# 강성저하 실험식 및 연성계수를 이용한 철근콘크리트 전단벽 구조시스템의 비탄성 하중-변위 관계식 예측

# Prediction of Inelastic Force-Displacement Relationships of Reinforced Concrete Shear Wall Systems Based on Prescribed Ductilities

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## 요 약

한 cycle의 이력곡선 loop을 완전히 표현하기 위해서는 pinch force, drift offset, effective stiffness, unloading, reloading, tangential stiffness 등의 변수가 필요하게 된다. 각 이력 loop에 대해 이들 변수들은 에너지 소산정도에 따라 변위와 축력의 함수로 표현될 수 있다. 본 논문에서는 먼저 16개의 전단벽 실험에서 얻어진 이력곡선 데이타를 분석하여 앞에 기술된 모든변수를 표준화된 변위($\Delta/\Delta y$)의 함수로 표현했으며 이를 바탕으로 이력곡선의 포락선으로 표현되는 힘-변위관계를 예측할 수 있는 6개의 step을 제시하였다. 제시된 기법으로 구해진 비탄성 힘-변위관계는 실험곡선과 비교되었으며 내진설계에 있어서 가장 중요한 요소중 하나인 구조물의 비탄성 힘-변위관계를 예측하는 편리한 기법으로 이용될 수 있음을 보였다.

## Abstract

The parameters describing a complete hysteresis loop include pinch force, drift offset, effective stiffness, unloading and reloading trangential stiffness. Analytical equations proposed to quantify the non-linear, inelastic behavior of reinforced shear walls can be used to predict these parameters as a function of axial load and drift rato. For example, drift offset, effective stiffness, and first and second unloading and reloading tangential stiffness are calculated using equations obtained from test data for a desired drift ratio or ductility level. Pinch force can also be estimated for a given drift ratio and axial load. The effective virgin stiffness at the first yield and its post yield reduction can be estimated. The load deflection response of flexural reinforced concrete shear walls can now be estimated based on the effective wall stiffness that is a function of axial force and drift ratio.

## 1. GENERAL PERFORMANCE OF REINFORCED CONCRETE SHEAR WALLS

### 1.1 General

A total of sixteen reinforced concrete shear wall specimens has been tested. As shown in Figure 1.1, the specimens were 6ft.(183cm) high and 6ft.(183cm)long and fabricated with a single wythe of 6×8×16(in) hollow concrete blocks. They were fully grouted with uni-

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이 논문에 대한 토론을 1996년 6월 31일까지 본 학회에 보내주시면 1996년 12월호에 그 결과를 게재하겠습니다.

formly distributed vertical and horizontal reinforcement. The horizontal reinforcement had 180 degree hooks around the extreme vertica steel. Each specimen had a reinforced concrete top beam and base slab. The vertical reinforcement ran continuously from the base slab to the top beam with 180 degree anchoring hooks. Bond-beam units were used througthout the wall panel to allow the placement of horizontal reinforecement and enhance the continuity of the grout. The construction of a typical specimen is shown in Figure 1.1 and the test setup is shown in Figure 1.2.

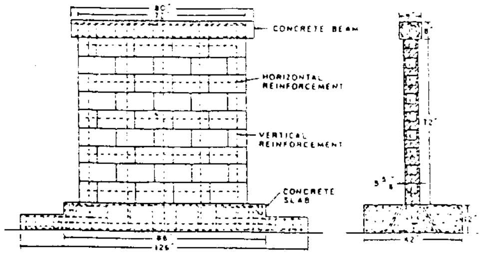

Figure 1.1 Test specimen

Table 1.1 Axial load and steel variables

| Wall | Axial Load(psi) | Vertical Steel | Horizontal Steel |
|---|---|---|---|
| 1 | 200 | 5×#5 | 5×#4 |
| 2 | 270 | 5×#5 | 5×#3 |
| 3 | 270 | 5×#7 | 5×#3 |
| 4 | 0 | 5×#7 | 5×#3 |
| 5 | 100 | 5×#7 | 5×#3 |
| 6 | 0 | 5×#5 | 5×#3 |
| 7 | 100 | 5×#7 | 5×#4 |
| 8 | 0 | 5×#5 | 5×#4 |
| 9 | 270 | 5×#5 | 5×#3 |
| 10 | 100 | 5×#5 | 5×#3 |
| 11 | 0 | 5×#7 | 5×#4 |
| 12 | 100 | 5×#5 | 5×#4 |
| 13 | 270 | 5×#6 | 5×#4 |
| 14 | 270 | 5×#6 | 5×#3 |
| 15 | 100 | 5×#6 | 5×#4 |
| 16 | 270 | 5×#7 | 5×#4 |

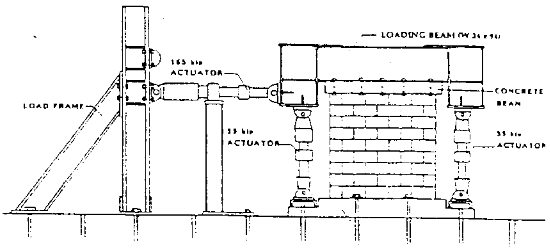

Figure 1.2 Test setup

The sixteen walls that were tested are listed in Table 1.1 with the information of the axial stress, the vertical and the horizontal reinforcement.

A structure must be designed to have the necessary ductility capacity to undergo the deformation required to prevent sudden and premature failure. It was shown by experimental and analytical research conducted in New Zealand by Priestley(1982) that walls designed without confinement of vertical steel, but with adequate shear reinforcement, can be expected to perform in a predictable manner with small but significant ductility.[1,2] Lateral load capacities at limit states characterizing the performance of shear walls were accurately calculated and experimentally verified in previous paper.[3]

The development of a force-displacement relationship that describes the overall response characteristics of shear walls also requires an analytical method for estimating deflections. the deflection estimation based on flexural deformation and shear deformatiion neglects the contribution of slipping deformation on the walls, resulting in underestimated deflection. The analytical approach suggested here will account for three factors including flexural deformation, shear deformation and slipping deformation on the base of the walls in estimating the total deflection of reinforced concrete shear walls with one to one aspect ratio. The research is to develop an analytical model for use in design pratice. This will be arrived at by predicting the inelastic behavior of flexural shear walls. This paper will thoroughly examine the performance of flexural shear walls in terms of the following parameters

