

# Knowledge Acquisition on Scheduling Heuristics Selection Using Dempster-Shafer Theory(DST)

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**Dempster-Shafer Theory를 이용한 스케줄링 휴리스틱선정 지식습득**

## ABSTRACT

Most of solution methods in scheduling attempt to generate good solutions by either developing algorithms or heuristic rules. However, scheduling problems in the real world require considering more factors such as multiple objectives, different combinations of heuristic rules due to problem characteristics. In this respect, the traditional mathematical approach showed limited performance so that new approaches need to be developed. Expert system is one of them. When an expert system is developed for scheduling one of the most difficult processes faced could be knowledge acquisition on scheduling heuristics.

In this paper we propose a method for the acquisition of knowledge on the selection of scheduling heuristics using Dempster-Shafer Theory(DST). We also show the examples in the multi-objectives environment.

Key Words : knowledge acquisition, job shop scheduling, Dempster-Shafer Theory (DST), multi-objectives scheduling, statistical reasoning

## 1. Introduction

Scheduling problem involves allocating resources over time to perform a set of tasks(Baker, 1974), and it is also a decision process that arranges the necessary activities and controls resources to achieve a goal(Han, 1991). Scheduling has been applied to various domains such as job shop scheduling, flexible manufacturing

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systems(FMS) scheduling, robot scheduling, personnel scheduling, and so on.

Scheduling, in general, requires solving a problem under various constraints such as limited resources and precedence relationships. Therefore, scheduling cannot be performed without taking such constraints into account and the use of those constraints guides the schedule generator by reducing the search space.

Subject to both the precedence constraints and the resource constraints, the schedule decision is to determine when each operation should start to optimize the given objectives. There can be many kinds of objectives in scheduling depending upon the managerial perspectives. Rinnooy Kan(1976) found 26 different objectives and classified them into 6 objectives as follows :

- ① Minimization of the makespan(or project duration)
- ② Minimization of the sum of completion times
- ③ Minimization of the weighted sum of completion times
- ④ Minimization of the sum of late times
- ⑤ Minimization of the weighted sum of late times
- ⑥ Minimization of the maximum late time

Since the late 1950s, many different scheduling approaches have been applied to resource-constrained scheduling problems, including integer programming (Wagner, 1959), dynamic programming (Held and Karp, 1962), branch and bound methods (Carlier and Pinson, 1989), PERT/CPM techniques (Wiest, 1969), and various heuristic procedures (Adams *et al.*, 1988 ; Davis and Patterson, 1975).

Numerous researches have been reported to be successful for the problems of single objective and moderate size, and some heuristic methods attempted to tackle more realistic problems with an acceptable computational time limit. However, these approaches are limitedly applied only to those problems that are constrained by a single and unchanging objective. In those problems resource conflicts are usually resolved by applying a single heuristic rule throughout the whole scheduling horizon.

Recently, new approaches such as simulated annealing(Laarhoven *et al.*, 1988) and expert systems(Kusiak, 1990) have been applied to resource-constrained scheduling problems. Particularly the expert system provides a completely different way of solving the problems, compared to analytical ones, in the sense that it utilizes knowledge of human expertise that can't be quantified. In scheduling problems, knowledge of human expertise is related to the way to sequence the activities according to certain rules. When an expert system is developed, however, the most critical issue is how to acquire the scheduling knowledge and describe it systematically in the system.

In this paper we suggest a method for the selection of heuristic rule(s) if multi-objectives are to be considered like real world problems. The problem we choose is tardiness-oriented job shop scheduling and dispatching

heuristic rules are taken from several papers such as Weeks(1979) and Baker and Bertrand(1982). The performances of each rule are analyzed for the problems generated artificially. And then the most appropriate heuristic rule for the problems with multi-objectives is selected using Dempster-Shafer Theory(DST).

This paper is organized as follows. In the following section, the heuristic rules employed in this paper are described. The section 3 explains how to acquire the most basic knowledge on the performance behavior of each rule using simulation. The section 4 shows how to select the most promising rule if multiple objectives are considered simultaneously. The section 5 presents conclusions.

## 2. Tardiness-Oriented Rules For Job Shop Scheduling

In a job shop problem,  $n$  jobs have to be processed on  $m$  machines assuming that : ① a machine can process only one job at a time, ② the processing of a job on a machine is called an operation, ③ an operation cannot be interrupted, ④ a job consists of at most  $m$  operations, ⑤ the processing order of a job is given according to this job, ⑥ the operation sequence on the machines are unknown and have to be determined in order to optimize given performance measure(s) (Carlier and Pinson, 1989).

