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Bayesian Inference for the Two-Parameter Exponential Models : Type-II Censored Case †

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ABSTRACT

Suppose that we have $k(k \geq 2)$ populations(or systems), say $\pi_1, \ldots, \pi_k$, to be tested. Under the type-II censored testing without replacement we consider the problem of estimating the unknown parameters of interests and the reliability for a given time t for each population. Also we compare the performances of the proposed Bayes estimators with another estimators under the Jeffrey-type noninformative prior distribution.

KEYWORDS: Type-II censoring, Reliability, Bayes estimation, Noninformative prior, Conjugate prior, Two-parameter exponential model.

1. INTRODUCTION

In the reliability, the exponential distribution is one of the most frequently used distributions(Barlow and Proschan(1965)). Hence there is a huge body

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of literatures concerned with exponential models in the life-time and reliability analysis. Also many works have been done under the Bayesian approach since the middle of 1970. Among such studies, the cases that the failure time X follows one or two-parameter exponential distribution for a single population were discussed by Sinha and Guttman(1976) under the Bayesian setting. The classical statistical estimation of the reliability function was considered by Kurkjian and Karson(1987). Also for the type-II censored case of two-parameter exponential distribution, Kambo(1978) studied the maximum likelihood estimation of the location parameter, scale parameter and the reliability function. For the type-II doubly censored samples from k exponential distributions with one-parameter, Shetty and Joshia(1987) derived classical estimators of the reliability function.

In this paper, we consider $k(\geq 2)$ independent two-parameter exponential populations consisting of $(k-1)$ treatment groups and one control group. A random sample of size n from each population is taken and the test for each system will be terminated when the first r failures among n random samples are observed. That is, the data are taken under the type-II censored(or item-censored) testing without replacement. Under this scheme we consider the problem of estimating the unknown paramters of interests and comparing the efficiency of proposed estimator with other estimators. Also we study the problem of selecting the better treatment population than the control group. We consider these problems under the Bayesian framework because of utilization of the prior knowledge. Noninformative prior and conjugate prior are considered for the prior distribution of unknown parameter and the squared error loss is used.

2. SOME PROPOSED ESTIMATORS

Suppose that given the reliability model, a life time X^i follows a two-parameter exponential distribution, denoted by $\mathcal{E}(\mu_i, \sigma)$, with a location parameter μ_i, $i = 1, \ldots, k$, respectively, a common scale parameter σ, and probability density function(pdf), $f(\cdot|\mu_i, \sigma)$, given by

$$f(x^i|\mu_i, \sigma) = \frac{1}{\sigma}\exp\Big[-\frac{x^i - \mu_i}{\sigma}\Big], \quad i = 1, \ldots, k, \tag{2.1}$$

where the k-th population is the control group, first $k-1$ populations are the treatment groups and $0 < \mu_i \le x^i$, $0 < \sigma < \infty$. Note that the μ_i is regarded as the guaranteed-life time of the system i. In this model the parameters of interest are location parameters μ_i, $i = 1, \ldots, k$, the common scale parameter σ and the reliability function R_t^i given by

$$\begin{aligned} R_t^i &= \Pr(X^i \ge t) = \int_t^\infty f(x|\mu_i, \sigma)dx \\ &= \exp\Big[-\frac{t - \mu_i}{\sigma}\Big], \quad t \ge \mu_i. \end{aligned} \tag{2.2}$$

A random sample of size n from each system is subjected to test and the test for each system is terminated when the first $r(\le n)$ items fail. Let X_1^i, X_2^i, ..., X_r^i be the failure times and let $X_{(1)}^i \le X_{(2)}^i \le \cdots \le X_{(r)}^i$ be ordered failure times from the system i, $i = 1, \ldots, k$, respectively. Then the likelihood function of $\underline{\mu} = (\mu_1, \mu_2, \ldots, \mu_k)$ and σ given $\underline{x} = \{X_{(1)}^i = x_{(1)}^i, \ldots, X_{(r)}^i = x_{(r)}^i\}$ is

$$\begin{aligned} L(\underline{\mu}, \sigma|\underline{x}) &= \prod_{i=1}^k \left(\exp\Big[-\frac{x_{(r)}^i - \mu_i}{\sigma}\Big]\right)^{n-r} \prod_{j=1}^r \frac{1}{\sigma}\exp\Big[-\frac{x_{(j)}^i - \mu_i}{\sigma}\Big] \\ &= \prod_{i=1}^k \exp\Big[-r\log\sigma - \frac{1}{\sigma}\big(S_r(i) + n(x_{(1)}^i - \mu_i)\big)\Big] \\ &= \exp\left\{-kr\log\sigma - \frac{1}{\sigma}\left(\sum_{i=1}^k S_r(i) + n\sum_{i=1}^k (x_{(1)}^i - \mu_i)\right)\right\}, \end{aligned} \tag{2.3}$$

where $S_r(i) = \sum_{j=1}^r (x_{(j)}^i - x_{(1)}^i) + (n-r)(x_{(r)}^i - x_{(1)}^i)$. Note that $x_{(1)}^i$, $i = 1, \ldots, k$ and $\sum_{i=1}^k S_r(i)/kr$ are maximum likelihood estimators(MLE) of μ_i, $i = 1, \ldots, k$ and σ, respectively.

For the prior distribution, the Jeffrey-type noninformative prior given by

$$\pi(\underline{\mu}, \sigma) \propto \sigma^{-a}, \quad a \ge 0 \tag{2.4}$$

is considered. Then the joint posterior distribution of μ_i, $i = 1, \ldots, k$ and σ given $\underline{x} = \{X^i_{(1)} = x^i_{(1)}, \ldots, X^i_{(r)} = x^i_{(r)},\ i = 1, \ldots, k\}$ is

$$\begin{aligned} \pi(\underline{\mu}, \sigma|\underline{x}) &\propto L(\underline{\mu}, \sigma|\underline{x})\pi(\underline{\mu}, \sigma) \qquad (2.5) \\ &\propto \exp\left\{-(kr+a)\log\sigma - \frac{1}{\sigma}\left(\sum_{i=1}^{k} S_r(i) + n\sum_{i=1}^{k}(x^i_{(1)} - \mu_i)\right)\right\}, \\ & 0 < \mu_i \le x^i_{(1)}, \qquad i = 1, \ldots, k, \qquad 0 < \sigma < \infty, \end{aligned}$$

and the joint marginal posterior distribution of μ_i, $i = 1, \ldots, k$ is

$$\pi(\underline{\mu}|\underline{x}) \propto \Gamma(kr+a-1)S_r^{-(kr+a-1)}\left[1 + \frac{\sum_{i=1}^{k}(x^i_{(1)} - \mu_i)}{S_r/n}\right]^{-(kr+a-1)}, \qquad (2.6)$$

where $x^i_{(1)}$, $i = 1, \ldots, k$ and $\sum_{i=1}^{k} S_r(i)/(kr+a) \equiv S_r/(kr+a)$ are generalized maximum likelihood estimators(GMLE) of μ_i, $i = 1, \ldots, k$ and σ, respectively. Note that the GMLE is the Bayesian version of the MLE with the posterior density.

Remark. One can see that the GMLE's are the same as the MLE's when $a = 0$.