(1) stiffness and strength degradation
(2) pinch force
(3) drift offset
(4) unloading tangential stiffness of hysteresis loops
(5) reloading tangential stiffness of hysteresis loops

### 1.2 Description of the Behavior of Reinforced Concrete Shear Walls

Each wall was subjected to a standard lateral displacement history under a constant axial load. The displacement amplitude of the first cycle of each loading sequence was identical to the maximum amplitude of the previous loading sequence. It was then followed three to four cycles of an increased amplitude and, finally, by three cycles of decaying amplitudes. Figure 1.3 shows thirteen parameters used to describe each hysteresis loop. Table 2 introduces the name and the definition of each parameter and Tables 1.3-(a) through (d) give their values by cycle for Walls #1, #2, #6 and #8. The performance of flexural shear walls in terms of these parameters is investigated for both virgi and stabilized cycles and compared.[4)]

The parameters of four flexural walls are analyzed and trends as a function of design parameters are investigated to obtain an analytical model that can quantify the inelastic and inplane characteristics of reinforced concrete shear walls. These behaviors of reinforced concrete shear walls examined in more depth in terms of the hysteretic characteristics of the wall. The degradation of stiffness and strength and the variation of both pinch force and drift offset as a function of displacement are investigated in detail. The influence of axial load, vertical and horizontal steel on the flexural capacities, the failure mechanisms, the ductilities and the energy dissipation capacities of shear walls are also studied in this section. Recommendations for the precise assessment of the inplane and inelastic response of typical flexural reinforced concrete shear walls are presented in the each subsection.

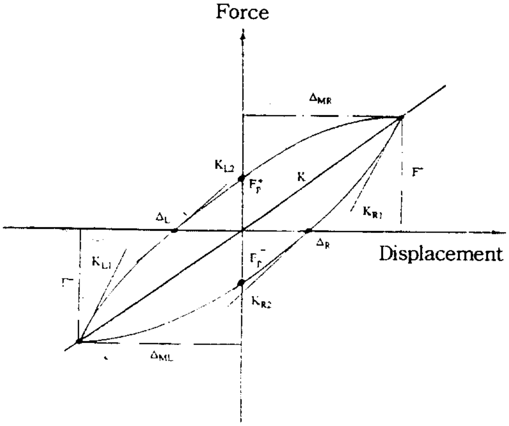

Figure 1.3 Parametric study parameters

Table 1.2 Description of hysteresis loop parameters

$F_P^+$ = Positive pinch force(positive Y-offset)
$F_P^-$ = Negative pinch force(negative Y-offset)
$\Delta_R$ = Positive drift offset(positive X-offset)
$\Delta_L$ = Negative drift offset(negative X-offset)
$F^+$ = Positive peak force
$F^-$ = Negative peak force
$\Delta_{MR}$ = Positive peak drift ratio
$\Delta_{ML}$ = Negative peak drift ratio
$K_{R1}$ = First unloading slope
$K_{R2}$ = Second unloading slope
$K_{L1}$ = First reloading slope
$K_{L2}$ = Second reloading slope

### 1.3 Reduction in Effective Stiffness

The effective stiffness, as the measure of structural degradation, is defined to be the maximum positive load to the maximum negative load secant stiffness. The equation for this is

$$K_{eff} = \frac{(F^{+} - F^{-})}{(\Delta_{MR} - \Delta_{ML})} \qquad (1)$$

In Figure 1.4 and 1.5 the reduction in the effective stiffness for virgin and stabilized cycles is depicted by showing the normalized effective stiffness plotted against the normalized drift ratio.[5)]

The normalized effective stiffness is defined to be the effective stiffness divided by the effective stiffness at the first yield of vertical steel. The normalized drift ratio is obtained by

**Table 1.3(a) Hysteresis parametric values by cycle for Wall #1**

| Cycle # | 6 | 10 | 12 | 17 | 19 | 25 | 26 |
|---|---|---|---|---|---|---|---|
| $F_P^{+}$(kips) | 6.8 | 24.6 | 29.3 | 31.3 | 20.2 | 11.3 | 11.6 |
| $F^{-}$(kips) | 60.9 | 81.5 | 74.6 | 86.7 | 77.0 | 62.3 | 59.5 |
| $\Delta_{MR}$(%) | 0.231 | 0.775 | 0.777 | 1.138 | 1.218 | 1.697 | 1.711 |
| $K_{R1}$(kip/in) | 553 | 244 | 237 | 327 | 200 | 126 | 122 |
| $\Delta_R$(%) | 0.044 | 0.182 | 0.098 | 0.251 | 0.163 | 0.086 | 0.043 |
| $K_{R2}$(kip/in) | 441 | 194 | 174 | 85 | 79 | 37 | 13 |
| $F_P^{-}$(kips) | −16.1 | −23.9 | −12.4 | −14.3 | −8.5 | −0.9 | −0.4 |
| $F^{-}$(kips) | −59.7 | −77.7 | −61.7 | −57.0 | −42.3 | −20.0 | −6.3 |
| $\Delta_{ML}$(%) | −0.205 | −0.769 | −0.857 | −1.226 | −1.200 | −1.733 | −1.549 |
| $K_{L1}$(kip/in) | 454 | 488 | 307 | 322 | 474 | 95 | 32 |
| $\Delta_L$(%) | 0.007 | −0.260 | −0.338 | −0.556 | −0.516 | −1.157 | −1.246 |
| $K_{L2}$(kip/in) | 384 | 193 | 166 | 89 | 72 | 25 | 17 |
| K(kip/in) | 384 | 143 | 122 | 84 | 69 | 33 | 28 |
| $K_{norm}$ | 1.00 | 0.37 | 0.32 | 0.22 | 0.18 | 0.09 | 0.07 |
| Comments | Yield | Virgin | Stblzd | Virgin | Stblzd | Virgin | Stblzd |