In solving those scheduling problems a number of heuristic rules could be employed. Among them, those to be used in this paper are to be briefly introduced as follows.

### 2.1 Shortest Processing Time (SPT)

SPT rule is one of the most popular rules for any performance measures. It has been employed for various objectives and showed excellent performances in various problems. SPT sequences are made by assigning all jobs in the order of nondecreasing processing time as follows :

$$p_{[1]} \leq p_{[2]} \leq \dots \leq p_{[n]}$$

where  $p_{[i]}$  is the processing time of the job that is processed  $i^{\text{th}}$

### 2.2 Earliest Due Date (EDD)

EDD rule is another rule that is used often for the due date-oriented scheduling. EDD sequences are made by assigning all jobs in the order of nondecreasing due dates. Since due dates for individual operations

are usually unknown in the general job shop scheduling problems, they need to be computed from the due date of each job. As an extension of EDD rule for single machine scheduling (Han *et al.*, 1995), the due date of an individual operation can be computed as

$$d_{ij} = \begin{cases} d_j, & \text{if operation } i \text{ is the last operation in the job } j. \\ d_{jk} - p_{jk}, & \text{otherwise.} \end{cases}$$

where, operation  $k$  is the direct successor of operation  $i$  of job  $j$ ,  $p_{jk}$  is the processing time of operation  $k$  of job  $j$ ,  $d_j$  is the due date of job  $j$ , and  $d_{jk}$  and  $d_i$  are due dates for operation  $k$  and  $i$  of job  $j$ , respectively.

### 2.3 Job-based Modified Due Date (MDD)

At time  $t$ , modified due date of an operation  $i$  of job  $j$  is computed in the following way :

$$d_{ji} = \text{Max} \left\{ d_j, t + \sum_{k=1}^m p_{jk} \right\}$$

where,  $d_j$  is the due date of job  $j$ ,  $m$  is the number of operations in job  $j$ , and  $p_{jk}$  is the processing time of operation  $k$  of job  $j$ .

MDD sequences are made by assigning all jobs in the order of nondecreasing modified due dates (Baker and Bertrand, 1982).

### 2.4 Operation-based Modified Due Date (MOD)

MOD is computed in the similar way as MDD rule except that MOD takes into account each operation's processing time such that

$$d_{ji} = \text{Max} \left\{ \sum_{k=1}^i p_{jk} * \left( \frac{d_j}{P_j} \right) t + p_{ji} \right\}$$

where,  $P_j$  is the sum of the processing time of all operation for job  $j$ .

### 2.5 New Operation-based Modified Due Date (NDD)

Another modified due dates (NDD) can be obtained as follows (Hwang, 1995) :

$$d_{ji} = \begin{cases} d_j, & \text{if operation } i \text{ is the last operation in the job } j, \\ \text{Max}(d_{jk} - p_{jk}, t + p_{jk}), & \text{otherwise.} \end{cases}$$

where, operation  $k$  is the direct successor of operation  $i$  of job  $j$ ,  $p_{jk}$  is the processing time of operation  $k$  of job  $j$ ,  $d_j$  is the due date of job  $j$ , and  $d_{jk}$  and  $d_{ji}$  are due dates for operation  $k$  and  $i$  of job  $j$ , respectively.

### 3. Comparisons of Each Rule

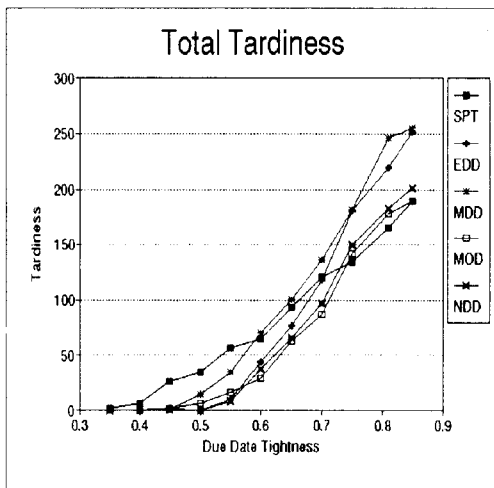
#### 3.1 Computational Experiences

Computational comparisons were conducted to acquire knowledge on the performance behavior of each rule over a range of due date tightness, that is, the ratio of the total processing time to the due date. A simulation program was developed in ART-IM, one of expert system development tools, and run on an IBM PC/486.

