma_j}{dt} &= |u_{\theta j}|\,\Delta t\,\frac{du_{\theta j}}{dt} \\ &= |u_{\theta j}|\,\Delta t\,\Im\left\{\frac{i}{2\pi}z_{cj}\sum_{k=1}^{M_t}\Gamma_k\left[\frac{w_k}{(z_{cj}-z_k)^2}-\frac{w_{Ik}}{(z_{cj}-z_{Ik})^2}\right]\right\}. \end{aligned} \tag{20}$$

8 Calculated results and discussion

Success of vortex shedding simulation by the DVM depends unfortunately quite sensitively on the values of the discretization parameters. Among the four parameters Δt, δ, shifting time step t_s and shifting distance ξ, the best working values for Δt and δ are known from experiences to be 0.2 and 0.1[8] respectively. These values are used in the present simulation. As for the other two parameters concerned with stimulating asymmetry, it was found that the most suitable values were twelve and 1.0 respectively through a large number of numerical experiments. The vortex distribution patterns so obtained are shown in Fig.2 through Fig.5 sequentially in the time step interval 10. The patterns observed in the Karman vortex street are very well reproduced. The instability due to discretization has been successfully suppressed up to the present time step and can be done so a little further. Up until this time the foremost vortex cluster convects about twenty five radii. If the computation proceeds still further, the effect of instability creeps in rapidly and destroys the orderly vortex street structure.

The force coefficient curves obtained with the same values of the parameters are shown in Fig.6 and Fig.7. In accordance with the stipulation of the DVM that the measured force and the predicted one are to be directly compared in spite of the frictional drag, this being usually negligibly small compared with the pressure drag, included in the former, Sarpkaya's experimental measurements[9] are shown in Fig.8 and Fig.9 for comparison. Although no definitive statements can be formulated about the results as the amplitude of oscillation of the lift coefficient and the magnitude of the drag coefficient appear to be growing with progress of the time step, a few points may be noticed: firstly the magnitude of drag coefficient agrees with the measured values within the experimental accuracy, secondly the predicted period of lift coefficient oscillation agrees better with that of the measured curve for lower Reynolds number and lastly the lift coefficient is greatly over-predicted. This last point is one of the general trends of the DVM analysis. In addition, the fact that the force coefficients were more sensitive to the effect of the instability due to discretization than the vortex distribution pattern may be worth noting.

Other combination of values of the discretization parameters may work as well. The workability is to be assessed on both the simulated vortex distribution pattern and the force coefficient curves. No rational principle has been formulated for the choice of suitable values of the shifting time step and the shifting distance. A rough working guideline found through the experience of the present study is that for a smaller shifting time step, a smaller shifting distance is likely to match and, to reduce the effect of artificial disturbance, the smaller shifting distance is the more preferrable.

As for the value of Δt, compromise should be sought between the two facts that it is the fundamental parameter of the problem discretization and as such cannot become so small as to lose meaning of discrete treatment of the problem and that it is concerned with how detail the flow is to be represented. In this respect, the value 0.2 is thought to be a choice based on a little more weight to the first fact than the latter. A tentative comment on the value of δ is that it should be adjusted proportionally with the value of Δt. This is only a conjecture not systematically explored in the present study, but a possible topic for the future research.

Acknowledgements

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Figure 2: Vortex distribution at the time step $t = 100$