Also the marginal posterior densities of μ_i, $i = 1, \ldots, k$ and σ can be obtained and are given by

$$\begin{aligned} \pi(\mu_i|\underline{x}) \propto & \left(1 + \frac{n(x^i_{(1)} - \mu_i)}{S_r}\right)^{-(kr+a-k)} \qquad (2.7) \\ & - \sum_{j=1, j' \ne i}^{k}\left(1 + \frac{n(x^i_{(1)} - \mu_i)}{S_r} + \frac{nx^j_{(1)}}{S_r}\right)^{-(kr+a-k)} \\ & + \sum_{j,j'=1, j\ne j', j\ne i}^{k}\left(1 + \frac{n(x^i_{(1)} - \mu_i)}{S_r} + \frac{n(x^j_{(1)} + x^{j'}_{(1)})}{S_r}\right)^{-(kr+a-k)} \\ & - \sum_{j,j',j''=1, j\ne j'\ne j'', j\ne i}^{k}\left(1 + \frac{n(x^i_{(1)} - \mu_i)}{S_r} + \frac{n(x^i_{(1)} + x^{j'}_{(1)} + x^{j''}_{(1)})}{S_r}\right)^{-(kr+a-k)} \end{aligned}$$

$$\vdots$$

$$+ (-1)^{k+1}\left(1+\frac{n(x^i_{(1)}-\mu_i)}{S_r}+\frac{n\sum_{j=1,j\neq i}^k x^j_{(1)}}{S_r}\right)^{-(kr+a-k)},$$

$$0<\mu_i\le x^i_{(1)}, \qquad i=1,\ldots,k,$$

and

$$\pi(\sigma|\underline{x}) \propto \sigma^{-(kr+a-k)}\exp\Big[-\frac{S_r}{\sigma}\Big]\prod_{i=1}^k\Big(1-\exp\Big[-\frac{n}{\sigma}x^i_{(1)}\Big]\Big), \qquad 0<\sigma<\infty. \quad (2.8)$$

In particular, for $k=2$, the joint posterior distribution is given by

$$\pi(\mu_1,\mu_2,\sigma|\underline{x}) \propto \sigma^{-(2r+a)}\exp\Big[-\frac{1}{\sigma}(S_r+n(x^1_{(1)}-\mu_1)+n(x^2_{(1)}-\mu_2))\Big], \quad (2.9)$$

where

$$S_r=S_r(1)+S_r(2), \text{ and } S_r(i)=\sum_{j=1}^r(x^i_{(j)}-x^i_{(1)})+(n-r)(x^i_{(r)}-x^i_{(1)})$$

with normalizing constant $N_{\underline{\mu}}=\dfrac{n^2}{\Gamma(2r+a-3)}S_r^{2r+a-3}N_0$, where

$$N_i=\Big[1-\Big(1+\frac{nx^1_{(1)}}{S_r}\Big)^{-2r-a+3+i}-\Big(1+\frac{nx^2_{(1)}}{S_r}\Big)^{-2r-a+3+i}$$
$$+\Big(1+\frac{nx^1_{(1)}}{S_r}+\frac{nx^2_{(1)}}{S_r}\Big)^{-2r-a+3+i}\Big]^{-1}.$$

The joint marginal posterior distribution of $\underline{\mu}=(\mu_1,\mu_2)$ is given by

$$\pi(\underline{\mu}|\underline{x}) \propto \Big[1+\frac{n}{S_r}\Big((x^1_{(1)}-\mu_1)+(x^2_{(1)}-\mu_2)\Big)\Big]^{-(2r+a-1)} \quad (2.10)$$

with normalizing constant $K_\mu=n^2(2r+a-2)(2r+a-3)S_r^{-2}N_0$.

Also the marginal posterior densities of μ_i, $i=1,2$, respectively and σ are as follows :

$$\pi(\mu_i|\underline{x}) \propto \Big(1+\frac{nx^i_{(1)}}{S_r}-\frac{n\mu_i}{S_r}\Big)^{-(2r+a-2)} \quad (2.11)$$

$$- \Big(1 + \frac{nx^i_{(1)}}{S_r} + \frac{nx^j_{(1)}}{S_r} - \frac{n\mu_i}{S_r}\Big)^{-(2r+a-2)},$$

$$0 < \mu_i \le x^i_{(1)}, \quad i,j = 1,2, \quad i \neq j,$$

$$\pi(\sigma|\underline{x}) \propto \sigma^{-(2r+a-2)} \exp\Big[-\frac{S_r}{\sigma}\Big]\Big(1 - \exp\Big[-\frac{nx^1_{(1)}}{\sigma}\Big]\Big)\Big(1 - \exp\Big[-\frac{nx^2_{(1)}}{\sigma}\Big]\Big), \quad 0 < \sigma < \infty, \tag{2.12}$$

where $K_{\mu_i} = n(2r + a - 3)S_r^{-1}N_0$ and $K_\sigma = \dfrac{N_0 S_r^{2r+a-3}}{\Gamma(2r + a - 3)}$ are normalizing constants. Hence with the squared error loss function the following theorem can be obtained.

Theorem 1. Suppose that $\underline{x} = \{X^i_{(1)} = x^i_{(1)}, X^i_{(2)} = x^i_{(2)}, \ldots, X^i_{(r)} = x^i_{(r)}, \ i = 1,2\}$ is a type-II censored sample from $\mathcal{E}(\mu_1, \sigma)$ and $\mathcal{E}(\mu_2, \sigma)$. Let the joint prior of μ_1, μ_2 and σ be $\pi(\mu_1, \mu_2, \sigma) \propto 1/\sigma^a$, $a \ge 0$. Then under the squared error loss the Bayes estimators $\widehat{\mu}_i$ and $\widehat{\sigma}$ of μ_i, $i = 1,2$ and σ, respectively, are

$$\widehat{\mu}^n_i = \frac{N_0}{n(2r + a - 4)} \tag{2.13}$$

$$\times \Big\{n(2r + a - 4)x^i_{(1)}\Big[1 - \Big(1 + \frac{nx^j_{(1)}}{S_r}\Big)^{-(2r+a-3)}\Big] - S_r N_1^{-1}\Big\},$$

$$i,j = 1,2, \quad i \neq j,$$

and

$$\widehat{\sigma}^n = \frac{N_0 S_r}{2r + a - 4} N_1^{-1}, \quad 2r + a > 4. \tag{2.14}$$

Proof. Under the squared error loss, the Bayes estimators of μ_i, $i = 1,2$ are the posterior means. Thus by using the inverted gamma function,

$$\widehat{\mu}^n_i = E^{\mu_i|\underline{x}}(\mu_i)$$

$$= \int_0^{x^i_{(1)}} \mu_i \pi(\mu_i|\underline{x})\, d\mu_i$$

$$= n(2r+a-3)S_r^{-1}N_0$$
$$\times\Big[\int_0^{x^i_{(1)}} \mu_i\Big(1+\frac{n(x^i_{(1)}-\mu_i)}{S_r}\Big)^{-2r-a+2} d\mu_i$$
$$-\int_0^{x^i_{(1)}} \mu_i\Big(1+\frac{n(x^i_{(1)}-\mu_i)}{S_r}+\frac{nx^j_{(1)}}{S_r}\Big)^{-2r-a+2} d\mu_i\Big].$$
$$= \frac{N_0}{n(2r+a-4)}$$
$$\times\Big\{n(2r+a-4)x^i_{(1)}\Big[1-\Big(1+\frac{nx^j_{(1)}}{S_r}\Big)^{-(2r+a-3)}\Big]-S_rN_1^{-1}\Big\}.$$

Also $\widehat{\sigma}^n$ can be obtained easily and hence is omitted. Hence the proof is complete.

Now we consider the Bayes estimator of the reliability function R^i_t of the system i.