Normalized stiffness = stiffness/stiffness at yield = $K_{norm}$

Stiffness at yield = 384.5 kip/in

1 kip = 4.448 KN, 1 in = 2.54 cm

**Table 1.3(b) Hysteresis parametric values by cycle for Wall #2**

| Cycle # | 8 | 12 | 13 | 15 | 20 | 22 | 26 | 27 |
|---|---|---|---|---|---|---|---|---|
| $F_P^{+}$(kips) | 20.4 | 21.0 | 25.7 | 25.1 | 18.8 | 14.5 | 12.1 | 2.3 |
| $F^{-}$(kips) | 77.8 | 73.8 | 83.1 | 75.4 | 65.9 | 45.8 | 42.1 | 39.7 |
| $\Delta_{MR}$(%) | 0.320 | 0.289 | 0.690 | 0.770 | 1.123 | 1.262 | 1.620 | 1.914 |
| $K_{R1}$(kip/in) | 406 | 467 | 353 | 252 | 237 | 195 | 102 | 115 |
| $\Delta_R$(%) | 0.084 | 0.040 | 0.158 | 0.081 | 0.398 | 0.302 | 0.375 | 1.529 |
| $K_{R2}$(kip/in) | 487 | 423 | 255 | 142 | 50 | 36 | 30 | 19 |
| $F_P^{-}$(kips) | −33.1 | −14.4 | −29.1 | −13.7 | −26.5 | −15.0 | −12.2 | −26.4 |
| $F^{-}$(kips) | −68.4 | −77.7 | −97.6 | −87.4 | −83.1 | −69.4 | −62.3 | −60.0 |
| $\Delta_{ML}$(%) | −0.200 | −0.302 | −0.744 | −0.762 | −1.224 | −1.285 | −1.313 | −1.475 |
| $K_{L1}$(kip/in) | 697 | 590 | 378 | 318 | 266 | 169 | 159 | 153 |
| $\Delta_L$(%) | −0.045 | −0.063 | −0.062 | −0.175 | −0.178 | −0.260 | −0.095 | −0.257 |
| $K_{L2}$(kip/in) | 543 | 483 | 233 | 219 | 96 | 84 | 65 | 8 |
| K(kip/in) | 390 | 3456 | 175 | 8147 | 88 | 63 | 49 | 41 |
| $K_{norm}$ | 1.00 | 0.91 | 0.45 | 0.38 | 0.23 | 0.16 | 0.13 | 0.10 |
| Comments | Virgin&Yield | Stblzd | Virgin | Stblzd | Virgin | Stblzd | Virgin | Stblzd |

Normalized stiffness = stiffness/stiffness at yield = $K_{norm}$

Stiffness at yield = 390.5 kip/in

1 kip = 4.448 KN, 1 in = 2.54 cm

Table 1.3(c) Hysteresis parametric values by cycle for Wall #6

| Cycle # | 7 | 8 | 10 | 15 | 17 | 22 | 24 | 29 | 31 | 36 | 38 | 43 | 45 | 50 | 52 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $F_P^+$(kips) | 2.3 | 3.7 | 5.8 | 8.4 | 5.7 | 7.7 | 4.7 | 8.8 | 4.0 | 3.7 | 4.7 | 2.9 | 2.2 | 4.6 | 3.2 |
| $F^-$(kips) | 39.8 | 43.7 | 40.3 | 52.0 | 42.8 | 48.2 | 39.3 | 43.9 | 33.8 | 38.1 | 30.5 | 35.4 | 30.0 | 35.0 | 29.2 |
| $\Delta_{MR}$(%) | 0.166 | 0.240 | 0.245 | 0.560 | 0.592 | 0.916 | 0.946 | 1.264 | 1.298 | 1.631 | 1.667 | 1.993 | 2.012 | 2.374 | 2.342 |
| $K_{R1}$(kip/in) | 649 | 419 | 521 | 543 | 236 | 288 | 252 | 179 | 150 | 183 | 160 | 112 | 82 | 108 | 75 |
| $\Delta_R$(%) | 0.000 | 0.034 | 0.027 | 0.251 | 0.201 | 0.429 | 0.413 | 0.640 | 0.683 | 0.883 | 0.962 | 1.062 | 1.189 | 1.489 | 1.292 |
| $K_{R2}$(kip/in) | 358 | 202 | 74 | 78 | 35 | 25 | 15 | 14 | 20 | 7 | 18 | 4 | 13 | 3 | 7 |
| $F_P^-$(kips) | −1.40 | −4.90 | −1.50 | −14.7 | −3.90 | −8.6 | −4.1 | −6.8 | −3.8 | −5.3 | −3.3 | −4.4 | −2.7 | −3.3 | −2.8 |
| $F^-$(kips) | −37.9 | −42.1 | −41.4 | −47.9 | −43.5 | −40.7 | −39.9 | −44.7 | −44.1 | −44.0 | −40.1 | −43.1 | −39.2 | −41.4 | −33.7 |
| $\Delta_{ML}$(%) | −0.170 | −0.242 | −0.253 | −0.604 | −0.596 | −0.915 | −0.942 | −1.305 | −1.423 | −1.626 | −1.646 | −1.967 | −1.958 | −2.312 | −2.361 |
| $K_{L1}$(kip/in) | 556 | 439 | 415 | 327 | 370 | 214 | 211 | 152 | 150 | 140 | 130 | 135 | 104 | 130 | 123 |
| $\Delta_L$(%) | −0.00 | −0.049 | −0.047 | −0.257 | −0.227 | −0.444 | −0.396 | −0.635 | −0.657 | −0.858 | −0.868 | −1.016 | −1.095 | −1.261 | −1.471 |
| $K_{L2}$(kip/in) | 355 | 107 | 78 | 38 | 33 | 30 | 9 | 10 | 7 | 4 | 12 | 4 | 2 | 2 | 2 |
| K(kip/in) | 314 | 247 | 227 | 119 | 101 | 67 | 58 | 48 | 40 | 35 | 30 | 28 | 24 | 23 | 19 |
| $K_{norm}$ | 0.98 | 0.77 | 0.71 | 0.37 | 0.31 | 0.21 | 0.18 | 0.15 | 0.12 | 0.11 | 0.09 | 0.09 | 0.08 | 0.07 | 0.06 |
| Comments | Yield | Yirgin | Stblzd | Virgin | Stblzd | Virgin | Stblzd | Virgin | Stblzd | Virgin | Stblzd | Virgin | Stblzd | Virgin | Stblzd |

Normalized stiffness = stiffness / stiffness at yield = $K_{norm}$
Stiffness at yield = 321.0 kip/in
1 kip = 4.448 KN, 1 in = 2.54 cm

