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임의 배열 안테나로 입사하는 협대역 코히어런트 신호의 분리를 위한 새로운 알고리즘

(A New Algorithm for Resolving Narrowband Coherent Signals Incident on a General Array)

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요 약

본 논문에서는 임의의 배열 형태를 갖는 안테나 어레이에 입사하는 협대역 신호의 도래각을 추정하기 위한 새로운 알고리즘 즉, SDVTA 알고리즘을 제안한다. 제안된 알고리즘에서는 가상적으로 변환된 스티어링 행렬을 구성하여 신호의 고유벡터 영역을 변환함으로써 협대역 신호들의 상관관계를 제거한다. 본 알고리즘의 특징으로 1) m-1개의 코히어런트 신호를 분리할 수 있으며 2) 어레이 형태에 제한을 받지 않으며 3) 상관관계를 제거하는 효과가 CSM기법과 달리 신호들의 위상차에 영향을 받지 않음을 들 수 있다.

제안된 알고리즘의 우수함을 입증하기 위해 MUSIC 알고리즘과 비교한 시뮬레이션 결과를 제시하며 코히어런트 신호환경에서 변환 행렬의 선택에 따른 성능을 비교, 분석한다.

Abstract

In this paper, we propose a new algorithm, so called the Signal Decorrelation via Virtual Translation of Array (SDVTA) algorithm, for estimating the directions of arrival(DOA's) of narrowband coherent signals incident on a general array. An effective procedure is composed of transforming the steering matrix of the original array into that of the virtually translated sensor array and taking the average of the transformed covariance matrices in order to decorrelate the coherent signals. The advantage of this approach is in that 1) it can estimate the DOA's of m-1 coherent signals(m : the number of array sensors) since the effective aperture size is never reduced. 2) a geometry of array is unrestricted for solving the narrowband coherency problem. 3) the efficiency of signal decorrelation does not depend on the phase differences between coherent signals unlike the Coherent Signal Subspace Method (CSM).

Simulation results are illustrated to demonstrate the superior performance of this new algorithm in comparison with the normal MUSIC and examine the comparative performance with the various choices of the optimal transformation matrix under coherent signal environments.

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I. INTRODUCTION

In recent years, extensive efforts have been given in developing superresolution algorithms for estimating the DOA's of multiple signals incident on a sensor array[1]. These algorithms, in general, make use of the specific eigen-structure properties of covariance matrix estimates and are known to have better performance than Capon's MLM[2] and Burg's MEM[3], and so on. However, the performances of these algorithms are severely degraded when some of the incident signals are highly correlated since the source covariance matrix becomes almost singular.

In order to circumvent the coherency problem, Schmidt proposed the generalized MUSIC algorithm[4] for resolving coherent signals by replacing the normal DOA search vector with the vector composed of the electrical parameters and multiple angular parameters, and Zoltowski presented the vector space approach[5] which reduces the computational load by involving only the angular parameters in combinational search process. However, the large computational burden due to combinational search in the algorithms above makes them impractical in real situations like Maximum Likelihood Estimators[6][7]. Shan and Williams, et al., proposed the Spatial Smoothing Techniques[8][9] to solve the coherency problem, but these techniques are applicable only to uniform or translational equivalent array with the effective aperture size reduced due to subarray averaging. As an alternative approach for solving the problem, the Coherent Signal Subspace Method(CSM)[10] which takes an average of the covariance matrix estimates in the frequency domain was proposed by Wang and Kaveh, but this algorithm is applicable only to wideband signals with bandwidths comparable to the center frequency of the signal.

In this paper, we propose a new algorithm, so called the SDVTA, which can estimate the DOA's of narrowband coherent signals incident on a general array. The basic idea of the algorithm proposed herein is similiar to that of the CSM. However, while the CSM in wideband processing combines the signal subspaces at different frequencies in one coherent signal subspace, SDVTA combines the signal subspaces of the virtually transformed steering matrices associated with that of the original array to decorrelate the coherent signals. Furthermore, the efficiency of signal decorrelation in the SDVTA does not depend on phase delays between signals unlike the CSM and can resolve m-1 coherent signals since it does not reduce the effective aperture size of the array.

This paper is organized as follows: In section II, signal model is formulated for coherent signal environments. Section III presents the brief review of the CSM along with a discussion on signal reconstruction. In section IV, the Signal Decorrelation via Virtual Translation of Array(SDVTA) algorithm is presented for estimating the DOA's of multiple narrowband signals incident on a general array. The fundamental problem of finding the optimal transformation matrix is addressed with the theoretical analysis for establishing the SDVTA. Simulation results are also illustrated in section V in order to demonstrate that the proposed algorithm provides a superior performance compared with that of the MUSIC.