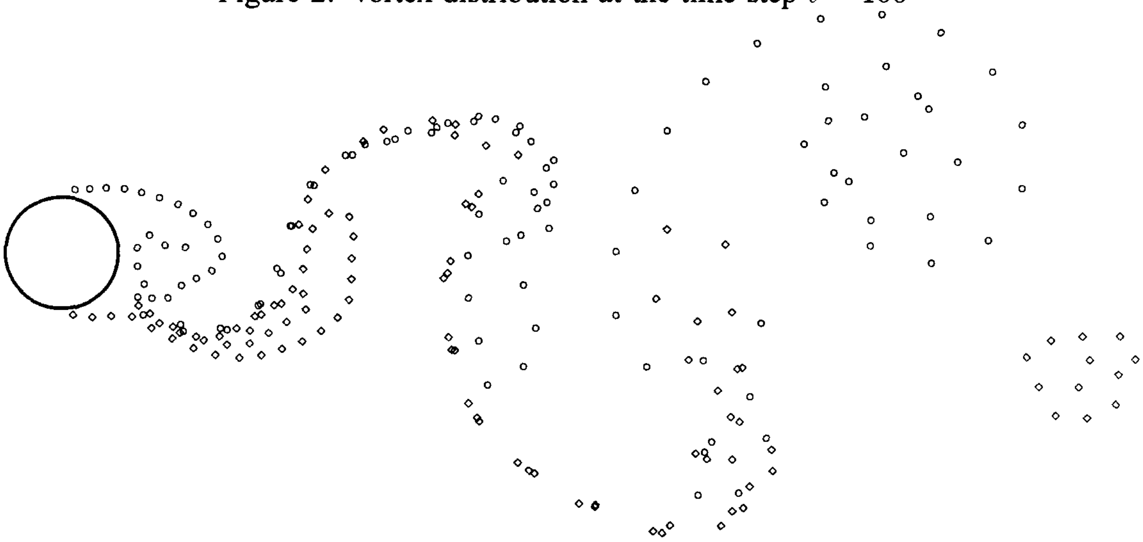

Figure 3: Vortex distribution at the time step $t = 110$

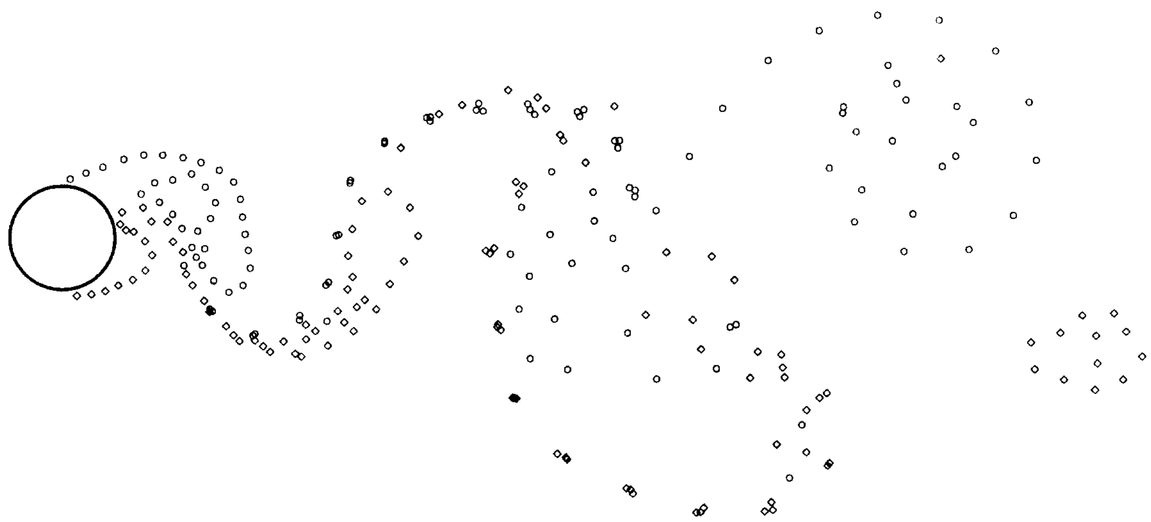

Figure 4: Vortex distribution at the time step $t = 120$

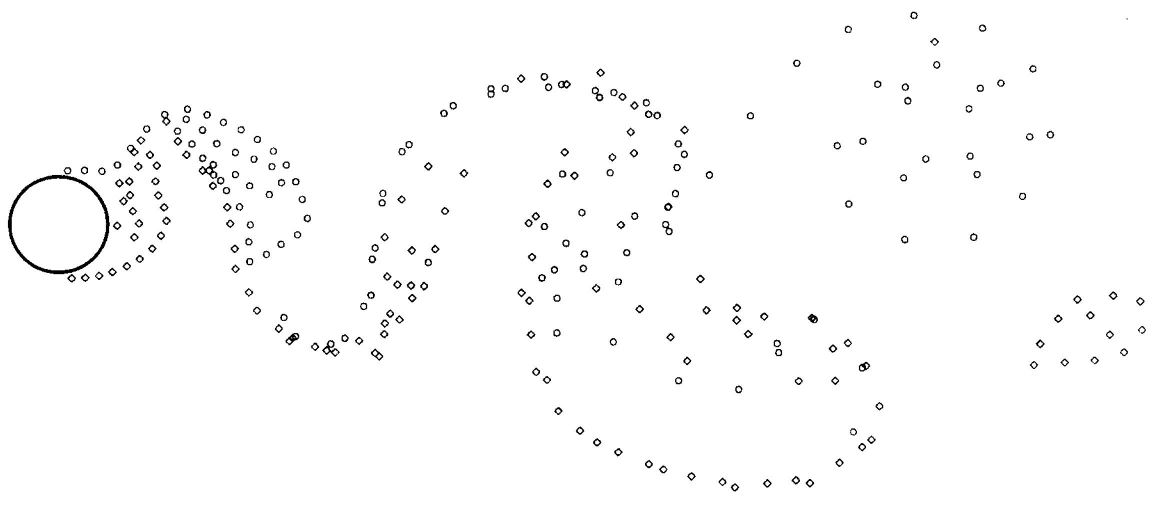