Table 1.3(d) Hysteresis parametric values by cycle for Wall #8

| Cycle # | 7 | 8 | 10 | 15 | 17 | 22 | 24 | 29 | 31 | 36 | 38 | 43 | 45 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $F_P^+$(kips) | 1.2 | 1.5 | 5.5 | 6.2 | 4.1 | 7.2 | 5.2 | 7.4 | 6.6 | 6.1 | 4.6 | 2.7 | 2.1 |
| $F^-$(kips) | 37.5 | 43.7 | 36.3 | 50.2 | 40.6 | 49.7 | 41.6 | 49.0 | 32.3 | 39.3 | 26.6 | 25.0 | 16.3 |
| $\Delta_{MR}$(%) | 0.166 | 0.240 | 0.252 | 0.562 | 0.591 | 0.920 | 0.935 | 1.201 | 1.296 | 1.652 | 1.664 | 2.047 | 2.038 |
| $K_{R1}$(kip/in) | 550 | 310 | 325 | 375 | 325 | 145 | 331 | 222 | 204 | 220 | 102 | 171 | 99 |
| $\Delta_R$(%) | 0.028 | 0.055 | 0.046 | 0.303 | 0.278 | 0.518 | 0.535 | 0.822 | 0.823 | 1.119 | 1.071 | 1.526 | 1.461 |
| $K_{R2}$(kip/in) | 361 | 193 | 127 | 62 | 60 | 28 | 31 | 33 | 11 | 19 | 11 | 7 | 5 |
| $F_P^-$(kips) | −3.10 | −5.60 | −1.70 | −12.3 | −6.60 | −11.3 | −7.6 | −10.6 | −7.4 | −8.7 | −6.0 | −4.2 | −2.9 |
| $F^-$(kips) | −35.5 | −42.5 | −31.6 | −46.9 | −40.9 | −46.3 | −40.8 | −46.1 | −43.7 | −47.5 | −44.2 | −33.5 | −26.2 |
| $\Delta_{ML}$(%) | −0.169 | −0.249 | −0.268 | −0.591 | −0.585 | −0.933 | −0.945 | −1.265 | −1.386 | −1.619 | −1.725 | −2.010 | −2.013 |
| $K_{L1}$(kip/in) | 506 | 441 | 424 | 416 | 375 | 307 | 179 | 209 | 165 | 241 | 232 | 208 | 167 |
| $\Delta_L$(%) | −0.015 | −0.015 | −0.078 | −0.224 | −0.196 | −0.414 | −0.424 | −0.708 | −0.707 | −0.904 | −0.950 | −1.162 | −1.219 |
| $K_{L2}$(kip/in) | 286 | 205 | 111 | 58 | 45 | 25 | 35 | 16 | 15 | 14 | 9 | 5 | 2 |
| K(kip/in) | 302 | 245 | 181 | 117 | 96 | 72 | 61 | 54 | 39 | 37 | 29 | 20 | 15 |
| $K_{norm}$ | 1.00 | 0.81 | 0.60 | 0.39 | 0.32 | 0.24 | 0.20 | 0.18 | 0.13 | 0.12 | 0.10 | 0.07 | 0.05 |
| Comments | Yield | Virgin | Stblzd | Virgin | Stblzd | Virgin | Stblzd | Virgin | Stblzd | Virgin | Stblzd | Virgin | Stblzd |

Normalized stiffness = stiffness / stiffness at yield = $K_{norm}$
Stiffness at yield = 302.0 kip/in
1 kip = 4.448 KN, 1 in = 2.54 cm

similar calculations. The normalized drift ratio can be thought of as the displacement ductility of the wall. For example, a ratio of 5 implies that the lateral wall displacement is 5 times the displacement at first yield. Two analytical equations, as shown in Equation (1) and (2), are proposed to predict the reduction in the effective stiffness for virgin cycles.[6] As shown in Figure 1.4, Equation (2) is easier to use but Equation (3) is better suited to the experimental data.

$$K_{eff}/K_{eff}(\text{at yield}) = EXP[-0.42[A-1.0]] \tag{2}$$

$$K_{eff}/K_{eff}(\text{at yield}) = \langle 0.11 + (1.0-0.11)\times \text{EXP}[-0.59[A-1.0]]\rangle \qquad (3)$$

where,

A=DRIFT RATIO / DRIFT RATIO(at yield)

These equations are used to analytically estimate the reduction in the effective stiffness for a specified ductility level for virgin cycles. For example, a reduction of 80% in the effective stiffness with respect to the effective stiffness at the first yield is obtained from both equations when a ductility level of 5.0 is considered. Equations (4) and (5) show the reduction in the effective stiffness for the stabilized cvcles. These analytical equations are used as an important tool in the analytical estimation of the force-displacement relationship and the development of a degrading modified Bouc model.

$$K_{eff}/K_{eff}(\text{at yield}) = \text{EXP}[-0.48[A-1.0]] \qquad (4)$$

$$K_{eff}/K_{eff}(\text{at yield}) = \langle 0.10 + (1.0-0.10)\times \text{EXP}[-0.69[A-1.0]]\rangle \qquad (5)$$

where,

A=DRIFT RATIO / DRIFT RATIO(at yield)

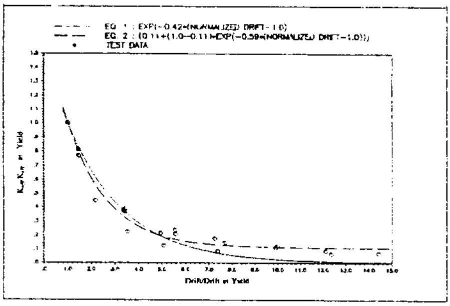

Figure 1.4 Normalized virgin effective stiffness vs drift ratio

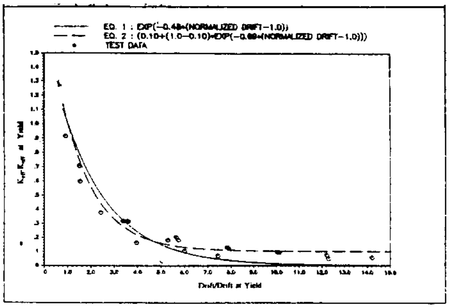

Figure 1.5 Normalized stabilized effective stiffness vs drift ratio

1.4 Unloading and Reloading Tangential Stiffness

To describe a complete hysteresis loop, a degrading stiffness slope along which reloading or unloading of shear walls occurs must be obtained in addition to the parameters identified in the previous section. Figures 1.6 through 1.9 show first and second unloading tangential stiffness denoted by $K_{R1}$ and $K_{R2}$ and reloading tangential stiffness defined by $K_{L1}$ and $K_{L2}$ defined in Figure 1.3 and Table 1.2 for virgin cycles. In these figures tangential stiffness normalized with respect to the stiffness at the first tension yield is plotted against the drift ratio normalized by the drift ratio at the first tension yield as well. It is shown from these figures that identical analytical curves can be used both for unloading and reloading tangential stiffness and are shown in Equations (6) and (7) for the first tangential stiffness ($K_{R1}$ or $K_{L1}$) and the second tangential stif fness ($K_{R2}$ or $K_{L2}$), respectively.