II. PROBLEM FORMULATION

Let us consider a general array composed of m-identical sensors located at the points z_1,

z_2, ..., z_m, in real three dimensional space and receiving d (d<m) narrowband signals coming from directions $\{k_1, k_2, \ldots\ldots, k_d\}$. The vector k_j is a unit direction vector and a function of the azimuth angle(θ) and elevation angle(ψ) as specified by $k_j = [\sin\theta_j\cos\psi_j, \cos\theta_j \cos\psi_j, \sin\psi_j]'$. In this case, the received signal at the i-th sensor can be written as

$$x_i(t) = \sum_{j=1}^{d} s_j(t)e^{j(\omega_0 k_j \cdot z_i/c + \phi_j)} + \eta_i(t) \qquad (1)$$

where c is the propagation velocity of signals, ω_0 the center frequency, ϕ_j the random initial phase of the j-th signal, and $\eta_i(t)$ the additive Gaussian noise at the i-th sensor. In this expression, the symbol " ' " denotes the transpose operator. The outputs of the m -sensors can be represented by the vector notation as follows :

$$\boldsymbol{x(t)} = \boldsymbol{A}(k)\, \boldsymbol{s}(t) + \boldsymbol{\eta}(t), \qquad (2)$$

where x(t), s(t) and $\eta(t)$ are given by

$$\boldsymbol{x(t)} = [x_1(t), x_2(t), \ldots\ldots, x_m(t)]' \in C^m$$

$$\boldsymbol{s(t)} = [s_1(t)e^{j\phi_1}, s_2(t)e^{j\phi_2}, \ldots\ldots, s_d(t)e^{j\phi_d}]' \in C^d$$

$$\boldsymbol{\eta(t)} = [\eta_1(t), \eta_2(t), \ldots\ldots, \eta_m(t)]' \in C^m \qquad (3)$$

A(k) is the *m x d* steering matrix whose columns are steering vectors and written as

$$\boldsymbol{A}(k) = [a(k_1), a(k_2), \ldots\ldots, a(k_d)] \in C^{mxd}$$

$$a(k_j) = [e^{j\omega_0 k_j Z_1/c}, \ldots\ldots, e^{j\omega_0 k_j' Z_m/c}] \in C^m \qquad (4)$$

In the analysis to follow throughout the paper, it is assumed that the signals and additive noises are zero mean wide-sense stationary and ergodic complex-valued Gaussian random processes which are pairwisely uncorrelated. From our assumption, it follows that the covariance matrix R_x is given by

$$\begin{aligned} R_x &= E[x(t)x^*(t)] \\ &= A(k)R_s A^*(k) + \sigma^2 R_\eta \end{aligned} \qquad (5)$$

in which $R_s = E\{s(t)s^*(t)\}$, $\sigma^2 R_\eta = E\{\eta(t)\eta^*(t)\}$. In the expression above, the symbols "E" and "*" denote the expectation and complex conjugate transpose operator, respectively. It is assumed that the noise covariance matrix R_η is known but the noise variance σ^2 is unknown. If the incident signals are not perfectly correlated, the source covariance matrix R_s remains nonsingular and $A(k)R_sA^*(k)$ is of rank d. From the generalized eigen-characteristics of the matrix pencil (R_x, R_η), it is seen that the multiplicity of the smallest eigenvalue is *m-d* and the value is equal to noise variance σ^2. Also, the eigenvectors ν_j corresponding to these smallest eigenvalues are orthogonal to the columns of matrix *A(k)*, i.e.,

$$A^*(k)v_j = 0 \quad (d+1 < j \le m) \qquad (6)$$

The fundamental multiple source location problem can be solved using this formula. When the incident signals are coherent, however, Eq. (6) is no longer valid. Now, we briefly address the coherency problem which may arise in multipath environments. Let us assume that two signals are coherent, i.e., $s_2(t)e^{j\phi_2} = \alpha s_1(t)e^{j\phi_1}$, where α is a complex constant describing the amplitude and phase relationship between signals. In this case, s(t) and A(k) can be rewritten as

$$s(t) = [s_1(t)e^{j\phi_1}, s_3(t)e^{j\phi_3}, \ldots\ldots, s_d(t)e^{j\phi_d}]'$$

$$A(k) = [a(k_1) + \alpha a(k_2), a(k_3), \ldots\ldots, a(k_d)] \qquad (7)$$

From Eq. (7), we can see that the source covariance matrix R_s is of rank d-1 and the multiplicity of the smallest eigenvalue becomes m-d+1. Also, the eigenvectors corresponding to the smallest eigenvalues are orthogonal to the columns of matrix A(k). Since the first column of matrix A(k) is not an ordinary steering vector, we can not estimate the DOA's of coherent signals with a straightforward application of eigen-analysis based algorithms. More detailed discussion about the coherency problem can be found in [8] [9] [11]

Ⅲ. Coherent Signal Subspace Method

Wang and Kaveh first introduced the CSM in order to improve a resolution capability of the MUSIC [10], which was further developed by Hung and Kaveh [12]. In order to achieve a significant improvement in estimation performance, it is essential that we suitably combine the informations at all frequencies for which the SNR is reasonably large. With this in mind, it is now assumed that the estimates of the steering matrices $A(k, w_j)$ at frequencies w_j for j=0, 1, ..., J-1, have been seperately determined where J is the number of frequency bins. Since the ranges of the composite steering matrices (i.e., signal subspaces) are different for distinct frequencies, we can not simply take an average of the covariance matrix estimates to improve the angle estimation. Thus, this approach selects the transformation matrix $T(k,\omega_j)$ which satisfies the following set of equations,

$$A(k,\omega_0) = T(k,\omega_j)A(k,\omega_j), \quad \text{for } j=0,1,2,\ldots,J-1. \quad (8)$$

The transformation matrices are approximated by using the initial angle estimates on **k** and the knowledge of the steering matrix. A coherent signal subspace is constructed by taking an average of the covariance matrix estimates at different frequencies. From Eq. (8), we provide another interpretation in order to explain the motivation from which the SDVTA is derived in next section. Let us consider the array output vector at the j-th frequency bin which is expressed as

$$x(\omega_j) = A(k,\omega_j)s(\omega_j) + \eta(\omega_j) \quad \text{for } j=0,1,2,\ldots,J-1. \quad (9)$$

