論文95-32B-7-1

투영제어 기법을 이용한 제어기의 저차수화 설계

(Reduced-order Controller Design using Projective Controls)

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요 약

본 논문에서는 저차수화된 제어기가 상태궤환 기준 제어시스템의 동특성 모드를 간직하도록 고안된 기존의 투영제어(Projective Controls) 기법을 확장하여 일반적인 출력궤환 기준 제어시스템의 동특성 모드까지도 간직할 수 있도록 하였다. 이 확장된 투영제어 기법에 의해 고차 기준제어기의 폐루프 특성을 근사화 하는 저차수화 된 제어기 설계가 가능하다. 특히 구현 가능한 최상의 기준 제어시스템이 출력궤환 제어 시스템인 경우 본 논문에서 제안된 기법이 유용하며, 기존의 상태궤환 기준 제어시스템 방식보다 더 많은 설계 자유도를 가지게 되어 더 낮은 차수의 제어기 설계가 가능함을 예제를 통하여 보여준다.

Abstract

In this paper the projective controls, previously derived to preserve the dynamic modes of a state-feedback reference system, are extended to allow the preservation of the modes of a general output-feedback reference system. In general, the extension allows projective controls to be used as a controller approximation technique, where a reduced-order controller is designed to approximate the closed-loop behavior of the higher-order reference controller. This extension is useful if the best available reference control for the system is an output-feedback control. An example shows that the increased design freedom of proposed design method allows the stabilization of a given plant using a lower-order controller than the projective controls with state-feedback reference.

I. Introduction

The projective controls method provides a systematic procedure for designing static and low-order dynamic output-feedback controllers for linear time-invariant multivariable systems.[1-3] The projective controls approach involves the approximation of a reference control system. The reference control law may be determined by standard multi-input/multi-output(MIMO) feedback synthesis methods, and should meet all the control system specifications, but may be difficult or impossible to implement.

The objective of the projective control is to match the reference control when the plant state vector lies in a chosen invariant sub-

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接受日字:1994年12月26日, 수정완료일:1995年7月13日

space of the reference system. This subspace then becomes an invariant subspace of the projective control system as well. For initial conditions in this preserved invariant subspace, and in the absence of disturbances, the trajectories of the projective system and the reference system will be identical. For a static projective controller, the dimension of the invariant subspace preserved(i.e., the number of reference eigenvalues and eigenvectors retained by the projective control system) is equal to the number of measurements available for feedback. For a dynamic projective controller, the dimension of the preserved invariant subspace is increased by the order of the controller.

The projective controls approach is a systematic design methodology, where the order of the dynamic controller is gradually increased, until enough reference modes are retained and enough design freedom is available that the design is able to satisfy the control requirements. The order of satisfactory projective controllers are often much lower than can be achieved by reduction of full order controllers. (See, for example, [4].) The approach has also been generalized to apply to discrete-time systems.[5] A recent practical application of projective controls appears in [6].

The projective controls method has always assumed the use of a state-feedback reference system. In some cases, this may present a limitation of the approach. A state-feedback control does not account for the plant output structure or for any measurement disturbances or uncertainties. An output-feedback control, on the other hand, may account for the output structure and measurement disturbances, thereby guaranteeing optimum performance and robustness. Such properties may be essential to the final control system implementation, but are usually achieved by introducing an observer or some other high-order control structure. Therefore, it may be advantageous to design a low-order projective control system to approximate the behavior of a high-order output-feedback reference system.

This paper develops an extension of the projective controls approach to allow the use of a general output-feedback reference. The derivation is parallel and the results are quite similar to those for projective controls with a state-feedback reference. The extension allows the projective controls approach to be interpreted as a controller reduction technique. The low-order projective controller is designed to approximate the closed-loop behavior of the higher-order reference controller. Since the projective controller preserves the dynamic properties of the closed-loop reference system, the approximation method directly accounts for the interaction between the controller and the plant. The extension also allows increased freedom in the selection of the reference modes to be retained by the reduced-order closed-loop system. By use of this additional freedom, the projective controls may stabilize a given system with a lower-order controller than could be found using a state-feedback reference. An example problem is given to show the advantage of the new approach.