Theorem 2. Under the assumptions of Theorem 1, the joint posterior density of μ_i and σ is

$$\pi(\mu_i,\sigma|\underline{x}) \propto \sigma^{-(2r+a-1)}\exp\Big[-\frac{1}{\sigma}\Big(S_r+n(x^i_{(1)}-\mu_i)\Big)\Big]\Big(1-\exp\Big[-\frac{nx^j_{(1)}}{\sigma}\Big]\Big),$$
$$0<\mu_i<x^i_{(1)},\quad 0<\sigma<\infty,\quad i,j=1,2,\quad i\neq j, \tag{2.15}$$

with normalizing constant $K=\dfrac{n}{\Gamma(2r+a-3)}S_r^{2r+a-3}N_0$. Then under the squared error loss, the Bayes estimators of R^i_t, $i=1,2$, are given as follows.

$$\widehat{R^i_t}^n = \frac{n}{n+1}N_0\Big[\Big(1+\frac{t}{S_r}-\frac{x^i_{(1)}}{S_r}\Big)^{-2r-a+3}-\Big(1+\frac{t}{S_r}+\frac{nx^i_{(1)}}{S_r}\Big)^{-2r-a+3} \tag{2.16}$$
$$-\Big(1+\frac{nx^j_{(1)}}{S_r}+\frac{t}{S_r}-\frac{x^i_{(1)}}{S_r}\Big)^{-2r-a+3}+\Big(1+\frac{nx^i_{(1)}}{S_r}+\frac{nx^j_{(1)}}{S_r}+\frac{t}{S_r}\Big)^{-2r-a+3}\Big],$$
$$t>x^i_{(1)}.$$

Proof. The proof is analogous to that of Theorem 1 and hence is omitted.

Also one can obtain the posterior distribution of R_t^i, $i=1,2$ by using the following transformation, g :

$$R_t^i = \exp\Big[-\frac{t-\mu_i}{\sigma}\Big] \quad \text{and} \quad \nu = \sigma. \tag{2.17}$$

Theorem 3. Under the assumptions of Theorem 1, the posterior density of $R_t^i = \exp\Big[-\frac{t-\mu_i}{\sigma}\Big]$ is given by

$$\begin{aligned}\pi(R_t^i|\underline{x}) &= K(R_t^i)^{n-1}\Gamma(2r+a-3) \qquad (2.18)\\ &\times\Bigg[\frac{1}{W_{r1}^{2r+a-3}}\exp\Big(-\frac{W_{r1}}{\alpha_2}\Big)\sum_{m=0}^{2r+a-4}\frac{1}{m!}\Big(\frac{W_{r1}}{\alpha_2}\Big)^m\\ &-\frac{1}{W_{r1}^{2r+a-3}}\exp\Big(-\frac{W_{r1}}{\alpha_1}\Big)\sum_{m=0}^{2r+a-4}\frac{1}{m!}\Big(\frac{W_{r1}}{\alpha_1}\Big)^m\\ &-\frac{1}{W_{r2}^{2r+a-3}}\exp\Big(-\frac{W_{r2}}{\alpha_2}\Big)\sum_{m=0}^{2r+a-4}\frac{1}{m!}\Big(\frac{W_{r2}}{\alpha_2}\Big)^m\\ &+\frac{1}{W_{r2}^{2r+a-3}}\exp\Big(-\frac{W_{r2}}{\alpha_1}\Big)\sum_{m=0}^{2r+a-4}\frac{1}{m!}\Big(\frac{W_{r2}}{\alpha_1}\Big)^m\Bigg], \quad i=1,2,\end{aligned}$$

where $\alpha_1 = \dfrac{t-x_{(1)}^i}{\ln(R_t^i)^{-1}}$, $\quad \alpha_2 = \dfrac{t}{\ln(R_t^i)^{-1}}$,

and $W_{r1} = S_r + nx_{(1)}^1 - nt$, $\quad W_{r2} = S_r + nx_{(1)}^1 + nx_{(1)}^2 - nt$,

with the normalizing constrant $K = \dfrac{n}{\Gamma(2r+a-3)}S_r^{2r+a-3}N_0$.

Proof. The joint posterior density of μ_i and σ, $i=1,2$ is

$$\pi(\mu_i,\sigma|\underline{x}) = \frac{K}{\sigma^{2r+a-1}}\exp\Big[-\frac{1}{\sigma}\Big(S_r + n(x_{(1)}^i-\mu_i)\Big)\Big]\Big[1-\exp\Big(-\frac{n}{\sigma}x_{(1)}^j\Big)\Big],$$

for $0<\mu_i<x_{(1)}^i$, $\quad 0<\sigma<\infty$, $\quad i,j=1,2$, $\quad i\neq j$. Then g is one-to-one on

$S = \{(\mu_i, \sigma) : 0 < \mu_i < x^i_{(1)}, \sigma > 0\}$ and its range is

$$S_1 = \Big\{(R^i_t, \nu) : \exp\Big[-\frac{t}{\nu}\Big] < R^i_t < \exp\Big[-\frac{t - x^i_{(1)}}{\nu}\Big], \ \frac{t - x^i_{(1)}}{\ln(R^i_t)^{-1}} < \nu < \frac{t}{\ln(R^i_t)^{-1}}\Big\}.$$

Then the joint posterior density of R^i_t and ν is given by

$$\begin{aligned}\pi(R^i_t, \nu|\underline{x}) &= \frac{K}{\nu^{2r+a-2}}(R^i_t)^{n-1} \times \Big[\exp\Big(-\frac{1}{\nu}(S_r + nx^i_{(1)} - nt)\Big) \\ &\qquad - \exp\Big(-\frac{1}{\nu}(S_r + nx^i_{(1)} + nx^j_{(1)} - nt)\Big)\Big],\end{aligned}$$

where $\exp\Big[-\frac{t}{\nu}\Big] < R^i_t < \exp\Big[(\frac{t - x^i_{(1)}}{\nu}\Big]$ and $\frac{t - x^i_{(1)}}{\ln(R^i_t)^{-1}} < \nu < \frac{t}{\ln(R^i_t)^{-1}}$.
Using the identity connecting the incomplete gamma distribution and Poisson, one can obtain the posterior distribution. This completes the proof.

A conjugate prior of $\underline{\mu}$ and σ is

$$\pi(\underline{\mu}, \sigma) \ \propto \sigma^{-k(\nu_0+1)} \exp\Big[-\frac{1}{\sigma}\sum_{i=1}^{k}(\delta_0 - \lambda_0\mu_i)\Big], \tag{2.19}$$
$$0 < \mu_i \le \eta_0, \quad \sigma > 0,$$

where δ_0, $\eta_0 > 0$ and λ_0 and ν_0 are nonnegative integers such that $\nu_0 \le \lambda_0 \le \frac{\delta_0}{\eta_0}$. Then the joint posterior distribution of μ_i, $i = 1, \dots, k$ and σ given $\underline{x}$ is

$$\begin{aligned}\pi(\underline{\mu}, \sigma|\underline{x}) \ &\propto L(\underline{\mu}, \sigma|\underline{x})\pi(\underline{x}, \sigma) \qquad\qquad (2.20)\\ &\propto \exp\Big[-k(r + \nu_0 + 1)\log\sigma \\ &\quad - \frac{1}{\sigma}\Big(\sum_{i=1}^{k} S_r(i) + k\delta_0 + n\sum_{i=1}^{k} x^i_{(1)}\Big) + \frac{1}{\sigma}(n + \lambda_0)\sum_{i=1}^{k}\mu_i\Big] \\ &\overset{let}{\equiv} \exp\Big[-k(r + \nu_0 + 1)\log\sigma - \frac{1}{\sigma}(S^c_r - (n + \lambda_0)\sum_{i=1}^{k}\mu_i)\Big], \\ &\quad 0 < \mu_i < M_i, \qquad \sigma > 0,\end{aligned}$$