Under the assumption that $s(\omega_j)$ and $\eta(\omega_j)$ are an unknown deterministic vector and a normal random vector, respectively, the minimum variance unbiased estimate of $S(W_j)$ is the orthogonal projection of $s(\omega_j)$ onto the range (or column space) of $A(k,\omega_j)$ [13]. The estimate of $s(\omega_j)$ is given by

$$\begin{aligned}\hat{s}(\omega_j) &= \{A^*(k,\omega_j)A(k,\omega_j)\}^{-1}A(k,\omega_j)^*x(\omega_j)\\ &= A^{\#}(k,\omega_j)x(\omega_j)\end{aligned} \quad (10)$$

where $A^{\#}(k,\omega_j)$ is referred to as the pseudo-inverse of $A(k,\omega_j)$ which satisfies the four Moore-Penrose conditions [14]. The j-th transformed covariance matrix can be rewritten as

$$\begin{aligned}R_T(\omega_j) &= T(k,\omega_j)R_x(\omega_j)T^*(k,\omega_j)\\ &= A(k,\omega_0)A^{\#}(k,\omega_j)R_x(\omega_j)\{A^{\#}(k,\omega_j)\}^*A^*(k,\omega_0)\\ &= A(k,\omega_0)E[A^{\#}(k,\omega_j)x(\omega_j)\{A^{\#}(k,\omega_j)x(\omega_j)\}^*]\,A^*(k,\omega_0)\\ &= A(k,\omega_0)E[\,s(\omega_j)s^*(\omega_j)\,]\,A^*(k,\omega_0)\end{aligned} \quad (11)$$

It can be seen from Eq. (11) that the signal vector (or the covariance matrix) is reconstructed as the output of an another system corresponding to center frequency ω_0. This interpretation allows us to have the intuition that the covariance matrix measured in one sensor array can be transformed to the one measured in another virtually (or practically) translated array.

Ⅳ. Signal Decorrelation Via Virtual Translation Of Array (SDVTA)

By using the concept of the CSM and translating a sensor array virtually in the spatial domain, an effective procedure is herein developed for solving the narrowband coherency problem for a general array geometry. The idea is based on the fact that any steering matrix corresponding to one sensor array (or its subarray) can be transformed to another steering matrix associated with the virtually translated array by using the optimal transformation matrix in the least squares sense. Fig. 1 shows the translation of a sensor array to virtual positions by displacement vectors.

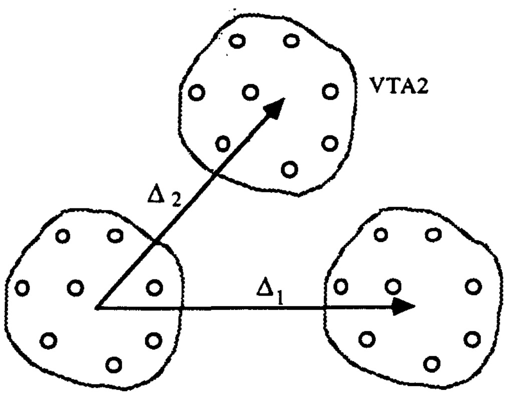


그림 1. 배열 안테나의 가상적인 이동
Fig. 1. Virtual translation of array geometry

1. Solution Procedure

Let $x_j(t)$ denote the virtual snapshot vector of the received signals at the j-th translated array by a displacement vector Δ_j. Using Eq. (2), $x_j(t)$ is given by

$$x_j(t) = A(k)D(k,\Delta_j)s(t)+\eta_j(t) \text{ for } j=0,1,2,\ldots\ldots,q-1 \quad (12)$$

where the d x d diagonal matrix $D(k,\Delta_j)$ is given by

$$D(k,\Delta_j) = diag[e^{j\omega_0 k_1 \Delta_j/c}, e^{j\omega_0 k_2' \Delta_j/c}, \ldots\ldots, e^{j\omega_0 k_d' \Delta_j/c}] \quad (13)$$

and (q-1) denotes the number of the virtually translated arrays. In this expression, $x_0(t)$ represents the original snapshot vector where Δ_0 is a three dimensional zero vector and Δ_j is chosen by a processor. Before examining the second-order statistics of the snapshot vector which provides a useful tool for estimating the DOA's, we first review the following lemma.

Lemma 1 : If the matrices A(k) and $D(k,\Delta_j)$ have full rank of d, then there exists the m x m matrix $T(k,\Delta_j)$ such that

$$A(k)D(k,\Delta_j) = T(k,\Delta_j)A(k) \text{ for } j=0,1,2,\ldots\ldots,q-1. \quad (14)$$

The simple proof is given in[10]. It should be noted that the range of A(k) is the same as that of $A(k)D(k,\Delta_j)$ since $D(k,\Delta_j)$ has full rank of d. Namely, the signal subspace corresponding to the original array is equal to that related to the virtually translated array while the signal subspaces are different for distinct frequencies in wideband array signal processing. A variety of approaches for selecting $T(k,\Delta_j)$ are given in later part of this section. Based on lemma 1, the j-th transformed covariance matrix is given by

$$E\{x_j(t)x_j^*(t)\} = T(k,\Delta_j)R_x T^*(k,\Delta_j), \quad (15)$$

in which the transformed noise covariance matrix is expressed as

$$E\{\eta_j(t)\eta_j^*(t)\} = \sigma^2 T(k,\Delta_j)R_\eta T(k,\Delta_j). \quad (16)$$

