부착되지 않은 텐돈을 갖는 프리스트레스트 콘크리트부재의 해석

Analytical Method of Prestressed Concrete Members with Unbonded Tendons

문 정 호*
Moon, Jeong-Ho
이 리 형**
Lee, Li-Hyung

요 약

본 연구의 목적은 부착되어 있거나 혹은 부착되어 있지 않는 텐돈을 갖고 있는 프리스트래스트 콘크리트 구조체의 해석이 가능한 프로그램을 개발하는데 있다. 이를 위해서 먼저 콘크리트, 철근, 그리고 텐돈등의 반복하중에 대한 모델을 선정 개발하고, 텐돈이 부착되어 있지 않는 경우에 그 응력 및 변형도를 보다 정확하게 해석할 수 있는 복합유한요소법을 유도하였다. 이러한 복합 유한요소법을 사용하였을때 요소의 변형 형상을 가정하지 않고도 각 단면들의 변형을 결정할 수 있어 요소의 길이가 기존의 유한요소법에 비하여 상당히 길어질 수 있다. 이러한 복합유한요소법을 가능하게 하기 위해서 다양한 형태의 적분 방법이 프로그램되었다. 그리고 텐돈이 부착되어 있지 않는 경우 그효과를 예측할 수 있는 방법이 개발되었다. 끝으로, 실제 구조체에 대한 해석결과 및 실제실험에 대한 해석결과를 비교하여 프로그램의 적용성을 검토하였다.

Abstract

The purpose of the present study is to develop a computer program which can be used to analyze prestressed concrete structures containing either bonded or unbonded tendons. To accomplish this, first, the concrete, nonprestressed, and prestressed steels are modeled with cyclic constitutive laws to take into account the various loading effects. Then, the hybrid-type element method is dervied to improve the computational capability of stresses and strains, especially for the unbonded tendon. Since it allows one to determine the cross-sectional deformations in an element without any assumptions for its deformed shape, the element length can be much longer than that of the conventional finite element method. In order to achieve such a long element, various integral schemes are examined to implement them into the program. Then, the computational method for prestressing effects is developed consistently with the analytical method for the structure. Finally, analytical studies for actual tests were carried out to verify the program developed in this study.

* 한양대학교 초대형구조시스템 연구센터 연구조교수
** 한양대학교 건축공학과 교수

이 논문에 대한 토론을 1996년 6월 31일까지 본 학회에 보내주시면 1996년 12월호에 그 결과를 게재하겠습니다.

1. INTRODUCTION

The application of unbonded tendons to concrete has been increasingly demanded for possibility of economic, simple, fast construction as well as easy replacement of defective tendons. Unbonded tendons can also provide an economic solution for strengthening and repairing of existing structures. However, the structural behavior of a member containing unbonded tendons is not yet fully investigated even though the use of unbonded tendons becomes popular in engineering profession.

In order to investigate the behavior of the prestressed concrete member with unbonded tendons accurately, every critical stage during the life of structure must be examined. The stages include: (1) jacking stage; (2) service load stage; (3) factored load stage. Since the tendon stress decreases gradually with time, cracking and load-deflection responses have to be checked during the jacking stage and the service load stage. At the factored load stage, the strength of member should be computed considering the effects of unbonded tendons.

However, those computations cannot be conducted only by the cross-sectional analysis at a few critical locations which is typical for the concrete structures prestressed with bonded tendons. Most of codified equations for flexural strength computation, however, were made to consider only critical sections instead of the global behavior of member. If unbonded tendons, for example, are provided to a continuous member, the tendon strains have to be determined while considering the interaction between the structural behavior and the continuous tendons. Currently no code equations can take into account the effects of continuous tendons. Experimental data for continuous members are also so limited that they are not enough to revise the current code equations.

The focus of this study is to develop an analytical method for the prestressed concrete structures containing unbonded tendons. The research objectives include: (1) the development of nonlinear cyclic constitutive laws for concrete and steels; (2) the development of an analytical method that gives a better evaluation of unbonded tendon strains; (3) the derivation of a computational method of unbonded tendon stresses; (4) the analytical studies for the members containing unbonded tendons. As a results, a computer program called TAPS (Time-dependent Analysis for Prestressed concrete Structures) has been developed in this study.