and the joint posterior distribution of μ_i, $i = 1, \ldots, k$ is

$$\begin{aligned}\pi(\underline{\mu}|\underline{x}) &\propto \int_0^\infty \frac{1}{\sigma^{k(r+\nu_0+1)}} \exp\Big[-\frac{1}{\sigma}\Big(S_r^c - (\lambda_0+n)\sum_{i=1}^k \mu_i\Big)\Big]\, d\sigma \qquad (2.21)\\ &\propto \Gamma(k(r+\nu_0+1)-1)(S_r^c)^{-k(r+\nu_0+1)}\Big[1 - \frac{(\lambda_0+n)}{S_r^c}\sum_{i=1}^k \mu_i\Big]^{-k(r+\nu_0+1)+1},\\ &\quad 0 < \mu_i < M_i,\end{aligned}$$

where $S_r^c = \sum_{i=1}^k S_r(i) + k\delta_0 + n\sum_{i=1}^k x_{(1)}^i$ and $M_i \equiv \min(\eta_0, x_{(1)}^i)$.

Also the marginal posterior density of μ_i, $i = 1, \ldots, k$ is

$$\begin{aligned}\pi(\mu_i|\underline{x}) \propto \Bigg\{&\Big[1 - \frac{\lambda_0+n}{S_r^c}\Big(\mu_i + \sum_{j=2}^k M_j\Big)\Big]^{-k(r+\nu_0)} \qquad (2.22)\\ &- \sum_{j=1, j\neq i}^k \Big[1 - \frac{\lambda_0+n}{S_r^c}\Big(\mu_i + \sum_{m=1, m\neq j}^k M_m\Big)\Big]^{-k(r+\nu_0)}\\ &+ \sum_{j=1, j\neq i}^k \Big[1 - \frac{\lambda_0+n}{S_r^c}\Big(\mu_i + \sum_{m=1, m\neq j}^k \sum_{l=1, l\neq i,j}^k M_l\Big)\Big]^{-k(r+\nu_0)}\\ &\vdots\\ &+ (-1)^{k+1}\Big[1 - \frac{\lambda_0+n}{S_r^c}\mu_i\Big]^{-k(r+\nu_0)}\Bigg\}, \quad 0 < \mu_1 < M_1\end{aligned}$$

and the marginal posterior density of σ is

$$\pi(\sigma|\underline{x}) \propto \sigma^{-k(r+\nu_0)} \exp\Big[-\frac{S_r^c}{\sigma}\Big] \prod_{i=1}^k \Big\{\exp\Big[\frac{\lambda_0+n}{\sigma} M_i\Big] - 1\Big\}, \quad \sigma > 0. \qquad (2.23)$$

In paticular for $k = 2$, the posterior distribution of $(\mu_1, \mu_2, \sigma|\underline{x})$ is given by

$$\begin{aligned}\pi(\mu_1, \mu_2, \sigma|\underline{x}) &= \frac{(\lambda_0+n)^2}{\Gamma(2(r+\nu_0)-1)}(S_r^c)^{2(r+\nu_0)-1} C_0 \qquad (2.24)\\ &\quad \times \sigma^{-2(r+\nu_0+1)} \exp\Big[-\frac{1}{\sigma}\Big(S_r^c - (\lambda_0+n)(\mu_1+\mu_2)\Big)\Big],\end{aligned}$$

where

$$C_m = \Big\{1 - \Big(1 - \frac{\lambda_0 + n}{S_r^c} M_1\Big)^{-2(r+\nu_0)+m+1} - \Big(1 - \frac{\lambda_0 + n}{S_r^c} M_2\Big)^{-2(r+\nu_0)+m+1}$$
$$+ \Big(1 - \frac{\lambda_0 + n}{S_r^c}(M_1 + M_2)\Big)^{-2(r+\nu_0)+m+1}\Big\}^{-1}$$

and $S_r^c = S_r(1) + S_r(2) + 2\delta_0 + n(x_{(1)}^1 + x_{(1)}^2)$.

With the squared error loss the following theorem can be obtained.

Theorem 4. Suppose that $\underline{x} = \{X_{(1)}^i = x_{(1)}^i, X_{(2)}^i = x_{(2)}^i, \ldots, X_{(r)}^i = x_{(r)}^i, i = 1, 2\}$ is a type-II censored sample from $\mathcal{E}(\mu_1, \sigma)$ and $\mathcal{E}(\mu_2, \sigma)$. Let the joint prior of μ_1, μ_2 and σ be

$$\pi(\underline{\mu}, \sigma) \quad \propto \quad \sigma^{-k(\nu_0+1)} \exp\Big[-\frac{1}{\sigma}\sum_{i=1}^{k}(\delta_0 - \lambda_0\mu_i)\Big],$$
$$0 < \mu_i \le \eta_0, \qquad \sigma > 0,$$

where δ_0, $\eta_0 > 0$ and λ_0 and ν_0 are nonnegative integers such that $\nu_0 \le \lambda_0 \le \frac{\delta_0}{\eta_0}$. Under the squared error loss the Bayes estimators $\widehat{\mu}_i$, $i = 1, 2$ and $\widehat{\sigma}$ are

$$\widehat{\mu}_i^c = C_0\Big\{M_i\Big[\Big(1 - \frac{\lambda_0 + n}{S_r^c}(M_i + M_j)\Big)^{-2(r+\nu_0)+1} - \Big(1 - \frac{\lambda_0 + n}{S_r^c} M_i\Big)^{-2(r+\nu_0)+1}\Big]$$
$$- \frac{1}{(2(r+\nu_0) - 2)} \frac{S_r^c}{\lambda_0 + n} C_1^{-1}\Big\}, \quad i, j = 1, 2, \quad i \ne j, \tag{2.25}$$

and

$$\widehat{\sigma}^c = \frac{(S_r^c) C_0 C_1^{-1}}{2(r + \nu_0) - 2}. \tag{2.26}$$

Proof. Under the squared error loss, the Bayes estimators of μ_i, $i = 1, 2$ and σ are the posterior means which can be obtained easily. Thus the proof is omitted.

Consider the Bayes estimator of R_t^i, $i = 1, 2$. From the joint density function of (μ_i, σ), $i = 1, 2$, given by

$$\pi(\mu_i, \sigma|\underline{x}) \propto \sigma^{-2(r+\nu_0)-1} \exp\Big[-\frac{1}{\sigma}\Big(S_r^c - (\lambda_0+n)\mu_i\Big)\Big]\Big(\exp\Big[\frac{\lambda_0+n}{\sigma}M_2\Big] - 1\Big),$$
$$0 < \mu_i < M_i, i = 1, 2, \quad 0 < \sigma < \infty, \tag{2.27}$$

one can obtain the following result.