Upon taking an average of the transformed covariance matrices, the averaged covariance matrix can be written as

$$\begin{aligned}\overline{R}_x &= \frac{1}{q}\sum_{j=0}^{q-1} T(k,\Delta_j)R_x T^*(k,\Delta_j) \\ &= \frac{1}{q}\sum_{j=0}^{q-1} T(k,\Delta_j)A(k)R_s A^*(k)T^*(k,\Delta_j) \\ &\quad + \frac{\sigma^2}{q}\sum_{j=0}^{q-1} T(k,\Delta_j)R_\eta T^*(k,\Delta_j) \\ &= A(k)\left[\frac{1}{q}\sum_{j=0}^{q-1} D(k,\Delta_j)R_s D^*(k,\Delta_j)\right] A^*(k) \\ &\quad + \frac{\sigma^2}{q}\sum_{j=0}^{q-1} T(k,\Delta_j)R_\eta T^*(k,\Delta_j) \\ &= A(k)\ \overline{R}_s A^*(k) + \sigma^2\ \overline{R}_\eta \end{aligned} \quad (17)$$

where $\overline{R}_s$ and $\overline{R}_\eta$ are given by

$$\begin{aligned}\overline{R}_s &= \frac{1}{q}\sum_{j=0}^{q-1} D(k,\Delta_j)R_s D^*(k,\Delta_j) \\ \overline{R}_\eta &= \frac{1}{q}\sum_{j=0}^{q-1} T(k,\Delta_j)R_\eta T^*(k,\Delta_j)\end{aligned} \quad (18)$$

and represent the spatially smoothed source covariance matrix and noise covariance matrix, respectively. We shall now formulate that the averaged source covariance matrix is nonsingular without regard to the coherency of signals.

Theorem 2 : Let $D(k,\Delta)$ and $\overline{R}_s$ denote the d×d diagonal matrix and the source covariance matrix, respectively. If the number of the virtually translated arrays of q is greater than or equal to the number of the sources of d, then $\overline{R}_s$ is of full rank.

The proof is given in references[8][9]. From theorem 2, the effective aperture size is not reduced since the size of the averaged covariance matrix is the same as the number of original elements. That is, the SDVTA algorithm can resolve m-1 coherent signals incident on a general array geometry. We can also use the SDVTA in conjunction with the widely employed algorithms, such as MUSIC [4], SEM[15], Minimum-Norm[16], and so on, since Eq. (17) has exactly the same form as Eq. (5).

Up to this point in this section, we have hypothesized that k are the same as true angles. However, k must be estimated in practical situations resulting that the efficiency of signal decorrelation may depend on the preliminary angle estimates on k, the displacement vector, and the transformation matrices. Wang and Kaveh introduced the preliminary processing to obtain good estimates on k. In the analysis to follow, it is assumed that the preliminary estimates are in the neighborhood of the true angles. From our empirical results, it has been found that twice of a wavelength is a proper displacement in terms of the probability of resolution, bias, and standard deviation, and the performance of the SDVTA does not depend heavily on the initial angle estimates. To address the problem of selecting the optimum transformation matrix, fundamental least squares problem will be first examined.

2. Fundamental Least Squares Problem

The problem of transforming a given matrix into another matrix often arises in many applications such as system identification, direction-of-arrival estimation, and statistical analysis, and so forth. We shall now consider the least squares problem of transforming a given matrix B into a given matrix A by a transformation matrix T so that sum of the squares of the residual matrix E = A-TB be minimized. Mathematically, this problem can be expressed as follows

$$TB = A + E \tag{19}$$

$$\| E \|_F^2 = tr[E^* E] = \min \tag{20}$$

where the matrices A, B and E are m x n complex matrices, T is a m x m complex matrix, Frobenius norm of matrix, and tr [ ] is the trace of the matrix. Eq. (19) represents the model and Eq. (20) the criterion. Several approaches for solving the least squares problem are now examined.

(1) Nonunitary transformation matrix

If we define the matrix $B^{\#}$ as $B^{\#} = V\Sigma^{\#}U^*$, then

$$\min \| A - TB \|_F \Rightarrow T = AB^{\#} \tag{21}$$

where

$$\Sigma^{\#} = diag[\sigma_1^{-1}, \sigma_2^{-1}, \ldots, \sigma_r^{-1}, 0, 0, 0, \ldots, 0] \in R^{n \times m}$$

$$r = rank(B)$$

In this expression, the orthogonal matrices U and V correspond to the SVD of B

$$B = U\Sigma V^*$$

$$U = [\, \boldsymbol{u}_1, \boldsymbol{u}_2, \ldots, \boldsymbol{u}_m \,], \quad V = [\, \boldsymbol{v}_1, \boldsymbol{v}_2, \ldots, \boldsymbol{v}_n \,]$$

where

$$\Sigma = diag[\sigma_1, \sigma_2, \ldots, \sigma_p]$$

$$\sigma_1 \geq \sigma_2 \geq \ldots \geq \sigma_r > \sigma_{r+1} = \ldots = \sigma_p = 0$$