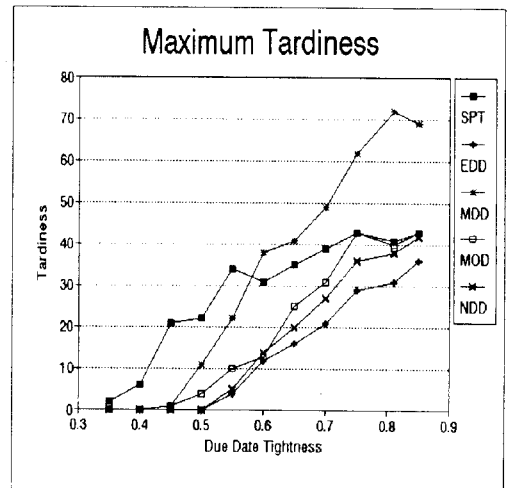
The experimental design for the simulation was followed from Posner's(Posner, 1985). The individual operation's processing time( $p_{jk}$ ) is generated from a discrete uniform distribution over  $[1, 10]$ . Due dates for each job are generated using two number  $\alpha$  and  $\beta$  with  $\alpha \in (a_1, a_2, a_3)$  and  $\beta \in (b_1, b_2, b_3)$ , where  $a_i$  and  $b_i$  are non-negative constants and from a discrete uniform distribution over integer interval of  $[\alpha * \sum p_{jk}, \beta * \sum p_{jk}]$ . Consequently,  $a_i$  and  $b_i$  determine the due date tightness for each problem.

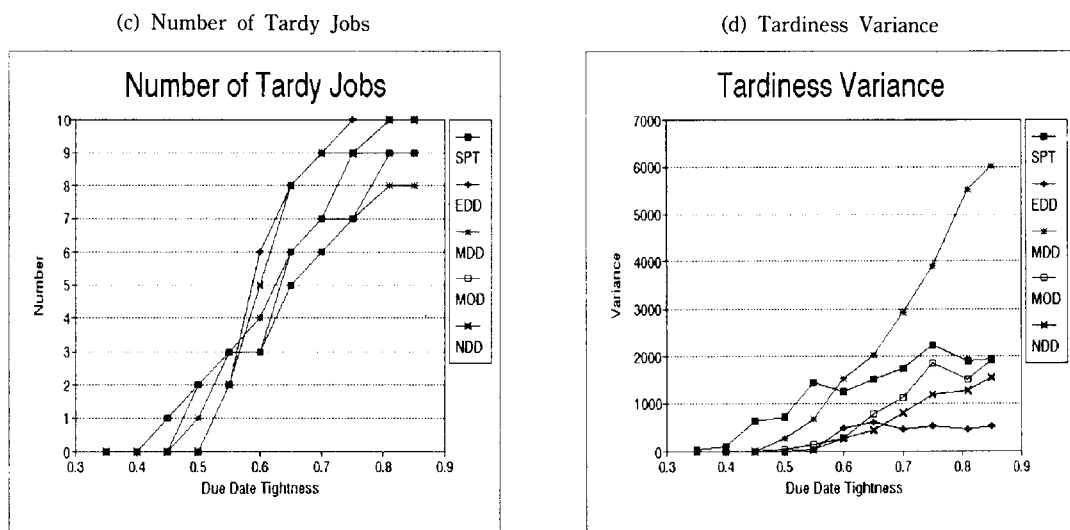
For each rule, 150 problems, sized of 10 jobs and 10 machines, were simulated. <Figure 1> shows the simulation results.

(a) Total Tardiness



(b) Maximum Tardiness





<Figure 1> Comparisons of Each Heuristic

From the simulation results, followings are observed :

- ① Operation-based modified due date(MOD) usually generates better solution than job-based modified due date(MDD) except for the problem of minimizing the number of tardy jobs.
- ② EDD and NDD outperform other rules in most of the problems.
- ③ None of the rules shows dominant performance behavior.
- ④ The best solution rule(s) depends upon scheduling objectives and due date tightness.