Theorem 5. Under the assumptions of Theorem 4 and the squared error loss, the Bayes estimators of R_t^i, $i = 1, 2$, are given by

$$\begin{aligned}\widehat{R_t^i}^c &= \frac{\lambda_0+n}{\lambda_0+n+1}C_0\Big\{[1+\frac{t}{S_r^c} - \frac{\lambda_0+n}{S_r^c}(M_i+M_j)]^{-2(r+\nu_0)+1} \\ &\quad - [1+\frac{t}{S_r^c} - \frac{\lambda_0+n}{S_r^c}M_j]^{-2(r+\nu_0)+1} \\ &\quad - [1+\frac{t}{S_r^c} - \frac{\lambda_0+n+1}{S_r^c}M_i]^{-2(r+\nu_0)+1} \\ &\quad + [1+\frac{t}{S_r^c}]^{-2(r+\nu_0)+1}\Big\}, \quad i, j = 1, 2, \quad i \neq j.\end{aligned} \tag{2.28}$$

Also the posterior density of R_t^i for $t > x_{(1)}^i$ is given by

$$\begin{aligned}\pi(R_t^i|\underline{x}) &= (\lambda_0+n)(S_r^c)^{2(r+\nu_0)-1}C_0(R_t^i)^{\lambda_0+n-1} \\ &\quad \times \Big\{\frac{1}{(W_{r1}^c)^{2(r+\nu_0)-1}}\exp\Big[-\frac{W_{r1}^c}{\beta_2}\Big]\sum_{m=0}^{2(r+\nu_0)-2}\frac{1}{m!}\Big(\frac{W_{r1}^c}{\beta_2}\Big)^m \\ &\quad - \frac{1}{(W_{r1}^c)^{2(r+\nu_0)-1}}\exp\Big[-\frac{W_{r1}^c}{\beta_1}\Big]\sum_{m=0}^{2(r+\nu_0)-2}\frac{1}{m!}\Big(\frac{W_{r1}^c}{\beta_1}\Big)^m \\ &\quad - \frac{1}{(W_{r2}^c)^{2(r+\nu_0)-1}}\exp\Big[-\frac{W_{r2}^c}{\beta_2}\Big]\sum_{m=0}^{2(r+\nu_0)-2}\frac{1}{m!}\Big(\frac{W_{r2}^c}{\beta_2}\Big)^m \\ &\quad + \frac{1}{(W_{r2}^c)^{2(r+\nu_0)-1}}\exp\Big[-\frac{W_{r2}^c}{\beta_1}\Big]\sum_{m=0}^{2(r+\nu_0)-2}\frac{1}{m!}\Big(\frac{W_{r2}^c}{\beta_1}\Big)^m\Big\},\end{aligned} \tag{2.29}$$

where

$$\beta_1 = \frac{t - M_i}{\ln(R_t^i)^{-1}}, \quad \beta_2 = \frac{t}{\ln(R_t^i)^{-1}}$$

and

$$W_{r1}^c = S_r^c - (\lambda_0 + n)(t + M_j), \quad W_{r2}^c = S_r^c - (\lambda_0 + n)t.$$

Proof. From the joint posterior density function of μ_1, μ_2 and σ, $\pi(\mu_i, \sigma|\underline{x})$ can be obtained easily. Therefore under the squared error loss, the Bayes estimators of the R_t^i are the posterior means. Also one can obtain the posterior distribution of R_t^i, $i = 1, 2$ by using the transformation g, given in equation (2.17).

3. THE PROPOSED BAYES SELECTION PROCEDURE

We consider the problem of selecting better populations among $(k-1)$ treatment groups than the control group in terms of the mean life time under the Bayesian setting. Since we want to select the better treatment group than the control group, we can reduce the problem to the case of one treatment group and the control group. So we consider the $0 - K_i$ loss(Berger(1985) page 63) which would be used in two-action decision problem and be defined as follows.

Definition. Let $L(\mu_i)$ be the loss to select a group having the mean life time μ_i. Then the $0 - K_i$ loss is given by

$$L(\mu_i) = \begin{cases} 0 & \text{if } \mu_i > \mu_j \\ K_i & \text{if } \mu_i \le \mu_j \end{cases}, \quad i \ne j, \quad i, j = 1, 2.$$

Under this loss the Bayes procedure $\mathcal{R}$ is as follows.

$\mathcal{R}$: Select the treatment group if

$$\frac{K_1}{K_2} < \frac{P(\mu_1 > \mu_2|\underline{x})}{P(\mu_2 \ge \mu_1|\underline{x})}, \tag{3.1}$$

Note that if $P(\mu_1 > \mu_2|\underline{x}) = \dfrac{K_1}{K_1+K_2}$, treatment group is selected with probability 1/2.

Now to evaluate the quantity $P(\mu_1 \geq \mu_2|\underline{x})$, the joint posterior density of μ_1 and μ_2 given $\underline{x}$ is derived as equation (2.10). Under the noninformative prior both case(i) $x^1_{(1)} < x^2_{(1)}$ and case (ii) $x^1_{(1)} > x^2_{(1)}$ are considered. For each case $P(\mu_1 \geq \mu_2|\underline{x})$ is computed and is given in the following theorem.

Theorem 6. Under the assumptions of Theorem 1, $P(\mu_1 \geq \mu_2|\underline{x})$ can be obtained as follows :

i) For the case $x^1_{(1)} < x^2_{(1)}$,

$$\frac{N_0}{2}\left\{\left(1-\frac{nx^1_{(1)}}{S_r}+\frac{nx^2_{(1)}}{S_r}\right)^{-(2r+a-3)}+\left(1+\frac{nx^1_{(1)}}{S_r}+\frac{nx^2_{(1)}}{S_r}\right)^{-(2r+a-3)} -2\left(1+\frac{nx^2_{(1)}}{S_r}\right)^{-(2r+a-3)}\right\}. \tag{3.2}$$

ii) For the case $x^1_{(1)} > x^2_{(1)}$,

$$\frac{N_0}{2}\left\{\left(1+\frac{nx^1_{(1)}}{S_r}+\frac{nx^2_{(1)}}{S_r}\right)^{-(2r+a-3)}+\left(1+\frac{nx^1_{(1)}}{S_r}\right)^{-(2r+a-3)} -2\left(1+\frac{nx^2_{(1)}}{S_r}\right)^{-(2r+a-3)}-2\left(1+\frac{nx^1_{(1)}}{S_r}-\frac{nx^2_{(1)}}{S_r}\right)^{-(2r+a-3)}+2\right\}. \tag{3.3}$$

Proof. For the case $x^1_{(1)} < x^2_{(1)}$,

$$\begin{aligned} P(\mu_1 \geq \mu_2|\underline{x}) &= \int_0^{x^1_{(1)}}\int_0^{\mu_1}\pi(\mu_1,\mu_2|\underline{x})\,d\mu_2\,d\mu_1 \\ &= c^*\int_0^{x^1_{(1)}}\int_0^{\mu_1}\left(1+\frac{n(x^1_{(1)}-\mu_1)}{S_r}+\frac{n(x^2_{(1)}-\mu_2)}{S_r}\right)^{-2r-a+1} d\mu_2\,d\mu_1, \end{aligned}$$

where $c^* = n^2(2r+a-2)(2r+a-3)S_r^{-2}N_0$. Therefore,

$$P(\mu_1 \geq \mu_2|\underline{x})$$

$$= c^* \frac{S_r}{n} \frac{1}{(2r+a-2)} \int_0^{x^1_{(1)}} \left[\left(1 + \frac{nx^1_{(1)}}{S_r} + \frac{nx^2_{(1)}}{S_r} - \frac{2n\mu_1}{S_r}\right)^{-2r-a+2} \right.$$