$$p = \min(m, n)$$

in which the σ_ks are real nonnegative singular values which are ordered in monotonically nonincreasing fashion $\sigma_k > \sigma_{k+1}$ and $\boldsymbol{u_k}$ and $\boldsymbol{v_k}$ are corresponding orthonormal m x 1 left and n x 1 right singular vectors, respectively. If rank(B)=m=n, then T is given by

$$T = AB^{-1} \tag{22}$$

(2) Unitary transformation matrix

A solution to the least squares problem addressed in section A is well known and employed in many applications. We shall herein consider the problem as that of selecting a unitary matrix T which minimizes tr[E*E] with T preserving the Frobenius norm of any matrix. In other words, the following constraint is given

$$T^*T = TT^* = I_{mxm} \tag{23}$$

in which I_{mxm} is m x m identity matrix. The solution to the constrained minimization problem is formulated in theorem 3.

Theorem 3 : Let A and B denote two given matrices contained in the vector space C^{mxn} of m x n complex matrices that have the same rank. Then the optimal unitary transformation matrix T which minimizes $\|A-TB\|_F$ subject to $T^*T = TT^* = I_{m\times m}$, is given by

$$T = VU^* \tag{24}$$

where the orthogonal matrices U and V correspond to the SVD of BA*, that is,

$$BA^* = U\Sigma V^*. \tag{25}$$

The proof is given in references[12][17]. The transformation matrixT is not unique when BA* is a rank deficient matrix[17]. Based on theorem 3, the following observation can be made

- o The mapping T:R(B)---->R(A) represents the closest rotation of the range of B into that of A with a possible preceeding reflection in some hyperplane.
- o If A is equal to B, then T is the identity matrix.
- o If B is equal to the identity matrix, then this problem is established as the orthogonalization of A..

It is to be noted in this optimization problem that the normalized error $\|A_{TB}\|_F / \|A-B\|_F$ may be relatively large even though R(A) matches R(TB) perfectly. This case results from the fact that the unitary transformation matrix T provides the invariant norm, that is

$$\|TB\|_F = \|B\|_F$$

3. Optimal Transformation Matrices for SDVTA

As previously discussed, it is apparent that the unitary and nonunitary transformation matrix can be optimally selected in the least squares senses. We shall now examine whether or not this solution is efficiently applicable to the DOA estimation problem. Since the choice of the transformation matrix plays an important role in establishing the SDVTA algorithm effectively, we will focus on how to select T(k,Δ) which enables us to improve the performance of the SDVTA algorithm.

Least Squares Problem : Given a composite steering matrix A(k) and A(k)D(k, Δ_j), develop a procedure for selecting $T(k,\Delta_j)$ which minimizes $\|A(k)D(k,\Delta_j) - T(k,\Delta_j)A(k)\|_F$, for j=0,1,2,.......,q-1. Several approaches for solving the problem have been discussed in section B. An effective application is made to entail in some manner the ability to solve the multiple coherency problem.

Method I : Least Squares Method

$$T(\beta,\Delta_j)=A(\beta)D(\beta,\Delta_j)A(\beta)^{\#} \text{ for } j=0,1,2,\ldots\ldots,q-1 \qquad (26)$$

Method II : Rotation of Signal Subspace Method (ROSS)[12][17]

$$T(\beta,\Delta_j)=VU^{*} \qquad (27)$$

where

$$A(\beta)D^{*}(\beta,\Delta_j)A(\beta)*=U\Sigma V^{*}$$

$$T^{*}(\beta,\Delta_j)T(\beta,\Delta_j)=I_{mxm}$$

Method III : Dummy Direction Vector Constrained Method (DDVC)

$$T(\beta,\gamma,\Delta_j)=[\ A(\beta),A(\gamma)\begin{bmatrix} D(\beta,\Delta_j),0 \\ 0,D(\gamma,\Delta_j)\end{bmatrix}[A(\beta),A(\gamma)]^{-1}$$

where β and γ are the sets of preliminary estimated vector and arbitrarily selected dummy vectors, respectively. Finally, the SDVTA is described as follows.

(1) Choose the initial angle estimates β (and γ).

(2) Select the displacement vector Δ_j for j = 0, 1, ..., q-1.

(3) Estimate $T(k,\Delta_j)$ by using Method I, II, III.

(4) Estimate $\overline{R}_x$ and $\overline{R}_\eta$ given in (17) and (18).

(5) Make a generalized eigenanalysis of matrix pencil ($\overline{R}_x$, $\overline{R}_\eta$).

(6) Apply the existing algorithms (e.g. MUSIC, SEM, MN, and so on).

(7) Estimate DOA's.