II. Projective Controls with Output-feedback Reference

Given a linear time-invariant system described by

$$\dot{x} = Ax + Bu \tag{2.1a}$$

$$y = Cx, \tag{2.1b}$$

where $x \in \mathbf{R}^n$, $u \in \mathbf{R}^m$, and $y \in \mathbf{R}^r$, consider the lth-order strictly proper reference controller

$$\dot{x}_c = A_{ref}x_c + B_{ref}y \ , \qquad (2.2a)$$

$$u = C_{ref}x_c \ , \qquad (2.2b)$$

where $x_c \in \boldsymbol{R}^l$. Then the reference closed-loop system is

$$\begin{pmatrix} \dot{x} \\ \dot{x}_c \end{pmatrix} = \begin{pmatrix} A & BC_{ref} \\ B_{ref}C & A_{ref} \end{pmatrix} \begin{pmatrix} x \\ x_c \end{pmatrix} . \qquad (2.3)$$

Defining

$$F = \begin{pmatrix} A & BC_{ref} \\ B_{ref}C & A_{ref} \end{pmatrix}, \qquad (2.4)$$

the invariant subspaces of the system (2.3) are described by

$$FX = X\Lambda, \qquad (2.5)$$

where X is the matrix of generalized eigenvectors and Λ is a block-diagonal matrix of eigenvalues in Jordan form. In case of complex-conjugate eigenvalues, it is convenient to use a 2×2 real block in Λ, and a corresponding pair of real vectors in X.[7,8] Projective controls will be designed such that the closed-loop system will preserve blocks of eigenvalues and corresponding eigenvectors of the reference system. That is, if X and Λ are decomposed as

$$X = [\ X_1 \ \ X_2\ ]\ ,\ \Lambda = \begin{pmatrix} \Lambda_1 & 0 \\ 0 & \Lambda_2 \end{pmatrix}\ , \qquad (2.6)$$

then the projective controls will preserve the eigenpairs (Λ_1, X_1).

1. Derivation of static projective controls

The static projective control system is to share r eigenpairs of the reference system (2.3), where r is the number of elements in the measurement vector y. Define the reference eigenpairs to be retained by

$$\begin{pmatrix} A & BC_{ref} \\ B_{ref}C & A_{ref} \end{pmatrix} \begin{pmatrix} X_r \\ W_r \end{pmatrix} = \begin{pmatrix} X_r \\ W_r \end{pmatrix} \Lambda_r, \qquad (2.7)$$

where $X_r \in \boldsymbol{R}^{n\times r}$, $W_r \in \boldsymbol{R}^{l\times r}$, and $\Lambda_r \in \boldsymbol{R}^{r\times r}$. The static projective control has the form

$$u = K_o y = K_o Cx\ , \qquad (2.8)$$

which gives the closed-loop system

$$\dot{x} = (A + BK_oC)x. \qquad (2.9)$$

For the projective control system to preserve the eigenpairs (2.7), we require

$$(A + BK_oC)X_r = X_r\Lambda_r\ , \qquad (2.10)$$

while (2.7) yields

$$AX_r + BC_{ref}W_r = X_r\Lambda_r\ . \qquad (2.11)$$

Comparing equations (2.10) and (2.11) yields

$$BK_oCX_r = BC_{ref}W_r\ . \qquad (2.12)$$

Assuming CXr is invertible, the relation (2.12) is satisfied by the feedback gain

$$K_o = C_{ref}W_r(CX_r)^{-1} \qquad (2.13)$$

The invertibility assumption requires first that the output y have the same number of elements as the set of eigenpairs to be retained, and second that each retained eigenvector correspond to an observable mode of the plant. While these two conditions are only necessary, and not sufficient, experience shows that CX_r is practically always invertible. In case it is not, then it can be made invertible by a different choice of the reference dynamics.

To relate this result to the projective controls with a state-feedback reference, consider the special case where the reference control is based on an observer. Then (2.2) becomes

$$\dot{x}_c = (A+BK_s-LC)x_c + Ly \ , \qquad (2.14a)$$

$$u = K_s x_c, \qquad (2.14b)$$

where L is the observer gain matrix, and K_s is a state-feedback gain matrix. Then the closed-loop system dynamics are described by the matrix

$$F = \begin{pmatrix} A & BK_s \\ LC & A+BK_s-LC \end{pmatrix} \qquad (2.15)$$

and the eigenvalues of the closed-loop system will be

$$\Lambda(F) = \Lambda(A+BK_s) \cup \Lambda(A-LC) \ , \qquad (2.16)$$

where $\Lambda(\cdot)$ denotes the spectrum of a matrix. Defining the reference eigenpairs by