$$K_{eff}/K_{eff}(\text{at yield}) = \langle 0.50 + (1.0-0.50)\times \text{EXP}[-0.50[A-3.0]]\rangle \qquad (6)$$

$$K_{eff}/K_{eff}(\text{at yield}) = \langle 0.50 + (1.0-0.50)\times \text{EXP}[-0.70[A-1.0]]\rangle \qquad (7)$$

where,

A=DRIFT RATIO/DRIFT RATIO(at yield)

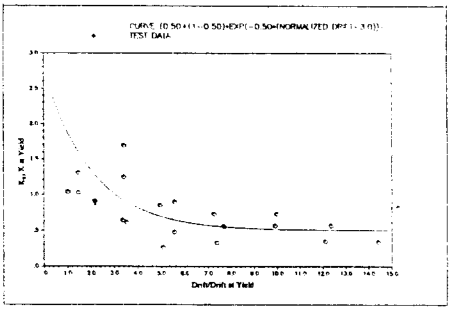

Figure 1.6 Normalized virgin $K_{R1}$ vs drift ratio

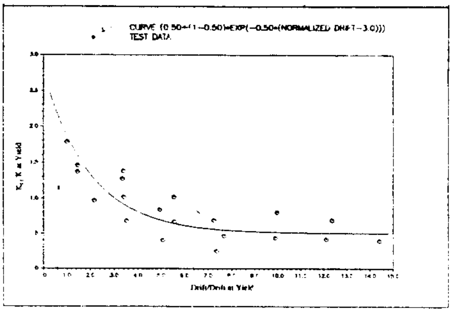

Figure 1.7 Normalized virgin $K_{L1}$ vs drift ratio

The same plots for stabilized cycles are presented in Figure 2.0 through 2.3 and Equation (8) and (9) are the analytical curves for the first tangential stiffness ($K_{R1}$ or $K_{L1}$) and the second stiffness ($K_{R2}$ or $K_{L2}$) tangential stiffness, respectively.

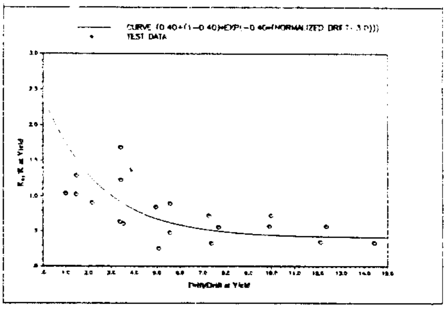

Figure 2.0 Normalized stabilized $K_{R1}$ vs drift ratio

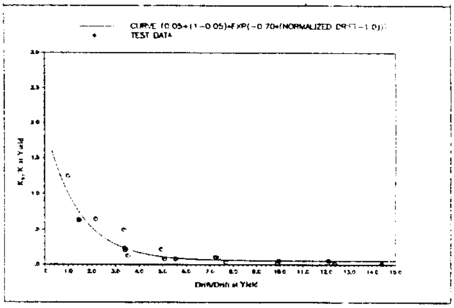

Figure 1.8 Normalized virgin $K_{R2}$ vs drift ratio

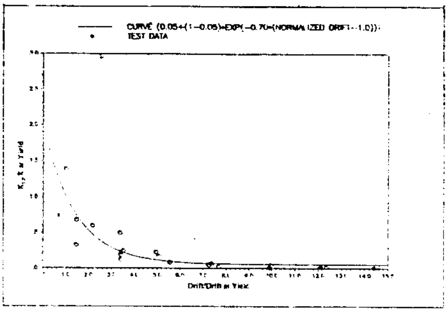

Figure 1.9 Normalized virgin $K_{L2}$ vs drift ratio

$$K_{eff}/K_{eff}(\text{at yield}) = \langle 0.40 + (1.0-0.40)\times \mathrm{EXP}[-0.40[A-3.0]]\rangle \tag{8}$$

$$K_{eff}/K_{eff}(\text{at yield}) = \langle 0.20 + (1.0-0.20)\times \mathrm{EXP}[-0.70[A-1.0]]\rangle \tag{9}$$

where,

A=DRIFT RATIO/DRIFT RATIO(at yield)

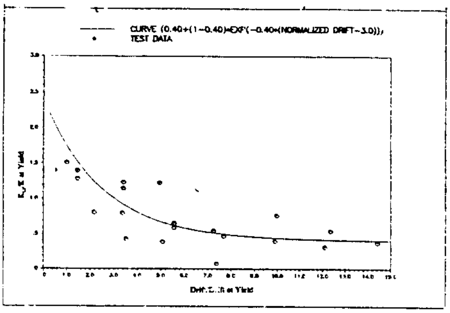

Figure 2.1 Normalized stabilized $K_{L1}$ vs drift ratio

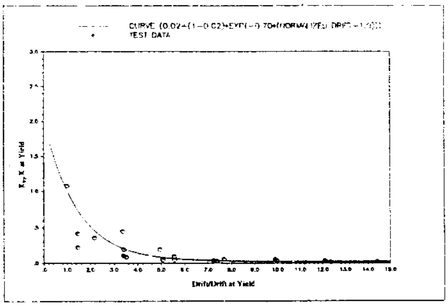

Figure 2.2 Normalized stabilized $K_{R2}$ vs drift ratio

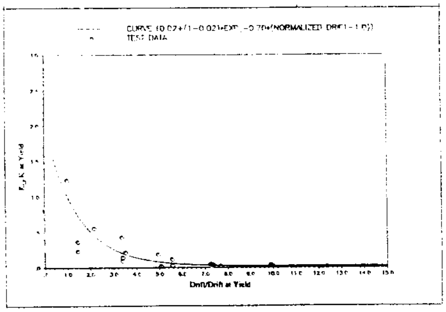

Figure 2.3 Normalized stabilized $K_{L2}$ vs drift ratio

## 2. DEFLECTION ESTIMATION USING STRUCTURAL ENGINEERING EQUATIONS

Equation (10) is used to calculate the delfection of a fixed end cantilever flexural member including shear deformation:

$$\Delta = \frac{Ph^3}{3EI} + \frac{Ph}{AG} \qquad (10)$$

where, P=lateral load

E=modulus of elasticity

I=moment of inertia of cross section

A=area of cross section

$G = E/2(1+\nu) = 0.4E$

$\nu$=poisson's ratio

However, the contribution of the slipping deflection of the wall on its base is not included in Equation (10). This section attempts to develop equations and procedures for estimating the deflection at the first tension steel yield, the deflection at the maximum strength and the larger post maximum strength deflections based on an effective wall stiffness that is a function of axial load and drift ratio.