Among the results, last two give us a following implication: since each heuristic rule performs differently depending upon scheduling objectives as well as due date tightness, it could be possible to single out a rule which outperforms the others as long as the information on the characteristics of a problem is known.

### 3.2 Summary of the Simulation Results

To get a more precise knowledge on the performance of each rule, solution behavior is further analyzed. First, in order to see the general performance behavior of each rule from a simple point of view, we classified the behavior patterns into three interval categories over which performance behaves differently from those of the other intervals as follows :

- category 1 named as 'Lower' level if due date tightness is less than 0.58,

- category 2 named as 'Medium' level if due date tightness is between 0.58 and 0.75, and
- category 3 named as 'High' level if due date tightness is greater than 0.75.

The points, 0.58 and 0.75, are intentionally selected by observing the cross points of each rule's performance over the range of due date tightness. Of course, the classification could be refined further if more simulations are proceeded or other information is given.

Second, performances of each rule in the three interval categories are summarized as in <Table 1>.

<Table 1> Percentage of Best Solution Found

(a) Total Tardiness (or Mean Tardiness)

Rules	Lower	Medium	High	Mean
EDD	91.6	20.6	3.2	38.5
SPT	38.8	2.9	61.3	34.3
MDD	64.7	2.9	0.0	22.5
MOD	68.2	50.0	29.0	49.1
NDD	92.9	26.5	12.9	44.1
MAX	92.9	50.0	61.3	68.1

(b) Maximum Tardiness (or Tardiness Variance)

Rules	Lower	Medium	High	Mean
EDD	95.3	58.8	64.5	72.9
SPT	35.3	2.9	0.0	12.7
MDD	64.7	50.0	12.9	42.5
MOD	68.2	2.9	12.9	28.0
NDD	94.1	29.4	22.6	48.7
MAX	95.3	58.8	64.5	72.9

(c) Number of Tardy Jobs

Rules	Lower	Medium	High	Mean
EDD	89.4	2.9	0.0	30.8
SPT	38.8	82.4	41.9	54.4
MDD	70.1	26.5	87.1	61.2
MOD	70.1	32.4	3.2	35.2
NDD	91.8	5.9	6.56	34.7
MAX	91.8	82.4	87.1	87.1

Then the percentage of best solutions for each rule generated as in <Table 1> is rephrased into the literal term of 'goodness' as in <Table 2>. Performance of each rule is expressed as 'BEST', 'GOOD', 'BAD' and 'WORST' according to their percentage of best solutions generated.

Applying the best rules for each due date tightness level, the percentage of the best solution found can be dramatically increased. For example, in total tardiness problem, MOD seems to be the best rule since 49.1% of the solutions it generated turned out to be the best solutions. However, if we could employ different heuristic rule in the different due date tightness level such as NDD for the 'LOWER' level, MOD for the 'Medium' level, and SPT for the 'High' level, the percentage of the best solutions generated can increase to 68.1%.

In the case that the percentage of the best solutions generated for each rule is unknown, possibility for <Table 2> Classifications of the Performances of Each Rule

(a) Total Tardiness (or Mean Tardiness)

Rules	Lower	Medium	High
EDD	BEST	GOOD	BAD
SPT	WORST	BAD	BEST
MDD	BAD	WORST	WORST
MOD	GOOD	BEST	GOOD
NDD	BEST	BEST	GOOD

(b) Maximum Tardiness (or Tardiness Variance)

Rules	Lower	Medium	High
EDD	BEST	BEST	BEST
SPT	WORST	BAD	BEST
MDD	BAD	WORST	WORST
MOD	GOOD	GOOD	BAD
NDD	BEST	GOOD	GOOD

(c) Number of Tardy Jobs

Rules	Lower	Medium	High
EDD	BEST	WORST	WORST
SPT	WORST	BEST	GOOD
MDD	GOOD	GOOD	BEST
MOD	GOOD	WORST	GOOD
NDD	BEST	BAD	WORST



the best rule as in (Table 2), rather than probability, could be obtained through interviewing experts, analyzing former studies, and so on.

Consequently, if there is only one objective we can easily generate the better schedules by classifying a given problem to one of those three due date tightness levels and applying the best rule selected for the category. However, if there are multi-objectives, we must proceed a little more complicated process. In the following section, a framework for finding the best rule in the job shop scheduling problems with multi-objectives will be discussed.