$$\begin{pmatrix} A & BK_s \\ LC & A+BK_s-LC \end{pmatrix}\begin{pmatrix} X_r \\ W_{rf} \end{pmatrix} = \begin{pmatrix} X_r \\ W_{rf} \end{pmatrix} \Lambda r \ , \qquad (2.17)$$

where now $W_{rf} \in R^{n\times r}$, the static projective control (2.13) becomes

$$K_o = K_s W_{rf}(CX_r)^{-1}. \qquad (2.18)$$

If, further, the retained eigenpairs (Λr,Xr) all correspond to the state-feedback spectrum $\Lambda(A+BK_s)$, then it turns out that the reference eigenvectors satisfy the relation

$$X_r = W_{rf}. \qquad (2.19)$$

Then the static projective control gain (2.18) becomes

$$K_o = K_s X_r (CX_r)^{-1} \qquad (2.20)$$

which is exactly the same as for the projective controls with state-feedback reference. Therefore, the projective control with output-feedback reference is seen as a more general version of the projective control with state-feedback reference.

2. Derivation of dynamic projective controls

Dynamic projective controls with output-feedback reference are now derived for the cases of both proper and strictly proper controllers. A pth-order proper controller preserves p reference eigenpairs in addition to the r already preserved by using static feedback of the r available measurements. By contrast, the pth-order strictly proper controller preserves only p reference eigenpairs, independent of the number of measurements.

(1) Proper controllers

Let the $r+p$ reference eigenpairs to be retained be given by

$$\begin{pmatrix} A & BC_{ref} \\ B_{ref}C & A_{ref} \end{pmatrix}\begin{pmatrix} X_r & X_p \\ W_{rf} & W_{pf} \end{pmatrix} = \begin{pmatrix} X_r & X_p \\ W_{rf} & W_{pf} \end{pmatrix}\begin{pmatrix} \Lambda_r & 0 \\ 0 & \Lambda_p \end{pmatrix}, \qquad (2.21)$$

which yields

$$AX_r + BC_{ref}W_{rf} + X_r\Lambda_r, \qquad (2.22a)$$

$$AX_p + BC_{ref}W_{pf} = X_p\Lambda_p, \qquad (2.22b)$$

where $X_r \in R^{n\times r}$, $X_p \in R^{n\times p}$, $W_{rf} \in R^{l\times r}$, $W_{pf} \in R^{l\times p}$, $\Lambda_r \in R^{r\times r}$, and $\Lambda_p \in R^{p\times p}$. Then the pth-order proper dynamic controller has the form

$$\dot{\xi} = A_c\xi + B_c y \ , \qquad (2.23a)$$

$$u = C_c\xi + D_c y \ , \qquad (2.23b)$$

which, with the plant (2.1), yields the closed-loop system

$$\begin{pmatrix} \dot{x} \\ \dot{\xi} \end{pmatrix} = \begin{pmatrix} A+BD_cC & BC_c \\ B_cC & A_c \end{pmatrix}\begin{pmatrix} x \\ \xi \end{pmatrix}. \qquad (2.24)$$

The closed-loop system is to retain the sets of eigenpairs (Λr,Xr) and (Λp,Xp), such that

$$\begin{pmatrix} A+BD_cC & BC_c \\ B_cC & A_c \end{pmatrix}\begin{pmatrix} X_r & X_p \\ W_r & W_p \end{pmatrix} = \begin{pmatrix} X_r & X_p \\ W_r & W_p \end{pmatrix}\begin{pmatrix} \Lambda_r & 0 \\ 0 & \Lambda_p \end{pmatrix}, \qquad (2.25)$$

where $W_r \in R^{p\times r}$, and $W_p \in R^{p\times p}$ are arbitrary matrices. Then from (2.25) we get

$$AX_r + BD_cCX_r + BC_cW_r = X_r\Lambda_r \ , \qquad (2.26a)$$

$$AX_p + BD_cCX_p + BC_cW_p + X_p\Lambda_p \, , \tag{2.26b}$$

$$B_cCX_r + A_cW_r = W_r\Lambda_r \, , \tag{2.26c}$$

$$B_cCX_p + A_cW_p = W_p\Lambda_p \, , \tag{2.26d}$$

Comparing equations (2.22a-b) and (2.26a-b) yields

$$D_cCX_r + C_cW_r = C_{ref}W_{rf} \, , \tag{2.27a}$$

$$D_cCX_p + C_cW_p = C_{ref}W_{pf} \, . \tag{2.27b}$$

Using (2.26c-d) and (2.27a-b) we can get an expression for the controller parameters as

$$\begin{pmatrix} D_C & C_c \\ B_c & A_c \end{pmatrix} = \begin{pmatrix} C_{ref}W_{rf} & C_{ref}W_{pf} \\ W_r\Lambda_r & W_p\Lambda_p \end{pmatrix} \begin{pmatrix} CX_r & CX_p \\ W_r & W_p \end{pmatrix}^{-1} . \tag{2.28}$$