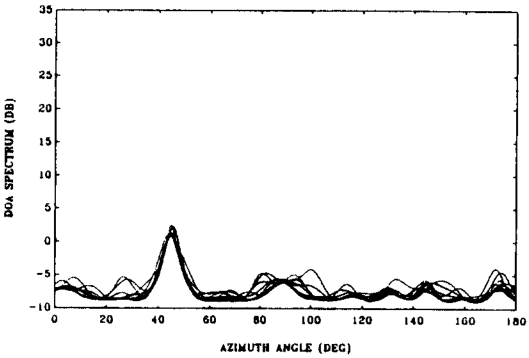
V. Simulation Results

Two examples are considered to test the effectiveness of the proposed SDVTA algorithm for a general array. The first example is presented to illustrate the performance of the proposed method relative to the normal MUSIC and investigate the statistical performance with the dispalcement vector and initial angles changed. The second example shows the comparison of the performances with the various choices of the transformation matrix. The array considered is composed of eight sensors in the (X-Y) plane at locations

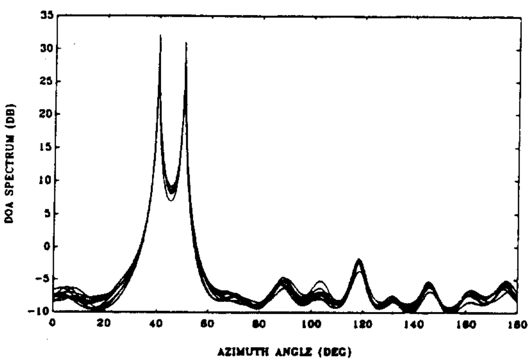
$$z_1=[0.0,2.0]',z_2=[1.3,1.3]',z_3=[1.9,0.0]',$$
$$z_4=[1.5,-1.5]',z_5=[0.0,-1.8]'z_6=[-1.7,-1.7]',$$
$$z_7=[-1.5,0.0]',z_8=[-1.6,1.6]'$$

where a sensor spacing unit is taken to be λ_0 which is the wavelength of the signal. The sensor noise vector $\eta(t)$ is taken as a complex valued additive white Gaussian random process whose components have identical variances and are statistically independent of the signals. The data length is taken to be 500 samples. In the simulation examples, we shall apply MUSIC to the SDVTA algorithm.

Example 1 : Two narrowband coherent plane waves were taken to have the azimuth angles of $\theta_1=40^0$ and $\theta_2=50^0(\phi=0^0)$. The sensor noise variance is selected so that two signal-to-noise ratio(SNR) levels of 10 dB are obtained. One virtually translated array is taken with the displacement vector $[2\lambda_0,0]'$ in order to resolve two coherent signals. In employing the SDVTA, a set of initial angles is taken to be $\{37.0^o,43.0^o,44.0^o,45.0^o,46.0^o,47.0^o,53.0^o,54.0^o\}$ for approximating the Dummy Direction Vector Constrained transformation matrix. It is to be noted that these initial angles are chosen to be apart from the true angles more than 3^o. The initial angles related to the dummy angles are determined by adding the multiples of 1BW/m to the preliminary angle estimates in real situations.



(a)



(b)

그림 2. 8개 센서로 구성된 임의배열 안테나를 이용한 두 코히어런트신호의 도래각 추정 (SNR=10dB)
(a) MUSIC (b) SDVTA-MUSIC

Fig. 2. Ten statistically independent superimposed angle estimates for two coherent narrowband plane waves at azimuth angles of 40° and 50°. These estimates were obtained using a general array which is composed of 8 elements at SNR = 10 dB.
(a) Normal MUSIC
(b) SDVTA-MUSIC in conjunction with the DDVC

One beamwidth(1BW) is chosen as λ_o/D in which D represents the aperture size of the array. Fig. 2 shows ten statistically independent superimposed estimates to illustrate the high resolution performance achieved with the SDVTA method relative to that obtained with the normal MUSIC. It is evident from Fig. 2 that the normal MUSIC fails to detect the two incident plane waves in any of the ten trial runs. Although not shown, this approach is unable to resolve two perfectly coherent sources in a higher SNR setting since the source covariance matrix is singular. On the other hand, it is seen that the SDVTA-MUSIC algorithm is able to consistently resolve two coherent signals.

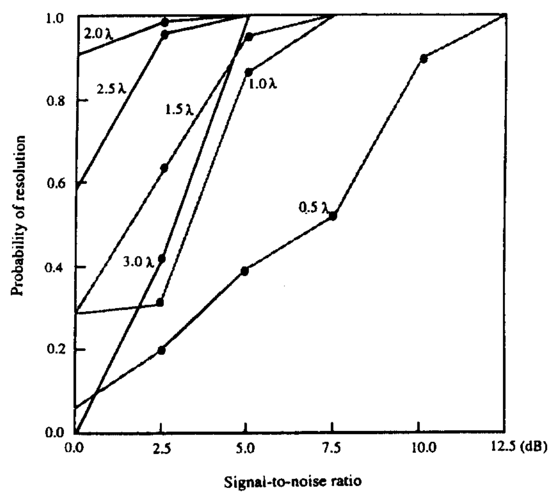


그림 3. 변위벡터 Δ_j에 따른 SDVTA-MUSIC의 분해능에 대한 확률

Fig. 3. Probability of resolution of the SDVTA-MUSIC with the various choices of array displacement vector. (Δ = $[\, l\lambda_o,\ 0\,]'$ for l = 0.5, 1.0, 1.5, 2.0, 2.5, 3.0).

To examine the sensitivity of angle estimates to displacement vector and initial angles, fifty independent generations of covariance matrix estimates are made, and Table I and Table II show the statistical performances in terms of the sampled bias, sampled standard deviation(STD), and root mean squared error(RMS). It can be seen from Table I that displacement of $2\lambda_o$ provides the best performance. It is also shown in Table II that the statistical performance of the SDVTA-MUSIC algorithm is not much sensitive to initial angle estimates, which makes it attractive in real situations. To test the resolution capability of the proposed method with the array displacement vector changed, the probability of resolution is herein defined as the probability that one source is estimated in the interval $[40^o-3.0^o, 40^o+3.0^o]$ and simultaneously the second source in the interval $[50^o-3.0^o, 50^o+3.0^o]$. The probability of resolution can not be considered theoretically reliable in the absolute sense, however, it is quite appropriate for the purpose of examination of the sensitivity of the method in resolving two plane waves. Fig. 3 shows the probability of resolution as a function of SNR for $\theta_1=40^o$ and $\theta_2=50^o$. It is apparent that the highest resolution capability is achieved in the case of displacement $2\lambda_o$ under the given condition.