$$\left. - \left(1 + \frac{nx^1_{(1)}}{S_r} + \frac{nx^2_{(1)}}{S_r} - \frac{n\mu_1}{S_r}\right)^{-2r-a+2} \right] d\mu_1$$

$$= \frac{c^* S_r}{n(2r+a-2)} \left[\frac{S_r}{2n} \int_{1+(nx^2_{(1)})/S_r-(nx^1_{(1)})/S_r}^{1+(nx^1_{(1)})/S_r+(nx^2_{(1)})/S_r} t^{-2r-a+2}\, dt \right.$$

$$\left. - \frac{S_r}{n} \int_{1+(nx^2_{(1)})/S_r}^{1+(nx^1_{(1)})/S_r+(nx^2_{(1)})/S_r} t^{-2r-a+2}\, dt \right]$$

$$= \frac{c^* S_r}{n(2r+a-2)} \frac{S_r}{2n} \frac{1}{2r+a-3}$$

$$\times \left[\left(1 + \frac{nx^2_{(1)}}{S_r} - \frac{nx^1_{(1)}}{S_r}\right)^{-2r-a+3} - \left(1 + \frac{nx^1_{(1)}}{S_r} + \frac{nx^2_{(1)}}{S_r}\right)^{-2r-a+3} \right.$$

$$\left. -2\left(1 + \frac{nx^2_{(1)}}{S_r}\right)^{-2r-a+3} + 2\left(1 + \frac{nx^1_{(1)}}{S_r} + \frac{nx^2_{(1)}}{S_r}\right)^{-2r-a+3} \right]$$

$$= \frac{N_0}{2} \left[\left(1 + \frac{nx^2_{(1)}}{S_r} - \frac{nx^1_{(1)}}{S_r}\right)^{-2r-a+3} + \left(1 + \frac{nx^1_{(1)}}{S_r} + \frac{nx^2_{(1)}}{S_r}\right)^{-2r-a+3} \right.$$

$$\left. -2\left(1 + \frac{nx^2_{(1)}}{S_r}\right)^{-2r-a+3} \right].$$

For the case $x^1_{(1)} < x^1_{(2)}$, one can derive the probability over $0 < \mu_2 < \mu_1$ and $0 < \mu_2 < x^2_{(1)}$. This completes the proof of the theorem.

Under the conjugate prior one can also obtain the quantity for the following four cases.

case (i) $x^2_{(1)} < x^1_{(1)} < \eta_0$, i.e., $M_1 = x^1_{(1)}$ and $M_2 = x^2_{(1)}$

case (ii) $x^2_{(1)} < \eta_0 < x^1_{(1)}$, i.e., $M_1 = \eta_0$ and $M_2 = x^2_{(1)}$

case (iii) $x^1_{(1)} < x^2_{(1)} < \eta_0$ or $x^1_{(1)} < \eta_0 < x^2_{(1)}$, i.e., $M_1 = x^1_{(1)}$ and $M_2 = x^2_{(1)}$ or $M_1 = x^1_{(1)}$ and $M_2 = \eta_0$

case (iv) $\eta_0 < x^1_{(1)},\ x^2_{(1)}$, i.e., $M_1 = M_2 = \eta_0$.

Theorem 7. Under the assumptions of Theorem 5, $P(\mu_1 \geq \mu_2 | \underline{x})$ can be obtained as follows :

case (i) For $x^2_{(1)} < x^1_{(1)} < \eta_0$, i.e., $M_1 = x^1_{(1)}$ and $M_2 = x^2_{(1)}$,

$$\frac{1}{2}(S^c_r)^{2(r+\nu_0)-1}C_0\Big\{S_r^{-2(r+\nu_0)+1} - \Big(S_r - 2(\lambda_0+n)x^2_{(1)}\Big)^{-2(r+\nu_0)+1} - 2\Big(S_r - (\lambda_0+n)x^1_{(1)}\Big)^{-2(r+\nu_0)+1} + 2\Big(S_r - (\lambda_0+n)(x^1_{(1)}+x^2_{(1)})\Big)^{-2(r+\nu_0)+1}\Big\}. \tag{3.4}$$

case (ii) For $x^2_{(1)} < \eta_0 < x^1_{(1)}$, i.e., $M_1 = \eta_0$ and $M_2 = x^2_{(1)}$,

$$\frac{1}{2}(S^c_r)^{2(r+\nu_0)-1}C_0\Big\{S_r^{-2(r+\nu_0)+1} - 2\Big(S_r - (\lambda_0+n)\eta_0\Big)^{-2(r+\nu_0)+1} - \Big(S_r - 2(\lambda_0+n)x^2_{(1)}\Big)^{-2(r+\nu_0)+1} + 2\Big(S_r - (\lambda_0+n)(\eta_0+x^2_{(1)})\Big)^{-2(r+\nu_0)+1}\Big\}. \tag{3.5}$$

case (iii) For $x^1_{(1)} < x^2_{(1)} < \eta_0$ or $x^1_{(1)} < \eta_0 < x^2_{(1)}$, i.e., $M_1 = x^1_{(1)}$ and $M_2 = x^2_{(1)}$ or $M_1 = x^1_{(1)}$ and $M_2 = \eta_0$,

$$\frac{1}{2}(S^c_r)^{2(r+\nu_0)-1}C_0\Big\{S_r^{-2(r+\nu_0)+1} + \Big(S_r - 2(\lambda_0+n)x^1_{(1)}\Big)^{-2(r+\nu_0)+1} - 2\Big(S_r - (\lambda_0+n)x^1_{(1)}\Big)^{-2(r+\nu_0)+1}\Big\}. \tag{3.6}$$

case (iv) For $\eta_0 < x^1_{(1)},\ x^2_{(1)}$, i.e., $M_1 = M_2 = \eta_0$,

$$\frac{1}{2}(S^c_r)^{2(r+\nu_0} - 1)C_0\Big\{S_r^{-2(r+\nu_0)+1} + \Big(S_r - 2(\lambda_0+n)\eta_0\Big)^{-2(r+\nu_0)+1} - 2\Big(S_r - (\lambda_0+n)\eta_0\Big)^{-2(r+\nu_0)+1}\Big\}. \tag{3.7}$$

Proof. By using the joint posterior distribution $\pi(\mu_1, \mu_2 | \underline{x})$ one can easily obtain these results. Thus the proof is omitted.

4. SIMULATION STUDIES

In this section, we compare the performances of the proposed Bayes estimators, GMLE's and Bayes estimators proposed by Sinha and Guttmans(1976) under the Jeffrey-type noninformative prior distribution.

We repeated 500 times to compute MSE's and biases of estimators. To obtain two-parameter exponential random numbers, we used the subroutine EPURN by McLeod(1984) to generate uniform random numbers and then transformed $X = \mu - \sigma \ln U$. The MSE's and bias of the estimators were computed for $a = 0,\ 0.5,\ 1, 2,\ \sigma = 1(2)5,\ (\mu_1, \mu_2) = (1.5, 1.0),\ (2.0, 1.0),\ (4.0, 1.0),\ (6.0, 1.0),\ (n, r) = (10, 5),\ (30, 15),\ (30, 20),\ (30, 25),\ (30, 30)$ under the noninformative prior and $\nu_0 = a,\ \delta_0 = 1(1)4,\ 8,\ 12,\ \lambda_0 = 0(1)4,\ \eta_0 = 1,\ 4$ under the conjugate prior and $t = 0.25,\ 3.0(1.0)5.0,\ 7.0,\ 12.0$.