2. NONLINEAR CONSTITUTIVE LAWS

Plain Concrete exhibits a softening behavior in which its compressive peak connot be maintained but it declines with a gradual increase of displacement. The ultimate load capacity of statically indeterminate structures is not correctly predicted without considering the softening response. A general model for concrete stress-strain relation also has to include post-peak and repeated-load behavior at large load reversals. Karsan and Jirsa[1] proposed a constitutive relation for the cyclic behavior of concrete in compression based on their experimental observations. Their model was modified in a slightly different way to reduce the number of internal variables. For the tensile stress-strain relationship under the cyclic loading, it was assumed that the unloading and reloading stiffness were indentical and the stiffness at zero tensile stress equals to the unload ing stiffness at the zero compressive stress. The tension stiffening effect was also modelled assuming the cracks be smeared out in a continuous manner.

The constititive models for reinforcing steel chosen in this study are the bilinear model, the power formula model, and the bounding surface model. The stress-strain curve which does not have a distinct yield point was characterized by the power formula of Devalapura and Tadros.[2] Cofie and Krawinkler's bounding surface model[3] was also selected for a complex cyclic loading in the inelastic range.

3. MATHEMATICAL FORMULATION

The deformed shape of beam element in conventional fiinite element method is approximated by interpolation functions such as a cubic polynomial function for a transverse displacment and a linear function for a longitudinal displacement. The cubic function implies a linear variation of curvature along the element. However, the analysis of prestressed concrete member with unbonded tendons requires an accurate evaluation of curvature variations since the compatibility equation should be formulated with the values of concrete strain at the level of tendon. Thus, a large number of short elements is necessary for an adequate evaluation of tendon strains. However, the hybrid-type element method is often used to improve the computational capability of curvatures. It uses the fact that analytical expressions for the cross sectional forces can be determined from nodal forces applied loads by the simple statics without being restricted by either strain displacement relations or material laws. If the cross sectional forces can be determined in advance, curvatures and axial deformations can be calculated in a theoretically exact manner regardless of shape variations of the element. The analytical expressions for the cross sectional forces can be considered as interpolation functions of forces rather than displacements. These expressions are exact instead of approximate since no assumptions are involved. Typical examples of the hybrid-type element method are found in reference 4, 5, 6. However, the mathematical formulation used in this study is more suitable for the analysis of unbonded tendons.

4. TENDON STRESSES

The design procedure for prestressed concrete structures if not as simple as that for nonprestressed concrete structures due to the loss of prestressing force. The total loss of prestressing force is attributed to the followings: (1) elastic shortening; (2) friction; (3) anchorage set. It is essential to include those effects for a refined analysis of prestressed concrete structure. If a member is pretensioned during construction, the elastic loss can be considered simultaneously during the computation. However, additional procedures are required for post-tensioned concrete structures. When more than a tendon is stressed in succession, the elastic loss for an individual tendon differs according to its order of the jacking sequence. Although the computation of such losses can be made in a theoretically accurate manner considering all the tendon profiles, the computed losses may not be necessarily accurate. Therefore, an approximate method is better for simplicity. The elastic loss in the i^{th} tendon due to the tensioning of $(i+1)^{th}$ to n^{th} tendons can be approximated by

$$\Delta f_{ps,i} = \sum_{j=i+1}^{n} \frac{f_{ps,j} A_{ps,j}}{[EA]_{c,eq}} (A_{ps,i} E_{ps}) \qquad (1)$$

with

$$[EA]_{c,eq} = \sum_{j=i}^{m} [EA]_{c,i} W_i / L_c \qquad (2)$$

in which $f_{ps,i}$ and $A_{ps,j}$ are the prestressing stress and the area of j^{th} tendon, respectively; E_{ps} is the young's modulus of tendon. $[EA]_{c,i}$ is the axial stiffness of section at i^{th} integration point; W_i is the weighting factor of the i^{th} section; m is the number of section that the tendon passes; L_c is the total length of member.

The relaxation is defined as the stress decrease in the tendon with time under a constant strain. The relaxation loss can be estimated accurately if the initial strain is sustained constantly. However, the strain in the tendon is altered continuously due to its interaction with its duct. Hernandez and Gamble[7] suggested a method to account for the effects of changes in tendon strain. The pinciple is to calculate the artificial initial stress at a time t_i with which the relaxation stress is estimated for the next time period. The artificial initial stress at the time t_i represents the initial stress that would be relaxed to the stress at the current time t. the equation for the artificial initial stress $f_{ps,i}$ is written as

$$af_{ps,i}^2 - f_{py}[1+0.55a]f_{ps,i} + f_{ps}(t)f_{py} = 0 \quad (3)$$

in which t is the current time period in hour; f_{py} is the yield stress; k=10 for stress-relieved steel; k=45 for low-relaxation steel; a is log $(t-t_i)/k$.