The method developed in this section combined with analytical approach for calculating the lateral load capacity of shear walls presents an accurate force displacement relationship. the drift ratio or deflection of shear walls, alternatively, at the first yield of vertical steel is an important structural design parmeter. As indicated in Table 1.4 this value can vary significantly with vertical steel reinforcement and axial load.[7)]

Equation (10) can be also rewritten as

$$\Delta = \frac{P}{K} \qquad (11)$$

where,

$$K(\text{stiffness}) = \frac{1}{\left[\frac{h^3}{3EI} + \frac{h}{AG}\right]}$$

Table 1.4 Drift ratio(%) of the flexural walls at first yield, peak load, 75% and 50% of peak load

| Wall | First Yield | Status | Peak | 75% of Peak | 50% of Peak |
|---|---|---|---|---|---|
| # 1 | 0.23 | Virgin | 1.14 | 1.60 | 2.08 |
| | | Stabilized | 1.22 | 1.65 | 2.10 |
| # 2 | 0.31 | Virgin | 0.76 | 1.35 | 1.60 |
| | | Stabilized | 0.77 | 1.40 | 1.75 |
| # 6 | 0.17 | Virgin | 0.56 | 2.65 | 2.65 |
| | | Stabilized | 0.59 | 2.70 | 2.70 |
| # 8 | 0.17 | Virgin | 0.56 | 1.60 | 2.05 |
| | | Stabilized | 0.94 | 1.65 | 2.10 |
| # 10 | N/A | Virgin | 0.68 | 1.60 | 2.00 |
| | | Stabilized | 0.68 | 1.60 | 2.10 |
| # 11 | N/A | Virgin | 1.16 | 1.50 | 1.55 |
| | | Stabilized | 0.83 | 1.50 | 1.60 |
| # 12 | N/A | Virgin | 0.87 | 1.95 | 2.35 |
| | | Stabilized | 0.89 | 1.95 | 2.35 |
| # 15 | N/A | Virgin | 1.25 | 1.68 | 2.50 |
| | | Stabilized | 1.28 | 1.75 | 2.55 |

Equation (11) can be used to estimate the stiffness of the wall at the first yielding of vertical steel since the slipping deflection on the foundation of the wall is very small at the yield limit state.

In column 1 and 2 in Table 1.5 the equivalent A and I for the cracked cross section is calculated for all flexural walls. Column 3 in this table gives the calculated value of stiffness using A and I based on a cracked cross section. If the test value of stiffness at yield in column 4 is divided by this calculated stiffness form column 3 the ratios noted in column 5 are obtained. These values are essentially the same with the range from a low of 0.26 to a maximum of 0.30. This ratio value is consistent with the elastic stiffness value presented by Priestley.

**Table 1.5 Effective stiffenss**

| | Calc Area ($in^2$) | Calc Moment of Inertia ($in^2$) | Calc K (kip/in) | Measured K at Yield (kip/in) | Measured K at Yield /Calc K |
|---|---|---|---|---|---|
| Wall # 1 | 171.8 | 50854.0 | 1264.3 | 384.1 | 0.30 |
| Wall # 2 | 181.4 | 52057.4 | 1321.1 | 389.9 | 0.30 |
| Wall # 6 | 109.9 | 40614.1 | 1085.6 | 321.1 | 0.30 |
| Wall # 8 | 96.8 | 34271.6 | 1157.7 | 302.5 | 0.26 |
| Wall # 10 | 120.2 | 33570.1 | 1322.0 | NA* | NA |
| Wall # 11 | 145.6 | 58225.7 | 1827.1 | NA* | NA |
| Wall # 12 | 120.2 | 33570.0 | 1322.0 | NA* | NA |
| Wall # 15 | 136.0 | 43398.4 | 1614.6 | NA* | NA |

NA* = Not available due to strain gauge malfunction

1 kip = 4.448 KN, 1 in = 2.54 cm

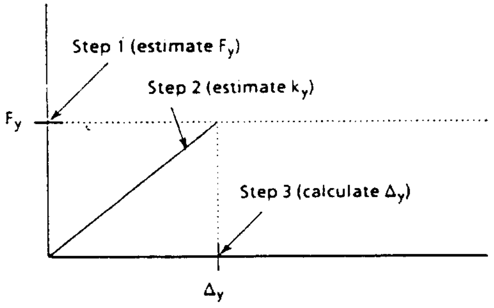

**Figure 2.4 Estimation of yield force and delfection**

Figure 2.4 shows how the deflection of shear walls at the first yield of vertical steel can be calculated. First, the yield load is estimated. In the second step, the stiffness is calculated based on a cracked cross section and scaled by approximtely 0.3. Then in step 3 Equation (12) is used to calculate the yield deflection of the walls.

$$\Delta_y = \frac{P_y}{K_y} \tag{12}$$

Figures 2.5 and 2.6 show the plots of the normalized stiffness versus the normalized drift ratio of shear walls. Equations are correlated through the data corresponding to virgin and stabilized curves and they are repeated in Equation (13) and (14) for virgin and stabilized curves, respectively.

$$\frac{K_{eff}}{K_{eff}(\text{at yield})} = [0.11 + (1.0-0.11)]\times \text{EXP}[-0.59(A-1.0)] \tag{13}$$

$$\frac{K_{eff}}{K_{eff}(\text{at yield})} = [0.10 + (1.0-0.10)]\times \text{EXP}[-0.69(A-1.0)] \tag{14}$$

where,

A=DRIFT RATIO/DRIFT RATIO(at yield)

Therefore, as shown in Figure 2.5 the force deflection curve can now be constructed for a given wall.