Table 1 to Table 3 are parts of the simulation results. The rest are available based upon the request. The Bayes estimators for μ_1, σ, and R_t proposed by Sinha and Guttman(1976) are denoted by $\widehat{\mu}_1^s$, $\widehat{\sigma}^s$, and $\widehat{R}_t^s$, respectively and GMLE's for μ_1, σ, and R_t are denoted by $\widetilde{\mu}_1$, $\widetilde{\sigma}$, and $\widetilde{R}_t$, respectively, in the tables. Also the proposed estimators for μ_1, σ and R_t are denoted by $\widehat{\mu}_1^n$, $\widehat{\sigma}^n$ and $\widehat{R}_t^n$ under the noninformative prior and $\widehat{\mu}_1^c$, $\widehat{\sigma}^c$ and $\widehat{R}_t^c$ under the conjugate prior, respectively. All computations were done by using IBM 486 personal computer. Note we confirm that these results with IBM 486 personal computer are identical to those with CRAY YMP-EL. From the tables one can observe the following facts :

For the estimators $\widehat{\mu}_1^n$, $\widehat{\mu}_1^c$ and $\widehat{\sigma}^n$, $\widehat{\sigma}^c$ of μ_1 and σ, respectively, one can see the followings :

1) the proposed Bayes estimators $\widehat{\mu}_1^n$ and $\widehat{\mu}_1^c$ of μ_1 perform better than the estimator proposed by Sinha and Guttman(1976) and GMLE. Also the proposed Bayes estimators $\widehat{\sigma}^n$ and $\widehat{\sigma}^c$ of σ perform better than the estimator proposed by Sinha and Guttman(1976). But for $a = 0$, *i.e.*, improper prior, GMLE performs better than the proposed Bayes estimators. For $a = 1$, though the MSE of the proposed Bayes estimators is

larger than that of GMLE , the bias is much smaller. On the other hand, $\hat{\sigma}^c$ is not better than $\hat{\sigma}^n$ and $\tilde{\sigma}$ for the cases I, II and III. It is found that the performance of $\hat{\sigma}^c$ is very sensitive to the choices of ν_0, δ_0, λ_0 and η_0. But as the rate of censoring decreases, $\hat{\sigma}^c$ performs better.

2) The proposed Bayes estimator $\hat{\sigma}^n$ is not robust to the changes of the values of a. This is an opposite result to that of Sinha and Guttman(1976). Thus further study may be desirable. For example, MSE's(biases) decrease as the value of a increases from 0 to 2.

3) The MSE's(biases) decrease as r increases except a few cases.

Also, for the estimators $\hat{R}_t^n$ and $\hat{R}_t^c$ of R_t, one can conclude as follows:

1) the proposed Bayes estimator of $\hat{R}_t^n$ always performs better than the estimator by Sinha and Guttman(1976). Also $\hat{R}_t^n$ performs better than the GMLE for small values of t. But for the larger values of t, GMLE performs better. The proposed Bayes estimator under the conjugate prior, $\hat{R}_t^c$ is not better than the others as the estimator of σ and underestimate the R_t in all t. But MSE's(biases) decrease as censoring rate decrease.

2) As the value of a increases, the proposed Bayes estimator performs better than the GMLE.

Table 1. MSE's and Biases(in the parenthesis) of the Proposed Estimators $\widehat{\mu}_1^n$, $\widetilde{\mu}_1$, $\widehat{\mu}_1^s$ and $\widehat{\mu}_1^c$ of μ_1 for $\sigma = 3$, $(\mu_1, \mu_2) = (2.0, 1.0)$.

$$\begin{aligned} \text{case I} &= (a, \nu_0, \delta_0, \lambda_0, \eta_0) = (0, 0, 8, 1, 4) \\ \text{case II} &= (a, \nu_0, \delta_0, \lambda_0, \eta_0) = (0.5, 0.5, 8, 2, 4) \\ \text{case III} &= (a, \nu_0, \delta_0, \lambda_0, \eta_0) = (1, 1, 12, 1, 4) \\ \text{case IV} &= (a, \nu_0, \delta_0, \lambda_0, \eta_0) = (2, 2, 12, 2, 4). \end{aligned}$$

case	(n, r)	$\widehat{\mu}_1^n$	$\widetilde{\mu}_1$	$\widehat{\mu}_1^s$	$\widehat{\mu}_1^c$
I	(10,5)	.093374	.158131	.110789	.093225
		(−.079570)	(.280155)	(−.152509)	(−.100391)
	(30,15)	.007711	.015522	.008483	.007782
		(−.014773)	(.092038)	(−.024236)	(−.018699)
	(30,25)	.009140	.019196	.009304	.009155
		(−.004193)	(.099789)	(−.010741)	(−.005247)
	(30,30)	.012734	.024605	.012868	.012728
		(.005803)	(.109427)	(.002424)	(.005491)
II	(10,5)	.113028	.186034	.125696	.108712
		(−.035842)	(.299194)	(−.075732)	(.008870)
	(30,15)	.008969	.018395	.009352	.008859
		(−.006195)	(.099521)	(−.009474)	(−.001393)
	(30,25)	.011885	.023123	.012331	.011925
		(.004462)	(.108063)	(.000921)	(.009784)
	(30,30)	.011066	.022660	.011286	.011151
		(.006322)	(.108712)	(.004157)	(.011712)
III	(10,5)	.094134	.183828	.101309	.093782
		(−.005689)	(.309922)	(−.026467)	(−.065002)
	(30,15)	.012132	.022409	.012930	.012152
		(−.000516)	(.103257)	(−.004652)	(−.009481)
	(30,25)	.009822	.019563	.009910	.009824
		(−.002089)	(.100268)	(−.005413)	(−.006265)
	(30,30)	.010083	.019889	.010168	.010085
		(−.001616)	(.099954)	(−.004137)	(−.004610)
IV	(10,5)	.081373	.161216	.082179	.080198
		(.004906)	(.289569)	(.016288)	(.012927)
	(30,15)	.010826	.020685	.011218	.010709
		(.001944)	(.102294)	(.001430)	(.002787)
	(30,25)	.008636	.017112	.008913	.008556
		(−.005828)	(.094324)	(−.005560)	(−.002932)
	(30,30)	.008887	.017455	.009072	.008826
		(−.006046)	(.094098)	(−.006184)	(−.002615)

Table 2. MSE's and Biases(in the parenthesis) of the Proposed Estimators $\widehat{\sigma}^n$, $\widetilde{\sigma}$, $\widehat{\sigma}^s$ and $\widehat{\sigma}^c$ of σ for $\sigma = 3$, $(\mu_1, \mu_2) = (2.0, 1.0)$.