An analytical model for unbonded members cannot be developed without considering the total compatibility requirement that the total elongation of an unbonded tendon must be equal to the integrated value of concrete deformations at the level of the tendon. Thus, it is critical to compute the correct distribution of curvatures along the member. The friction between tendon and its duct develops the stress loss which is independent with time. A widely accepted equation for the frictional loss is given as

$$f_i^u = f_u^{uo}\, \chi_{i,k} \quad (4)$$

where f_i^u is the stress at a point i; f_k^{uo} is the stress at k after all the frictional losses occur; $\chi_{i,k}$ is the frictional coefficient between the point k and the point i. the unbonded tendon stress at a point should be determined by considering all the influences from the other points. Since the stress in an unbonded tendon usually remains below the elastic limit even at the failure of member, it is possible to rewrite Eq. 4 in terms of incremental strain as

$$\Delta\varepsilon_i^u = \Delta\varepsilon_k^{uo}\, \chi_{i,k} \quad (5)$$

If a local deformation in concrete $\Delta\varepsilon_k^c$ is generated only at a point k while all the other points are underformed, the compatibility equation can be written as

$$\Delta\varepsilon_k^c w_k = \Delta\varepsilon_k^{uo}\, \pi_{i,k} \quad (6)$$

with

$$\pi_k = (\chi_{i,k}w_1 + \cdots\cdots + \chi_{i,k}w_i + \cdots\cdots + \chi_{n,k}w_n) \quad (7)$$

in which w_k is the weighting factor at the point k; n is the total number of integration points. The weighting factors are used to transform the strain increment to the length increment. For the overall deformation of concrete, the strain at a point i is obtained by considering all the deformations along the tendon length as

$$\Delta\varepsilon_i^u = \sum_{k=1}^{n} \Delta\varepsilon_k^{uo}\, \chi_{i,k} \quad (8)$$

By substituting Eq. 6 to Eq. 8, the total strain increment at the point i is computed as

$$\Delta\varepsilon_i^{u} = \sum_{k=1}^{n} \frac{\Delta\varepsilon_k^{c}\, w_k}{\pi_k}\, \chi_{i,k} \qquad (9)$$

The analytical procedure should be developed in a unified way regardless of the tendon types which are bonded or unbonded. Thus, the same procedure for the bonded member should be applicable once the strains in the unbonded tendon are determined. To this end, Eq. 9 was rewritten as

$$\Delta\varepsilon_i^{u} = \Delta\varepsilon_i^{ul} + \Delta\varepsilon_i^{uo} + \Delta\varepsilon_i^{ur} \qquad (10)$$

with

$$\Delta\varepsilon_i^{ul} = \sum_{k=1}^{i-1} \frac{\Delta\varepsilon_k^{c}\, w_k}{\pi_k}\, \chi_{i,k} \qquad (11)$$

$$\Delta\varepsilon_i^{uo} = \frac{\Delta\varepsilon_i^{c}\, w_i}{\pi_i} \qquad (12)$$

$$\Delta\varepsilon_i^{ur} = \sum_{k=i+1}^{n} \frac{\Delta\varepsilon_k^{c}\, w_k}{\pi_k}\, \chi_{i,k} \qquad (13)$$

where $\Delta\varepsilon_i^{ul}$ is the sum of tendon strain increments which come from the left hand side of point i; $\Delta\varepsilon_i^{uo}$ is the remaining tendon strain increment due to the deformation at its own location i after all the frictional losses occur; $\Delta\varepsilon_i^{ur}$ is the sum of tendon strain increments which come from the right hadn side of the point i. Since the frictional coefficients π and χ are the function of tendon geometry, they can be prepared at the beginning of computation. The geometrical changes in tendon profile during the member deformation were ignored. It has to be mentioned, however that the elastic behavior of the unbonded tendon is assumed only for the computation of frictional coefficients. The strain increment at each point along the unbonded tendon is obtained by the sum of individual strain increments from forward and backward calculations during computation. Once the strain increment of tendon is obtained, the remaining procedure is the same as the case of members with bonded tendons. Thus, additional procedure for the unbonded members is to compute the tendon strains by the backward and forward calculations.