Now, the controller parameters are determined, except that some freedom remains to choose W_r and W_p. Part of this freedom consists of the choice of coordinates for the controller, and therefore does not affect the control law. To reduce the remaining design freedom to a minimal number of parameters, introduce the definitions of V_r, V_p, Y_r, and Y_p as

$$\begin{pmatrix} CX_r & CX_p \\ W_r & W_p \end{pmatrix}^{-1} = \begin{pmatrix} V_r & V_p \\ Y_r & Y_p \end{pmatrix} . \tag{2.29}$$

Then (2.28) becomes

$$\begin{pmatrix} D_C & C_c \\ B_c & A_c \end{pmatrix} = \begin{pmatrix} C_{ref}W_{rf}V_r + C_{ref}W_{pf}Y_r & C_{ref}W_{rf}V_p + C_{ref}W_{pf}Y_p \\ W_r\Lambda_rV_r + W_p\Lambda_pY_r & W_r\Lambda_rV_p + W_p\Lambda_pY_p \end{pmatrix} . \tag{2.30}$$

We can eliminate V_r and V_p using $V_r = (CX_r)^{-1}(I_r = CX_pY_r)$ and $V_p = -(CX_r)^{-1}CX_pY_p$. Then, by changing the controller representation to an equivalent representation by using the fact

$$(AY_p, B, CY_p, D) \Leftrightarrow (Y_pA, Y_pB, C, D), \tag{2.31}$$

and substituting $Y_pW_p = I_p - Y_rCX_p$ and $Y_r = -Y_pW_r(CX_r)^{-1}$, we obtain the expressions for controller parameters in terms of fixed quantities and the free parameter matrix

$$L_p = Y_pW_r \, , \tag{2.32}$$

where $L_p \in \mathbf{R}^{p \times r}$.
The expressions are

$$A_c = \Lambda_p + L_pN_o \, , \tag{2.33}$$

$$B_c = (L_p\Lambda_r - \Lambda_pL_p - L_pN_oL_p)(CX_r)^{-1} \, , \tag{2.34}$$

$$C_c = K_p - K_rG_o \, , \tag{2.35}$$

$$D_c = (K_r - C_cL_p)(CX_r)^{-1} \, , \tag{2.36}$$

where

$$G_o = (CX_r)^{-1}CX_p, \; N_o = G_o\Lambda_p = \Lambda_rG_o, \; K_p = C_{ref}W_{pf}, \text{ and } K_rC_{ref}W_{rf} \tag{2.37}$$

The free parameter L_p may be chosen to position the remaining eigenvalues (residual spectrum) in the open left-half plane, and to minimize a chosen performance criterion for the closed-loop system.

If the reference controller is based on an observer, as (2.14), then the only change in the formulas is that the reference controller output matrix C_{ref} is replaced by the state-feedback gain matrix K_s. If only eigenvalues within $\Lambda(A+BK_s)$ are retained, then the formulas for projective controls with state-feedback reference are recovered as (2.33-36) with the revised definitions

$$G_o = (CX_r)^{-1}CX_p, \; No = G_o\Lambda_p - \Lambda_rG_o, \; K_p = K_sX_p, \text{ and } K_r = K_sX_r. \tag{2.38}$$

(2) Strictly proper controllers

Next, consider the design of a pth-order strictly proper projective controller.

$$\dot{\xi} = A_c\xi + B_cy \, , \tag{2.39a}$$

$$u = C_c\xi \, . \tag{2.39b}$$

The closed-loop system is given by

$$\begin{pmatrix} \dot{x} \\ \dot{\xi} \end{pmatrix} = \begin{pmatrix} A & BC_c \\ B_cC & A_c \end{pmatrix} \begin{pmatrix} x \\ \xi \end{pmatrix} . \tag{2.40}$$