$$\text{case I} = (a, \nu_0, \delta_0, \lambda_0, \eta_0) = (0, 0, 8, 1, 4)$$
$$\text{case II} = (a, \nu_0, \delta_0, \lambda_0, \eta_0) = (0.5, 0.5, 8, 2, 4)$$
$$\text{case III} = (a, \nu_0, \delta_0, \lambda_0, \eta_0) = (1, 1, 12, 1, 4)$$
$$\text{case IV} = (a, \nu_0, \delta_0, \lambda_0, \eta_0) = (2, 2, 12, 2, 4).$$

case	(n, r)	$\widehat{\sigma}^n$	$\widetilde{\sigma}$	$\widehat{\sigma}^s$	$\widehat{\sigma}^c$
I	(10,5)	2.432162	1.075501	12.480439	3.087871
		(.871949)	(-.569765)	(2.531909)	(1.448614)
	(30,15)	.359441	.288456	1.021725	.461598
		(.204421)	(-.222558)	(.489808)	(.432914)
	(30,25)	.203435	.177091	.522778	.239290
		(.119459)	(-.130018)	(.316036)	(.256103)
	(30,30)	.171158	.148541	.400588	.197840
		(.108715)	(-.098480)	(.210158)	(.222018)
II	(10,5)	1.585324	1.116305	6.607041	1.079067
		(.549002)	(-.725844)	(1.472144)	(.572747)
	(30,15)	.390965	.332927	.826203	.355562
		(.171560)	(-.244102)	(.270718)	(.229287)
	(30,25)	.200829	.179512	.473749	.191807
		(.108020)	(-.138087)	(.214401)	(.144936)
	(30,30)	.160467	.152746	.349902	.153271
		(.071710)	(-.131333)	(.136720)	(.104001)
III	(10,5)	1.262346	1.257724	3.597186	2.331396
		(.301508)	(-.838665)	(.801220)	(1.319298)
	(30,15)	.381618	.363155	.805747	.543507
		(.113239)	(-.288241)	(.238091)	(.494942)
	(30,25)	.187870	.184279	.436326	.253280
		(.070727)	(-.170047)	(.170560)	(.302542)
	(30,30)	.164314	.164876	.356249	.204669
		(.047102)	(-.152663)	(.122768)	(.241469)
IV	(10,5)	.932333	1.471060	2.019283	.566176
		(-.065052)	(-1.005863)	(-.075092)	(.368302)
	(30,15)	.319834	.378682	.618257	.278951
		(.010541)	(-.365609)	(.026333)	(.184221)
	(30,25)	.183062	.207304	.352227	.168840
		(.004583)	(-.226485)	(-.003421)	(.112218)
	(30,30)	.157468	.173719	.295025	.146843
		(.004315)	(-.189473)	(.008471)	(.094814)

Table 3. MSE's and Biases(in the parenthesis) of the Proposed Estimators $\widehat{R}_t^n$, $\widetilde{R}_t$, $\widehat{R}_t^s$ and $\widehat{R}_t^c$ of R_t for $(n,r) = (30,30)$, $(\mu_1,\mu_2) = (2.0,1.0)$.

$a = \nu_0 = 0,\ \delta_0 = 8,\ \lambda_0 = 1$ and $\eta_0 = 4$

σ	t	R_t	$\widehat{R}_t^n$	$\widetilde{R}_t$	$\widehat{R}_t^s$	$\widehat{R}_t^c$
3	2.5	.846482	.001070 (.000748)	.001702 (.022556)	.001825 (−.013854)	.168750 (−.406231)
	3.0	.716531	.001284 (.000466)	.001633 (.012199)	.003288 (−.028476)	.118873 (−.342040)
	5.0	.367879	.002551 (.007662)	.002710 (−.001547)	.005702 (−.033943)	.028500 (−.164407)
	12.0	.035674	.000295 (.006461)	.000205 (−.001759)	.000400 (−.001722)	.000214 (−.010334)
5	2.5	.904837	.000945 (.001818)	.001546 (.027066)	.001329 (−.006676)	.095658 (−.294151)
	3.0	.818731	.000937 (−.000572)	.001460 (.019092)	.002189 (−.020100)	.080337 (−.280891)
	5.0	.548812	.001786 (.003520)	.002021 (.003864)	.005112 (−.037170)	.035828 (−.184548)
	12.0	.135335	.001303 (.008651)	.001211 (−.004862)	.002167 (−.017860)	.002137 (−.037486)

$a = \nu_0 = 0.5,\ \delta_0 = 8,\ \lambda_0 = 2$ and $\eta_0 = 4$

σ	t	R_t	$\widehat{R}_t^n$	$\widetilde{R}_t$	$\widehat{R}_t^s$	$\widehat{R}_t^c$
3	2.5	.846482	.001149 (.003063)	.001938 (.026102)	.001795 (−.009855)	.176591 (−.415665)
	3.0	.716531	.001377 (−.000007)	.001761 (.013700)	.003070 (−.022565)	.126777 (−.353557)
	5.0	.367879	.002129 (.007427)	.002255 (.001390)	.005001 (−.025861)	.031117 (−.172681)
	12.0	.035674	.000336 (.006843)	.000240 (−.000370)	.000408 (.000449)	.000259 (−.012153)
5	2.5	.904837	.000664 (−.003198)	.001208 (.023416)	.001031 (−.011334)	.100969 (−.311457)
	3.0	.818731	.000993 (−.001673)	.001483 (.019128)	.001860 (−.017915)	.085633 (−.289270)
	5.0	.548812	.001972 (.001552)	.002234 (.004643)	.004543 (−.028893)	.038529 (−.190750)
	12.0	.135335	.001338 (.009555)	.001236 (−.001635)	.001992 (−.012017)	.002423 (−.041063)

(continued)

$a = \nu_0 = 1,\ \delta_0 = 12,\ \lambda_0 = 1$ and $\eta_0 = 4$

σ	t	R_t	$\widehat{R}_t^n$	$\widetilde{R}_t$	$\widehat{R}_t^s$	$\widehat{R}_t^c$
3	2.5	.846482	.001045	.001706	.001590	.161257
			(.000169)	(.024035)	(–.009156)	(–.397377)
	3.0	.716531	.001347	.001647	.002877	.116245
			(–.003573)	(.011889)	(–.019578)	(–.338406)
	5.0	.367879	.002282	.002450	.004844	.026266
			(.006179)	(.003279)	(–.020126)	(–.157949)
	12.0	.035674	.000279	.000222	.000422	.000200
			(.004750)	(–.001293)	(.000369)	(–.009285)
5	2.5	.904837	.001026	.001574	.001284	.095156
			(.000406)	(.026617)	(–.004865)	(–.292881)
	3.0	.818731	.001151	.001646	.001723	.082666
			(–.002739)	(.019268)	(–.012765)	(–.284142)
	5.0	.548812	.001952	.002207	.004301	.035603
			(–.000254)	(.005558)	(–.021573)	(–.183891)
	12.0	.135335	.001119	.001120	.002092	.002149
			(.004331)	(–.004545)	(–.007311)	(–.038969)

$a = \nu_0 = 2,\ \delta_0 = 12,\ \lambda_0 = 2$ and $\eta_0 = 4$

σ	t	R_t	$\widehat{R}_t^n$	$\widetilde{R}_t$	$\widehat{R}_t^s$	$\widehat{R}_t^c$
3	2.5	.846482	.001128	.001733	.001483	.177812
			(–.002271)	(.023511)	(–.005791)	(–.417227)
	3.0	.716531	.001412	.001817	.002497	.125907
			(–.002985)	(.016421)	(–.009450)	(–.352388)
	5.0	.367879	.002577	.002746	.004712	.033407
			(–.008986)	(–.006234)	(–.016440)	(–.178945)
	12.0	.035674	.000268	.000242	.000535	.000280
			(.002736)	(–.001192)	(.005921)	(–.013462)
5	2.5	.904837	.001162	.001620	.001324	.102707
			(–.000601)	(.026335)	(–.002998)	(–.301877)
	3.0	.818731	.001332	.002025	.001795	.089614
			(–.000035)	(.024543)	(–.005267)	(–.295012)
	5.0	.548812	.001942	.002136	.004056	.041098
			(–.005350)	(.005777)	(–.015186)	(–.198191)
	12.0	.135335	.001178	.001235	.002240	.002940
			(.000264)	(–.004080)	(.001835)	(–.047697)

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