Figure 5: Vortex distribution at the time step $t = 130$

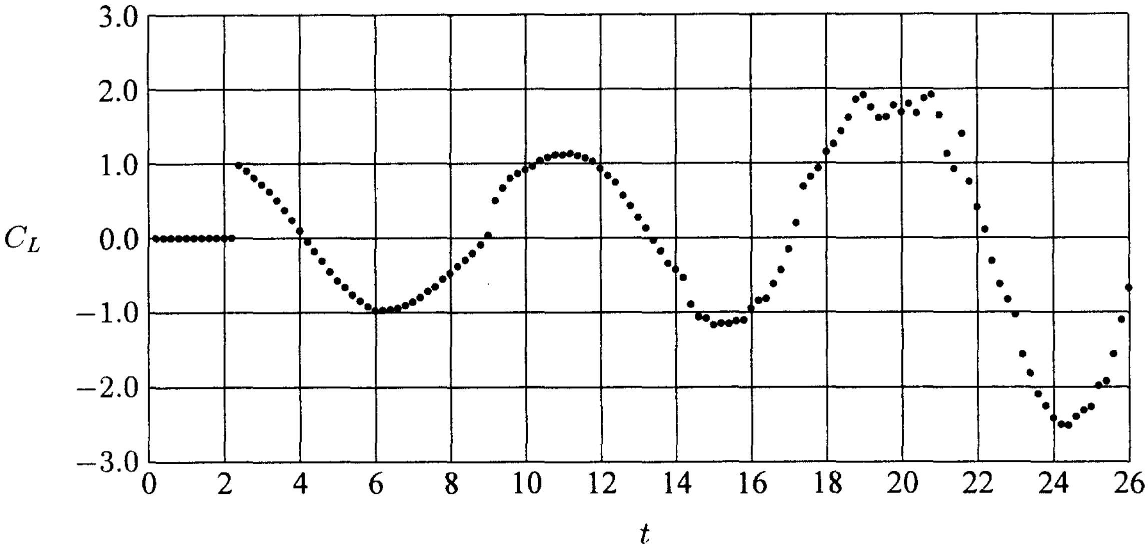

Figure 6: The lift coeficient curve

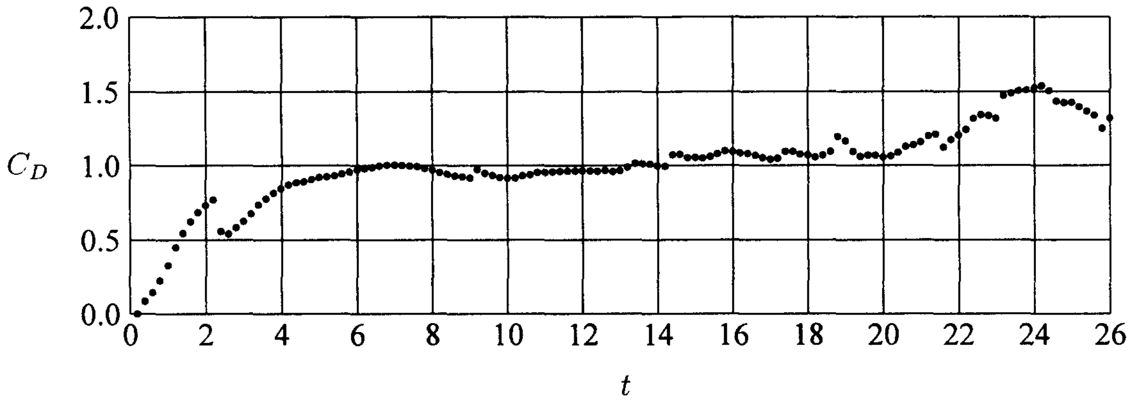

Figure 7: The drag coeficient curve