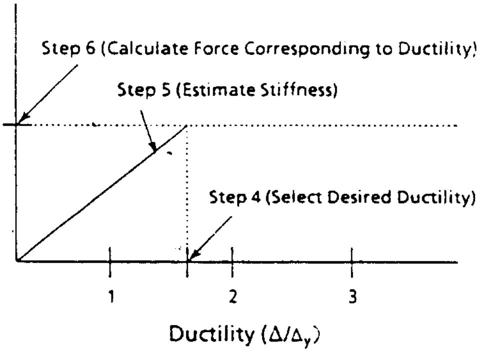

**Figure 2.5 Construction of force/ deflection curve**

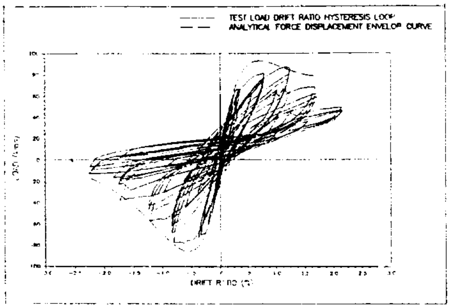

Figure 2.6 Analytical force displacement envelope curve for Wall ＃1

Step 4 involves selecting the desired displacement or ductility level for which the wall capacity is sought. Then using Equation (13) or (14), and the wall stiffness at th first tension yield, and the wall stiffness at desired displacement is calculated. Finally, using Equation (15) the force corresponding to the displacement is calculated.

$$P = K\Delta \qquad (15)$$

The repeating of this calculation process enables one to complete the force deflection curve for the walls.

## 3. COMPARSION OF ANALYTICAL PREDICTIONS AND EXPERIMENTAL VALUES

Table 1.6 and 1.9 provide the analytical predictions for the performance of reinforced concrete flexural shear walls in terms of the parameters examined in the previous sections.

Table 1.6 Analytical predictions of parameters for Wall ＃1

| Cycle ＃ | 6 | 10 | 12 | 17 | 19 | 25 | 26 |
|---|---|---|---|---|---|---|---|
| Peak Force (kips) | 60.9 | 81.5 | 74.6 | 86.7 | 77.0 | 62.3 | 59.5 |
| Pinch Force (kips) | 18.9 | 25.3 | 20.9 | 26.9 | 21.6 | 19.3 | 16.7 |
| Peak Drift Ratio(%) | 0.23 | 0.78 | 0.78 | 1.14 | 1.22 | 1.70 | 1.71 |
| Drift Offset (%) | 0.03 | 0.36 | 0.36 | 0.63 | 0.69 | 1.05 | 1.06 |
| Effective stiffness (kip/in) | 383.3 | 142.1 | 122.8 | 73.2 | 48.9 | 26.4 | 17.5 |
| First unloading and reloading tangential stiffness (kips/in) | 713.7 | 352.1 | 352.1 | 264.8 | 245.9 | 213.8 | 192.9 |
| Second unloading and reloading tangential stiffness (kips/in) | 384.4 | 88.8 | 79.0 | 42.3 | 26.3 | 23.4 | 11.8 |
| Remarks | yield | virgin | stblzd | virgin | stblzd | virgin | stblzd |

Stiffness at yield (kips/in) = 384.5
Drift ratio at yield (%) = 0.23
1 kip = 4.448 KN, 1 in. = 2.54 cm

Table 1.7 Analytical predictions of parameters for Wall ＃2

| Cycle ＃ | 8 | 12 | 13 | 15 | 20 | 22 | 26 | 27 |
|---|---|---|---|---|---|---|---|---|
| Peak Force (kips) | 77.8 | 73.8 | 83.1 | 75.4 | 65.9 | 45.8 | 42.1 | 39.7 |
| Pinch Force (kips) | 24.1 | 20.7 | 25.8 | 21.1 | 20.4 | 12.8 | 13.1 | 11.1 |
| Peak Drift Ratio(%) | 0.32 | 0.33 | 0.69 | 0.77 | 1.12 | 1.26 | 1.62 | 1.91 |
| Drift Offset (%) | 0.07 | 0.07 | 0.30 | 0.35 | 0.62 | 0.72 | 0.99 | 1.20 |
| Effective stiffness (kip/in) | 385.2 | 378.6 | 233.4 | 191.6 | 129.8 | 89.4 | 66.2 | 32.8 |
| First unloading and reloading tangential stiffness (kips/in) | 717.5 | 66.4.4 | 482.8 | 444.2 | 338.3 | 308.9 | 259.4 | 222.4 |
| Second unloading and reloading tangential stiffness (kips/in) | 382.2 | 378.6 | 176.8 | 143.2 | 78.7 | 52.4 | 38.8 | 18.1 |
| Remarks | yield | stblzd | virgin | stblzd | virgin | stblzd | virgin | stblzd |

Stiffness at yield (kips/in) = 390.5
Drift ratio at yield (%) = 0.31
1 kip = 4.448 KN, 1 in. = 2.54 cm

The total delfection, obtained form test was composed of three parts, i.e., the flexural deflection and the shear deflection of the walls as well as the slipping deflection on the foundation of the walls. The deflection at the fist tension yield, the deflection at the maximum strength, and the larger post maximum strength deflection are estimated based on the observed degrading effective stiffness through which the three types of deformations are accounted. The force deflection relationships can be obtained by combining the method devel-