The p reference eigenpairs to be retained are defined by

$$\begin{pmatrix} A & BC_{ref} \\ B_{ref}C & A_{ref} \end{pmatrix}\begin{pmatrix} X_p \\ W_{pf} \end{pmatrix} = \begin{pmatrix} X_p \\ W_{pf} \end{pmatrix}\Lambda p, \tag{2.41}$$

which yields

$$AX_p+BC_{ref}W_{pf}=X_p\Lambda_p, \tag{2.42}$$

where $X_p\in \boldsymbol{R}^{n\times p}$, $W_{pf}\in \boldsymbol{R}^{l\times p}$, and $\Lambda_p\in \boldsymbol{R}^{p\times p}$. The corresponding invariant subspace of the projective control system is described by

$$\begin{pmatrix} A & BC_c \\ B_cC & A_c \end{pmatrix}\begin{pmatrix} X_p \\ W_p \end{pmatrix} = \begin{pmatrix} X_p \\ W_p \end{pmatrix}\Lambda p, \tag{2.43}$$

which gives

$$AX_p+BC_cW_p=X_p\Lambda_p, \tag{2.44a}$$

$$A_cW_p+B_cCX_pW_p\Lambda_p, \tag{2.44b}$$

where $W_p\in \boldsymbol{R}^{p\times p}$ is an arbitrary matrix. Comparison of (2.42) and (2.44a) yields

$$C_cW_p=C_{ref}W_{pf}\ , \tag{2.45}$$

and hence the controller output matrix

$$C_c=C_{ref}W_{pf}W_p^{-1}. \tag{2.46}$$

The controller dynamics matrix is determined by postmultiplying (2.44b) by W_p^{-1} to obtain

$$A_c=W_p(\Lambda_p-W_p^{-1}B_cCX_p)W_p^{-1}. \tag{2.47}$$

The controller input matrix B_c is a free parameter in the design. A change of coordinates and the introduction of the free parameter matrix $L_p\in \boldsymbol{R}^{p\times r}$ yields

$$A_c=\Lambda_p-L_pCX_p, \tag{2.48}$$

$$B_c=L_p, \tag{2.49}$$

$$C_c=C_{ref}W_{pf}. \tag{2.50}$$

Again, for an observer-based reference control, Cref is replaced by K_s.

3. Making an initial choice of the free parameter

The completion of any dynamic projective control design will require the determination of the free parameter matrix L_p. This will normally be done by numerical optimization of a selected performance index, such as a norm of a given closed-loop transfer-function matrix. The H_2 norm and Frobenius-Hankel (FH) norm are often chosen, since either may be readily optimized using gradient methods. At each step of the optimization, including the first, the value of the free parameter must result in a stable closed-loop system. Therefore, some discussion is in order here concerning the initial choice of L_p for the optimization.

For the proper projective controller (2.33-36), the initial choice $L_p=0$ gives

$$A_c=\Lambda_p, \tag{2.51}$$

$$B_c=0, \tag{2.52}$$

$$C_c=C_{ref}W_{pf}-C_{ref}W_{rf}(CX_r)^{-1}CX_p, \tag{2.53}$$

$$D_c=C_{ref}W_{rf}(CX_r)^{-1}. \tag{2.54}$$

Then the proper projective controller reduces to the static projective feedback (2.13). If the static projective control is stabilizing, then the initial choice $L_p=0$ is admissible. If it is not, then finding a stabilizing initial choice is equivalent to an auxiliary output-feedback pole-placement problem,[8] which may be difficult to solve. Therefore, the existence of a stabilizing static projective control is valuable for finding a low-order dynamic projective controller.

An output-feedback reference control has more eigenvalues from which to choose than has a state-feedback reference control. Therefore, a greater number of static projective controls may be computed using an output-feedback reference. This increases the chances that one of them will be a stabilizing control, and will, if necessary, provide an easy starting point for computing a dynamic projective controller of the lowest possible order.