## 4. Statistical Reasoning for Multi-Objectives

In cases of the problems with multiple objectives, one of possible methods of selecting the best rule would be as follows :

- select the rule which have

$$Max_k \left\{ \sum_{i=1}^n w_i * p_{ki} \right\}$$

where,  $w_i$  is the weight on objective  $i$ ,  $p_{ki}$  is the percentage of the best solutions found for the heuristic rule  $k$  and the objective  $i$ .

However, it is not always possible to have the  $p_{ki}$ 's and also very likely to be controversial to determine the value of each  $w_i$ . Thus, to overcome such a limitation, statistical reasoning methods could be applied. There are several statistical reasoning methods : Bayesian network, certainty factor, Dempster-Shafer Theory (DST), fuzzy logic, and so on(Rich and Knight, 1991).

Here we decided to use DST because

- DST could easily manipulate propositional value such as 'BEST', 'GOOD', 'BAD', 'WORST', and further even 'Unknown',
- Computational procedure is simple and easy to understand, and is based on the theory of belief.

Even though DST has the problem of combinatorial explosion, it does not seem to cause serious problem because there are not so many candidate rules in most of real world problems. We show how to find the best rule using two examples in the following.

#### 4.1 An Example of Equally Weighted Multi-Objectives

Suppose that a problem has high due date tightness with objectives of minimizing the total tardiness and the number of tardy jobs. Among the several heuristic rules, the most promising rule can be selected by the following procedure (for more reference on DST see (Rich and Knight, 1991)) :

- (1) Assign a belief value  $b(h,l,r)$  for each heuristic rule and tightness level as follows :

$$b(h,l,r) = \begin{cases} 4, & \text{if } r = \text{BEST}, \\ 3, & \text{if } r = \text{GOOD}, \\ 2, & \text{if } r = \text{BAD}, \\ 1, & \text{if } r = \text{WORST}, \end{cases}$$

where,  $h$  is a heuristic,  $l$  is the level of tightness, and  $r$  is the performance. For each cell marked 'unknown', assign average value (In the above case, the average would be 2.5).

- (2) Compute belief masses from <Table 2 (a)> as follows

$$\begin{aligned} m_1(\text{SPT}) &= \frac{b(\text{SPT}, \text{high}, \text{BEST})}{b(\text{EDD}, \text{high}, \text{BAD}) + b(\text{SPT}, \text{high}, \text{BEST}) + b(\text{MDD}, \text{high}, \text{WORST}) + b(\text{MOD}, \text{high}, \text{GOOD}) + b(\text{NDD}, \text{high}, \text{GOOD})} \\ &= 4/(2+4+1+3+3) = 4/13 \\ m_1(\text{MOD}) &= 3/(2+4+1+3+3) = 3/13 \\ m_1(\text{NDD}) &= 3/(2+4+1+3+3) = 3/13 \\ m_1(\Theta) &= 3/(2+4+1+3+3) = 3/13 \end{aligned}$$

Here, the BAD and the WORST heuristic rules such as EDD and MDD were ignored for the simplicity of computation. The set  $\Theta$  consists of {EDD, SPT, MDD, MOD, NDD}. Thus the fact that the best rule set is  $\Theta$  means that the best rule would be one of those five rules, which means again that we do not have knowledge on the best rule.

- (3) Compute belief masses from <Table 2 (c)> as follows

$$m_2(\text{MDD}) = \frac{b(\text{MDD}, \text{high}, \text{BEST})}{b(\text{EDD}, \text{high}, \text{WORST}) + b(\text{SPT}, \text{high}, \text{GOOD}) + b(\text{MDD}, \text{high}, \text{BEST}) + b(\text{MOD}, \text{high}, \text{GOOD}) + b(\text{NDD}, \text{high}, \text{WORST})}$$

$$\begin{aligned}
 &= 4/(1+3+4+3+1) = 4/12 \\
 m_2(\text{SPT}) &= 3/(1+3+4+3+1) = 3/12 \\
 m_2(\text{MOD}) &= 3/(1+3+4+3+1) = 3/12 \\
 m_2(\Theta) &= 2/(1+3+4+3+1) = 2/12
 \end{aligned}$$

Again, the BAD and the WORST heuristic rules such as EDD and NDD were ignored for the simplicity of computation.

(4) Combining Evidences (Dempster's rule of combination)

To calculate the combinational possibility of being best rule with two equally weighted objectives, combine two evidences such as shown in <Table 3>.