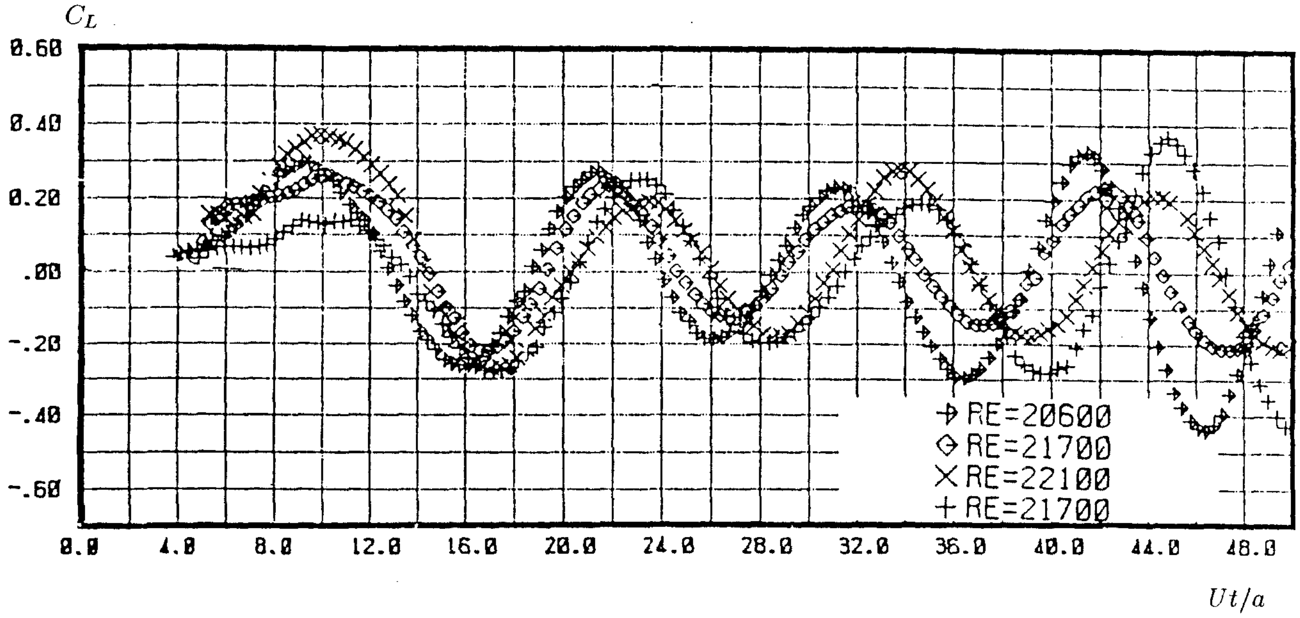

Figure 8: C_L vs Ut/a for Reynolds numbers 20600, 22100, and 21700

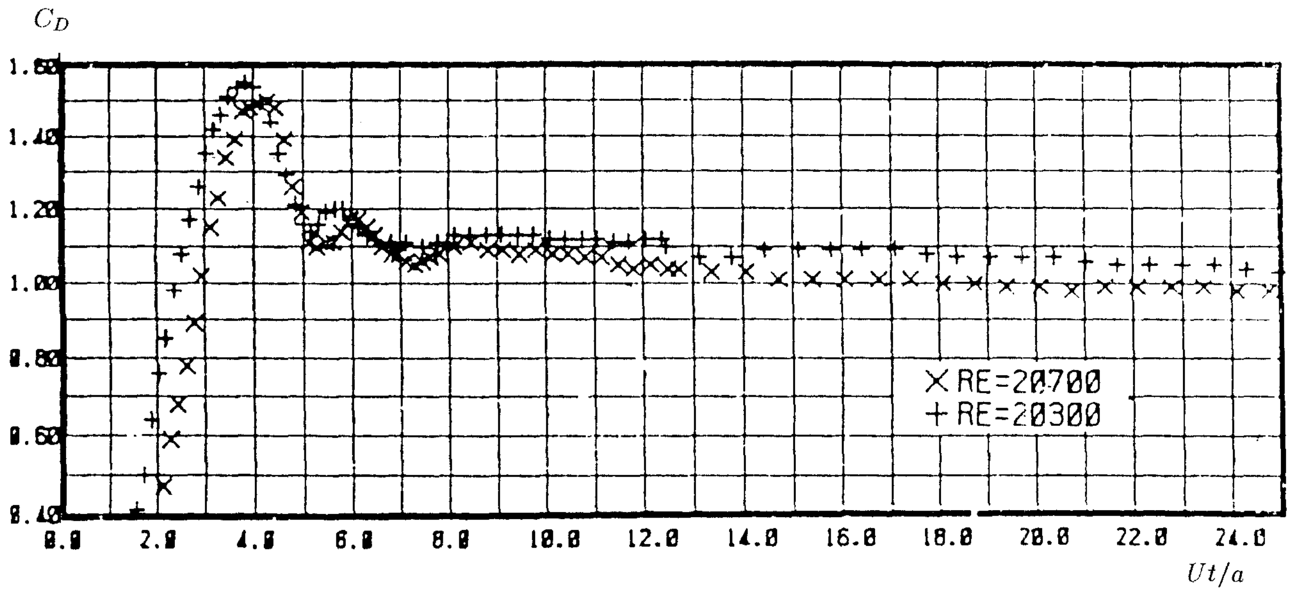

Figure 9: C_D vs Ut/a for Reynolds numbers 20300 and 20700