III. Example Problem

To illustrate the use of projective controls with an output-feedback reference, consider a system characterized by

$$A = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ -3947.80 & -5.50 & -17.90 & 5.50 & 10 \\ 0 & -1 & 0 & 1 & 0 \\ 0 & 0.55 & -1.79 & -0.55 & -1 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix},\ B = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix},$$

$$C = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0.55 & -1.79 & -0.55 & -1 \end{pmatrix}. \qquad (3.1)$$

This system represents a two-degree-of-freedom quarter-car model with a slow actuator used for active suspension control design in[9]. The open-loop poles are given by

$$\Lambda(A) = \{-2.75 \pm j62.89,\ -0.27 \pm j1.31,\ 0.00\}. \qquad (3.2)$$

A state-feedback control with gains

$$K_s = [\,18.23\ \ -0.005\ \ 21.23\ \ 15.25\ \ -6.37\,] \qquad (3.3)$$

yields a stable closed-loop system with the spectrum

$$\Lambda(A+BK_s) = -2.76 \pm j62.89,\ -1.56 \pm j2.48,\ -3.77. \qquad (3.4)$$

Since there are two measurements, two eigenvalues may be retained using static projective controls. Therefore, using a state-feedback reference, two different static projective controls are possible, one to retain each of the two complex-conjugate pairs of eigenvalues. Neither of these controls stabilizes the closed-loop system. It is also possible to compute a static projective control to preserve the single real reference eigenvalue by using either one of the two measurements. Such controls also fail to stabilize the system.

To continue with the design using the state-feedback reference, we would have to consider dynamic projective controls. Instead, we now proceed to use static projective controls with an output-feedback reference. By using an observer-based controller with the given state-feedback gain K_s (3.3) and the observer gain,

$$L = \begin{pmatrix} -0.02 & -0.23 \\ -0.44 & 51.97 \\ 0.62 & -0.25 \\ -0.22 & 0.02 \\ -0.28 & 0.22 \end{pmatrix} \qquad (3.5)$$

the spectrum of the new reference system is

$$\Lambda(F) = \{-2.76 \pm j62.89,\ -1.56 \pm j2.48,\ -3.77,\ -16.92 \pm j65.07,\ -0.47 \pm j1.04,\ -0.67\}, \qquad (3.6)$$

where

$$F = \begin{pmatrix} A & BK_s \\ LC & A+BK_s-LC \end{pmatrix} \qquad (3.7)$$

Now, in addition to the state-feedback eigenvalues, the reference spectrum contains the observer eigenvalues. Five different static projective controls are now possible, one corresponding to each complex-conjugate pair, and one corresponding to the two real eigenvalues. The spectrums of the closed-loop systems for the five possible choices are given in Table 1. The first two rows of the table correspond to the projective controls that retain the complex state-feedback eigenvalues. The controls are identical to those computed using the state-feedback reference, and do not stabilize the system. The next two rows of the table correspond to the controls that retain the complex observer eigenvalues, while the last row corresponds to that which retains one real state-feedback eigenvalue and one real observer eigenvalue. A stabilizing output-feedback gain,

$$K_o = [0.67 \quad 0.91], \qquad (3.8)$$

results from choosing to preserve a pair of observer eigenvalues.

This example has shown that the use of an output-feedback reference for projective controls can have advantages over the use of a state-feedback reference. A stabilizing static feedback has been found for the system, where none had been found previously.

Table 1. Spectrums of closed-loop systems

Selected Modes (2 eigenvalues)	Feedback Gains	Closed-loop Spectrum (5 eigenvalues)	Stability Property
$-2.76 \pm j62.89$	$[-5.45 \quad -0.01]$	$-2.76 \pm j62.89$, $-0.91 \pm j1.83$, 1.30	Unstable
$-1.56 \pm j2.48$	$[-18.78 \quad 0.46]$	$-1.56 \pm j2.48$, $-2.78 \pm j62.87$, 2.17	Unstable
$-16.92 \pm j65.07$	$[-11839 \quad 9.00]$	$-16.92 \pm j65.07$, $-4.95 \pm j18.37$, 28.58	Unstable
$-0.47 \pm j1.04$	$[0.67 \quad 0.91]$	$-0.47 \pm j1.04$, $-2.75 \pm j62.85$, -0.52	Stable*
-3.77, -0.67	$[-0.42 \quad 3.73]$	-3.77, -0.67, $-2.75 \pm j62.73$, 0.17	Unstable

* The only case that yields a stable closed-loop system

IV. Conclusion

An extension of the projective controls approach to allow the use of an output-feedback reference system is derived. The extension applies to both strictly proper and proper projective controllers. The original projective controls are recovered when an observer-based reference is used and only state-feedback modes are retained. An example shows that the proposed approach not only satisfies all the properties of the projective controls with state-feedback reference but also provides more freedom in the selection of the modes to be retained in the reduced-order closed-loop system. In addition, the approach makes it easier to find a dynamic projective controller if necessary to provide satisfactory robust performance of the system.

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