Table 1.8 Analytical predictions of parameters Wall #8

| Cycle # | 7 | 8 | 10 | 15 | 17 | 22 | 24 | 29 | 31 | 36 | 38 | 43 | 45 | 50 | 52 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Peak Force (kips) | 38.9 | 43.7 | 40.3 | 52.0 | 42.8 | 48.2 | 39.3 | 43.9 | 33.8 | 38.1 | 30.5 | 35.4 | 30.0 | 35.0 | 29.2 |
| Pinch Force (kips) | 6.0 | 6.6 | 4.8 | 7.8 | 5.1 | 7.2 | 4.7 | 6.6 | 4.1 | 5.7 | 3.7 | 5.3 | 3.6 | 5.3 | 3.5 |
| Peak Drift Ratio(%) | 0.17 | 0.24 | 0.25 | 0.56 | 0.59 | 0.92 | 0.95 | 1.26 | 1.30 | 1.63 | 1.67 | 1.99 | 2.01 | 2.37 | 2.34 |
| Drfit Offset (%) | 0.01 | 0.04 | 0.04 | 0.21 | 0.23 | 0.46 | 0.48 | 0.73 | 0.75 | 1.00 | 1.03 | 1.26 | 1.28 | 1.53 | 1.51 |
| Effective Stiffness (kip /in) | 324.2 | 270.0 | 259.7 | 122.5 | 97.5 | 50.8 | 35.9 | 21.5 | 13.3 | 8.7 | 4.7 | 3.6 | 1.8 | 1.4 | 0.7 |
| First unloading and reloading tangential stiffness (kips /in) | 601.9 | 515.6 | 487.7 | 299.1 | 287.2 | 209.1 | 197.4 | 178.0 | 158.6 | 166.4 | 141.1 | 162.5 | 134.0 | 161.2 | 131.0 |
| Second unloading and reloading tangential stiffness (kips /in) | 326.1 | 244.6 | 237.4 | 77.3 | 61.8 | 30.2 | 19.3 | 19.4 | 9.4 | 16.8 | 7.1 | 16.2 | 6.6 | 16.1 | 6.5 |
| Remarks | yield | virgin | stblzd | virgin | stblzd | virgin | stblzd | virgin | stblzd | virgin | stblzd | virgin | stblzd | virgin | stblzd |

Stiffness at yield (kips /in) = 321.0 Drfit ratio at yield (%) = 0.17 1 kip = 4.448 KN, 1 in. = 2.54 cm

Table 1.9 Analytical predictions of parameters Wall #8

| Cycle # | 7 | 8 | 10 | 15 | 17 | 22 | 24 | 29 | 31 | 36 | 38 | 43 | 45 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Peak Force (kips) | 37.5 | 43.7 | 36.3 | 50.2 | 40.6 | 49.7 | 41.6 | 49.0 | 32.3 | 39.3 | 26.6 | 25.0 | 16.3 |
| Pinch Force (kips) | 5.6 | 6.6 | 4.4 | 7.5 | 4.9 | 7.5 | 5.0 | 7.4 | 3.9 | 5.9 | 3.2 | 3.8 | 2.0 |
| Peak Drift Ratio(%) | 0.17 | 0.24 | 0.25 | 0.56 | 0.59 | 0.92 | 0.94 | 1.20 | 1.30 | 1.65 | 1.66 | 2.05 | 2.04 |
| Drfit Offset (%) | 0.01 | 0.04 | 0.04 | 0.21 | 0.23 | 0.46 | 0.48 | 0.68 | 0.75 | 1.02 | 1.02 | 1.30 | 1.29 |
| Effective Stiffness (kip /in) | 302.5 | 250.8 | 235.9 | 111.1 | 88.5 | 44.9 | 32.7 | 22.1 | 11.5 | 7.0 | 4.0 | 2.6 | 1.3 |
| First unloading and reloading tangential stiffness (kips /in) | 562.4 | 480.2 | 449.3 | 276.0 | 266.1 | 193.7 | 184.3 | 169.5 | 147.5 | 155.9 | 131.9 | 152.7 | 125.4 |
| Second unloading and reloading tangential stiffness (kips /in) | 302.5 | 225.5 | 212.3 | 69.2 | 55.4 | 27.1 | 17.6 | 18.8 | 8.6 | 15.7 | 6.6 | 15.2 | 6.2 |
| Remarks | yield | virgin | stblzd | virgin | stblzd | virgin | stblzd | virgin | stblzd | virgin | stblzd | virgin | stblzd |

Stiffness at yield (kips /in) = 302.5 Drfit ratio at yield (%) = 0.17 1 kip = 4.448 KN, 1 in. = 2.54 cm

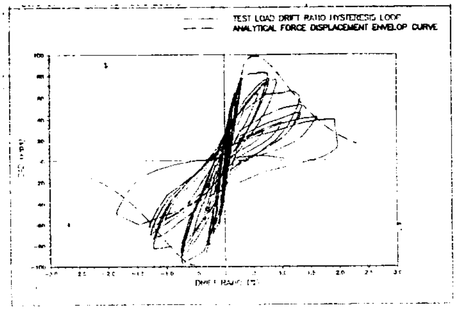

Figure 2.7 Analytical force displacement evelope curve for Wall #2

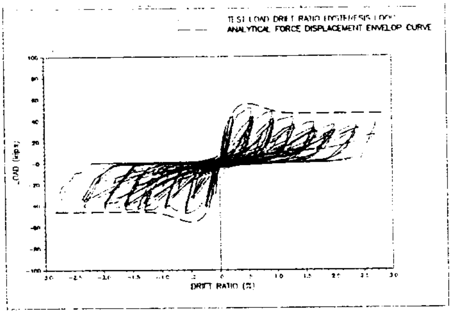

Figure 2.8 Analytical force displacment envelope curve for Wall #6

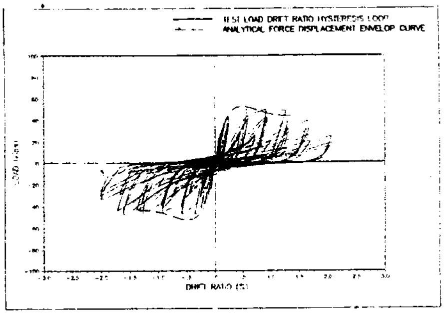

Figure 2.8 Analytical force displacement envelope curve for Wall #8

oped in this section with the model developed in previous paper. Figures 2.6 through 2.9 show the analytically generated force deflection envelope curves plotted on the top of the experimentally measured force deflection curves. These figures demonstrate that the analytical predictions accurately reproduce the experimental values.[7)]

## 4. CONCLUSIONS

The following steps summarize the estimation process of the load deflection response of reinforced concrete flexural walls based on flexural deformation, shear deformation.

Step 1 : Calculate the load at the yield limit state.

Step 2 : Calculate the effective virgin stiffness of the wall based on a cracked cross section.

Step 3 : Construct the load deflection curve up to the yield limit state and thus obtain deflection and drift retio at the first yield.

Step 4 : Calculate the value of the load at the maximum load limit state.

Step 5 : Calculate the deflection at the maximum load limit state.

Step 6 : Complete the calculation of the load-deflection curve.

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