<Table 3> Combining Evidences for the Problem of Equally Weighted

	$m_2(\{\text{MDD}\})$ = 4/12	$m_2(\{\text{SPT}\})$ = 3/12	$m_2(\{\text{MOD}\})$ = 3/12	$m_2(\Theta)$ = 2/12
$m_1(\{\text{SPT}\})$ = 4/13	$\Phi$ 16/156	{SPT} 12/156	$\Phi$ 12/156	{SPT} 8/156
$m_1(\{\text{MOD}\})$ = 3/13	$\Phi$ 12/156	$\Phi$ 9/156	{MOD} 9/156	{MOD} 6/156
$m_1(\{\text{NDD}\})$ = 3/13	$\Phi$ 12/156	$\Phi$ 9/156	$\Phi$ 9/156	{NDD} 6/156
$m_1(\Theta)$ = 3/13	{MDD} 12/156	{SPT} 9/156	{MOD} 9/156	$\Phi$ 6/156

$$\begin{aligned}
 m_1 \boxtimes m_2(\{\text{SPT}\}) &= \Sigma m_1(\{\text{SPT}\}) \cdot m_2(\{\text{SPT}\}) \\
 &= (12+9+8) / 156 = 29/156 \\
 m_1 \boxtimes m_2(\{\text{MOD}\}) &= (9+9+6) / 156 = 24/156 \\
 m_1 \boxtimes m_2(\{\text{MDD}\}) &= 12/156 \\
 m_1 \boxtimes m_2(\{\text{NDD}\}) &= 6/156 \\
 m_1 \boxtimes m_2(\Theta) &= 6/156 \\
 m_1 \boxtimes m_2(\cup) &= (29+24+12+6+6)/156 = 77/156
 \end{aligned}$$

Where,  $m_i \boxtimes m_j(Z) = \sum_{X \cap Y = Z} m_i(X) \cdot m_j(Y)$  and  $\cup$  is the union of the sets resulting from combining two evidences due to two different objectives.

## (5) Normalization

Since the resulting belief masses do not sum to one, normalization is required as follows.

$$m_1 \sqcap m_2(\{SPT\}) = 29/77$$

$$m_1 \sqcap m_2(\{MOD\}) = 24/77$$

$$m_1 \sqcap m_2(\{MDD\}) = 12/77$$

$$m_1 \sqcap m_2(\{NDD\}) = 6/77$$

$$m_1 \sqcap m_2(\Theta) = 6/77$$

(6) Repeat from (3) to (4) until all of the objectives are considered.

(7) Select a rule

From the above computation results, we select SPT rule in the problems of high due date tightness if two objectives, minimizing the total tardiness and the number of tardy jobs, are considered with equal weights.

## 4.2 An Example of Unequally Weighted Two Objectives

For the problem with multi-objectives of unequal weights, weight for each objective should be expressed in numerical values. Suppose that a manager considers the importance of the total tardiness and the number of tardy jobs in the job shop scheduling are 50% and 100%, respectively (there is no necessity that the total importance becomes 100%). Then, all the belief masses could be computed as follows.

$$m_1(SPT) = \frac{b(SPT, high, BEST) * w_1}{b(EDD, high, BAD) + b(SPT, high, BEST) + b(MDD, high, WORST) + b(MOD, high, GOOD) + b(NDD, high, GOOD)}$$

$$= 4/(2+4+1+3+3)*0.5 = 2.0/13$$

$$m_1(MOD) = 3/(2+4+1+3+3)*0.5 = 1.5/13$$

$$m_1(NDD) = 3/(2+4+1+3+3)*0.5 = 1.5/13$$

$$m_1(\Theta) = 3/(2+4+1+3+3)*0.5 = 1.5/13$$

$$m_2(MDD) = \frac{b(MDD, high, BEST)*w_2}{b(EDD, high, WORST) + b(SPT, high, GOOD) + b(MDD, high, BEST) + b(MOD, high, GOOD) + b(NDD, high, WORST)}$$

$$4/(1+3+4+3+1)*1.0 = 4.0/12$$

$$\begin{aligned}
 m_2(SPT) &= 3/(1+3+4+3+1)*1.0=3.0/12 \\
 m_2(MOD) &= 3/(1+3+4+3+1)*1.0=3.0/12 \\
 m_2(\Theta) &= 2/(1+3+4+3+1)*1.0=2.0/12
 \end{aligned}$$

Where,  $m_1(\Theta)$  and  $m_2(\Theta)$  are set by the value of the sum of all unassigned masses. From the recomputed values, we can combine evidences as shown in following <Table 4>.