Example 2 : In this example, we shall investigate the comparative performance of

표 1. 변위벡터에 따른 $\theta_1=40^0$에서의 통계적 성능

Table 1. The statistical performance at $\theta_1=40^0$ with various choices of the array displacement vector.

Displacement △	SNR=5dB			SNR=10dB			SNR=15dB		
	Bias	STD	RMS	Bias	STD	RMS	Bias	STD	RMS
0.5λ	* * *[1)]	* * *	* * *	* * *	* * *	* * *	$-9.3110\times^{-3}$	1.00×10^{-1}	1.01×10^{-1}
1.0λ	* * *	* * *	* * *	-1.78×10^{-2}	9.32×10^{-2}	9.49×10^{-2}	-7.99×10^{-3}	4.87×10^{-2}	4.93×10^{-2}
1.5λ	* * *	* * *	* * *	-1.65×10^{-2}	8.18×10^{-2}	8.34×10^{-2}	-4.29×10^{-3}	4.49×10^{-2}	4.51×10^{-2}
2.0λ	8.15×10^{-3}	1.75×10^{-1}	1.75×10^{-1}	2.79×10^{-3}	9.02×10^{-2}	9.03×10^{-2}	3.08×10^{-3}	5.02×10^{-2}	5.03×10^{-2}
2.5λ	4.35×10^{-2}	1.94×10^{-1}	1.99×10^{-1}	1.52×10^{-2}	1.02×10^{-1}	1.04×10^{-1}	7.40×10^{-3}	5.68×10^{-2}	5.73×10^{-2}
3.0λ	7.56×10^{-2}	1.90×10^{-1}	2.05×10^{-1}	2.84×10^{-2}	1.01×10^{-1}	1.05×10^{-1}	1.30×10^{-2}	5.63×10^{-2}	5.78×10^{-2}

1) '* * *' denotes that SDVTA-MUSIC does not resolve two signals at least one time out of 50 independent trials.

표 2. 초기값에 따른 $\theta_1=40^0$에서의 통계적 성능

Table 2. The statistical performance at $\theta_1=40^0$ with various choices of the nearest initial angles.

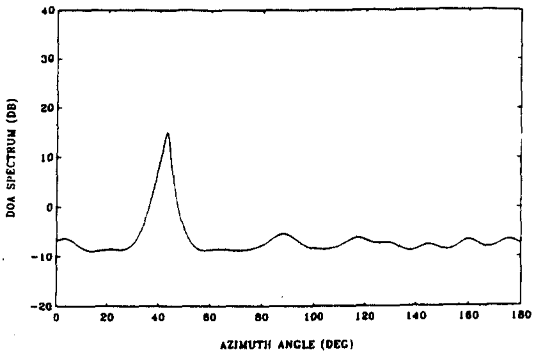
Cases SAD	SNR=2.5dB			SNR=5dB			SNR=7.5dB		
	Bias	STD	RMS	Bias	STD	RMS	Bias	STD	RMS
Case 1 6°	3.00×10^{-1}	3.15×10^{-1}	4.35×10^{-1}	2.43×10^{-1}	2.33×10^{-1}	3.37×10^{-1}	2.24×10^{-1}	1.86×10^{-1}	2.91×10^{-1}
Case 2 4°	1.36×10^{-1}	2.88×10^{-1}	3.18×10^{-1}	9.18×10^{-2}	2.06×10^{-1}	2.25×10^{-1}	7.93×10^{-2}	1.56×10^{-1}	1.75×10^{-1}
Case 3 2°	6.55×10^{-2}	2.73×10^{-1}	2.81×10^{-1}	2.91×10^{-2}	1.90×10^{-1}	1.92×10^{-1}	2.21×10^{-2}	1.41×10^{-1}	1.42×10^{-1}
Case 4 0°	4.25×10^{-2}	2.64×10^{-1}	2.68×10^{-1}	1.00×10^{-2}	1.82×10^{-1}	1.82×10^{-1}	2.93×10^{-3}	1.30×10^{-1}	1.30×10^{-1}

2) Initial angles for SDVTA-MUSIC

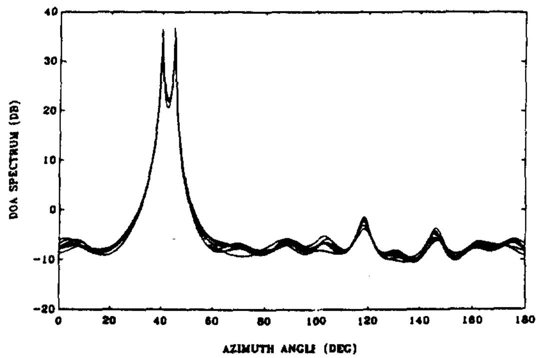
Case 1={28^0, 30^0, 32^0, 34^0, 56^0, 58^0, 60^0, 62^0}, Case 2={28^0, 30^0, 32^0, 36^0, 54^0, 58^0, 60^0, 62^0}

Case 3={280, 30^0, 32^0, 38^0, 52^0, 58^0, 60^0, 62^0}, Case 4={28^0, 30^0, 32^0, 40^0, 50^0, 58^0, 60^0, 62^0}

the Method II(ROSS) and III(DDVC) as examined in section IV-C in terms of the sensitivity to the initial angles.