<Table 4> Combining Evidences for the Problem of Unequally Weighted

	$m_2(\{MDD\})$ =4.0/12	$m_2(\{SPT\})$ =3.0/12	$m_2(\{MOD\})$ =3.0/12	$m_2(\Theta)$ =2.0/12
$m_1(\{SPT\})$ 4/13	$\Phi$ 8.0/156	$\{SPT\}$ 6.0/156	$\Phi$ 6.0/156	$\{SPT\}$ 4.0/156
$m_1(\{MOD\})$ 3/13	$\Phi$ 6.0/156	$\Phi$ 4.5/156	$\{MOD\}$ 4.5/156	$\{MOD\}$ 3.0/156
$m_1(\{NDD\})$ 3/13	$\Phi$ 6.0/156	$\Phi$ 4.5/156	$\Phi$ 4.5/156	$\{NDD\}$ 3.0/156
$m_1(\Theta)$ =3/13	$\{MDD\}$ 32.0/156	$\{SPT\}$ 24.0/156	$\{MOD\}$ 24.0/156	$\Phi$ 16.0/156

$$\begin{aligned}
 m_1 \sqcap m_2(\{SPT\}) &= \Sigma m_1(\{SPT\}) \cdot m_2(\{SPT\}) \\
 &= (6.0 + 24.0 + 4.0)/156 = 34.0/156 \\
 m_1 \sqcap m_2(\{MOD\}) &= (4.5 + 24.0 + 3.0)/156 = 27.5/156 \\
 m_1 \sqcap m_2(\{MDD\}) &= 32.0/156 \\
 m_1 \sqcap m_2(\{NDD\}) &= 3.0/156 \\
 m_1 \sqcap m_2(\Theta) &= 16.0/156 \\
 m_1 \sqcap m_2(\cup) &= (34.0 + 27.5 + 32.0 + 3.0 + 16.0)/156 = 112.5/156
 \end{aligned}$$

Where,  $m_i \sqcap m_j(Z) = \sum_{X \cap Y = Z} m_i(X) \cdot m_j(Y)$  and  $\cup$  is the union of the sets resulting from combining two evidences due to two different objectives.

Since the resulting belief masses do not sum to one, normalization is required as follows.

$$\begin{aligned}
 m_1 \sqcap m_2(\{SPT\}) &= 34.0/112.5 \\
 m_1 \sqcap m_2(\{MOD\}) &= 27.5/112.5 \\
 m_1 \sqcap m_2(\{MDD\}) &= 32.0/112.5
 \end{aligned}$$

$$m_1 \boxtimes m_2(\{NDD\}) = 3.0/112.5$$

$$m_1 \boxtimes m_2(\Theta) = 16.0/112.5$$

From the above results, we can find that SPT rule is still the best, and MDD the second.

## 5. Conclusions

Since late 1950s, many researches have addressed resource-constrained scheduling problems from different perspectives, and employed various heuristics and algorithms. Since most of the heuristics and algorithms were developed for single objective problems, none of these solution methods would be appropriate if they have multiple objectives.

Furthermore, in the real world problems that need to consider more constraints and have unquantifiable scheduling knowledge, traditional analytic methods could only be very limitedly applied. Expert system or knowledge-based system can be considered as an alternative in many studies since they can utilize unquantifiable scheduling knowledge which describes the complex scheduling situation such as multiple objectives, multiple heuristics, and so on.

In this paper we suggest that Dempster-Shafer Theory could be applied to acquiring knowledge on selecting the most appropriate heuristic rule when a scheduling expert system is developed. In addition, we show the applicability of DST by walking through the computational procedure of two examples: equally weighted case and unequally weighted one.

Even though DST might have the problem of combinatorial explosion, DST turned out be effective in considering multiple objectives in scheduling because

- DST could easily manipulate propositional value such as 'BEST', 'GOOD', 'BAD', 'WORST', and even 'Unknown', which would be often the case of knowledge expressed by human experts,
- Computational procedure is simple and easy to understand, and
- DST is based on the theory of belief so that unavoidable inexact reasoning process could be supported theoretically.

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