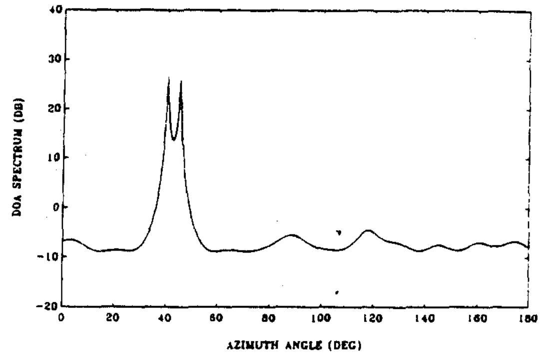
(a)



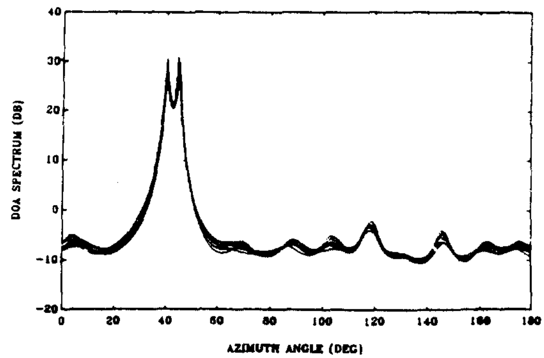
(b)

그림 4. 40° 와 45° 로 입사하는 두 협대역 코히어런트신호의 도래각 추정 (SNR =20dB) (a) SDVTA-MUSIC-ROSS (b) SDVTA-MUSIC-DDVC

Fig. 4. Ten statistically independent superimposed angle estimates for two coherent narrowband plane waves at azimuth angles of 40° and 45° . These estimates were obtained using a general array which is composed of 8 elements at SNR = 20 dB. (a) SDVTA-MUSIC in conjunction with the ROSS approach (a set of initial angles = {37.5° , 42.5° , 47.5° })
(b) SDVTA-MUSIC in conjunction with the DDVC approach (a set of initial angles = {35° , 37.5° , 42.5° , 47.5° , 50° , 52o, 53o, 54° })



(a)



(b)

그림 5. 40° 와 45° 로 입사하는 두협대역 코이어런트신호의 도래각 추정(SNR=15dB)
(a) SDVTA-MUSIC-ROSS
(b) SDVTA-MUSIC-DDVC

Fig. 5. Ten statistically independent superimposed angle estimates for two coherent narrowband plane waves at azimuth angles of 40° and 45° . These estimates were obtained using a general array which is composed of 8 elements at SNR = 15 dB.
(a) SDVTA-MUSIC in conjunction with the ROSS approach(a set of initial angles={39.3°, 42.5°, 45.7°})
(b) SDVTA-MUSIC in conjunction with the DDVC approach(a set of initial angles={35°, 37°, 39.3°,42.5°, 45.7°, 48°, 50°, 52°})

Two coherent plane waves are assumed to

have the azimuth angles of 40° and 45°. The number of samples and the displacement vector are the same as those given in example 1. Ten independent trials are made and the resultant plots obtained with the Methods II and III are shown in Fig. 4 at SNR 20 dB and in Fig. 5 at SNR 15 dB. In Fig. 4, a set of initial angles is taken to be {37.5°,42.5°,47.5°} for Method II and {35°,37.5°,42.5°,47.5°,50°,52°,53°, 54°} for Method III.

It is to be noted that the smallest angle difference between true angles and initial angles(SAD) is taken to be 2.5°. It is clear in Fig. 4 that Method III is able to resolve consistently two perfectly correlated signals while Method II fails to resolve them. In Fig. 5, Methods II and III chose the initial angles as {39.3°,42.5°,45.7°} and {35°,37°,39.3°,42.5°,45.7°,48°,50°,52°}, respectively. In this test, the SAD is taken to be 0.7°. It is seen that Method II provides the better performance than Method III. From Figs. 4 and 5, a truly significant increase in angle estimation performance is achieved in the DDVC approach to the SDVTA algorithm in terms of a less sensitivity to the initial angle estimates. On the other hand, the ROSS approach provides a more challenging test of resolution capability as the SNR is decreased under the condition that SAD is relatively small.

VI. CONCLUSION

The SDVTA algorithm has been presented for estimating the DOA's of multiple narrowband coherent signals incident on a general array. The fundamental concept is based on the virtual translation of the array for decorrelating the coherent signals. The proposed algorithm transforms the steering matrix corresponding to the original array to those of the virtually translated arrays in a fashion of multiple replica and takes an average of the transformed covariance matrix estimates. This approach is able to resolve m-1 coherent sources since the effective aperture size does not reduce.

Through the several examples, it has been found that the SDVTA algorithm provides a superior performance relative to the normal MUSIC. It has been also shown that the performance of the SDVTA algorithm in conjunction with the DDVC transformation matrix depends heavily on the array displacement vector but is relatively insensitive to initial angle estimates.

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