5. IMPLEMENTATION

The element length needs not to be short as in the conventional finite element method, but they can be long enough to model a member by one or two long elements. However, the governing equations for the nonlinear behavior consist of many integrals that cannot be evaluated in a closed form. It is necessary to employ numerical schemes to evaluate the integrals since the integral kernels are nonlinear functions. A significant amount of computing time is spent during the numerical integration process for a nonlinear problem. The numerical integration scheme, therefore, must be as efficient as possible, especially for the use of long elements. Furthermore, a suitable method should be chosen for the computation of transverse and rotational deformations within the element because the structural analysis gives only the nodal displacements of the element.

In this study, four types of integration schemes are used to evaluate various integrations. The integration schemes are rectangular rule, Simpon's rule, Gaussian integration rule, and Lobatto integration rule. However, no integration method can integrate accurately discontinouos functions which are usual in the nonlinear analysis with long elements. The discontinuity cannot be characterized by a single polynomial equation with a higher order. It is not a matter of the polynomial order, but a matter of the equation numbers used for the representation of integral kernel. Rather than increasing the polynomial order, i.e., the integration points, a better solution can be obtained by incresing the number of equations, i.e., the integration modules. An element may be divided into a certain number of modules considering the discontinuities, in which each module is represented by an individual polynomial equation. Each module is evaluated by an individual integration scheme.

If a few number of long elements are used to model a member, it is required to compute transverse and rotational deformations within the element. The moment area method and the finite difference method may be appropriate tools for solving those deformations. Theoretically, the moment area method gives an exact solution if curvature variations along the element are estimated correctly. When the finite difference method is used, only approximate solutions are obtained. However, the modulation of integration points allows one to improve the accuracy. As mentioned previously, the theoretically exact values of curvature can be evaluated for the given axial force and moment. If the values obtained by the moment area method are used as the boundary values for the finite difference method, the errors due to the approximation can be reduced. Since the Gaussian integration method can evaluate accurately the deformations at the end of the integration module, the moment area method can be combined with the Gaussian integration method. Therefore, the same distribution of integration points is still used for this purpose.

Although Newton-Raphson method is a powerful tool for nonlinear analysis, it may fail to find solution in the neighborhood of a critical point. With the load incremental approach, it is not possible to capture such deformational characteristics as the yield plateau or the softening behavior. The stiffness matrix approaches singularity resulting in an increasing number of iterations even with very small load steps. The Crisfield's arc length method is a famuous technique for the case. However, deformations such as snap-back or snap-through is not expected to occur significantly in the structures of interest in this study. These considerations lead one to conclude that the Rik's method,[8] called the displacement control method, is more suitable than the arc length method. At the end of iteration, finally, the current variables are stored for the next calculation while forgetting all the intermediate steps since concrete is a highly history dependent material.

6. COMPARISON WITH TESTS

6.1 Tao and Du's beams (TDs)

Tao and Du[9] tested twenty-six partially prestressed concrete beams with unbonded tendons that were divided into four groups according to various parameters. The main objective of the test program was to investigate the ultimate tendon stress at different levels of reinforcement. A group of nine beams were chosen in this study (see Table 1). Those beams were subdivided into three categories in which each beam in a category was designed for the unstressed steels to carry about 30, 50, and 70 percent of the total ultimate load. All the test beams were 6.3 in × 11 in of cross section and 165 in of span.

The comparison of load-deflection curves and the load-tendon stress increase curves (TAPS.1) are presented in Fig. 1. The program TAPS predicted the load-deflection behavior of the unbonded beams with good agreements. However, the beams with the medium level of reinforcing indices (TD.2, TD.5, and TD.8) were predicted to fail prior to reaching their maximum test deflections. The beams with the highest reinforcing indices (TD.3, TD.6, and TD.9) show that ductile failures did not occur experimentally and analytically. When the reinforcing indices were low (TD.1, TD.4, and TD.7), the load-deflection responses were predicted well with a sufficiently ductile mode.

Additionally, all the beams were also analyzed as if they had bonded tendons (TAPS.2) to investigate the effects of unbonded tendon.

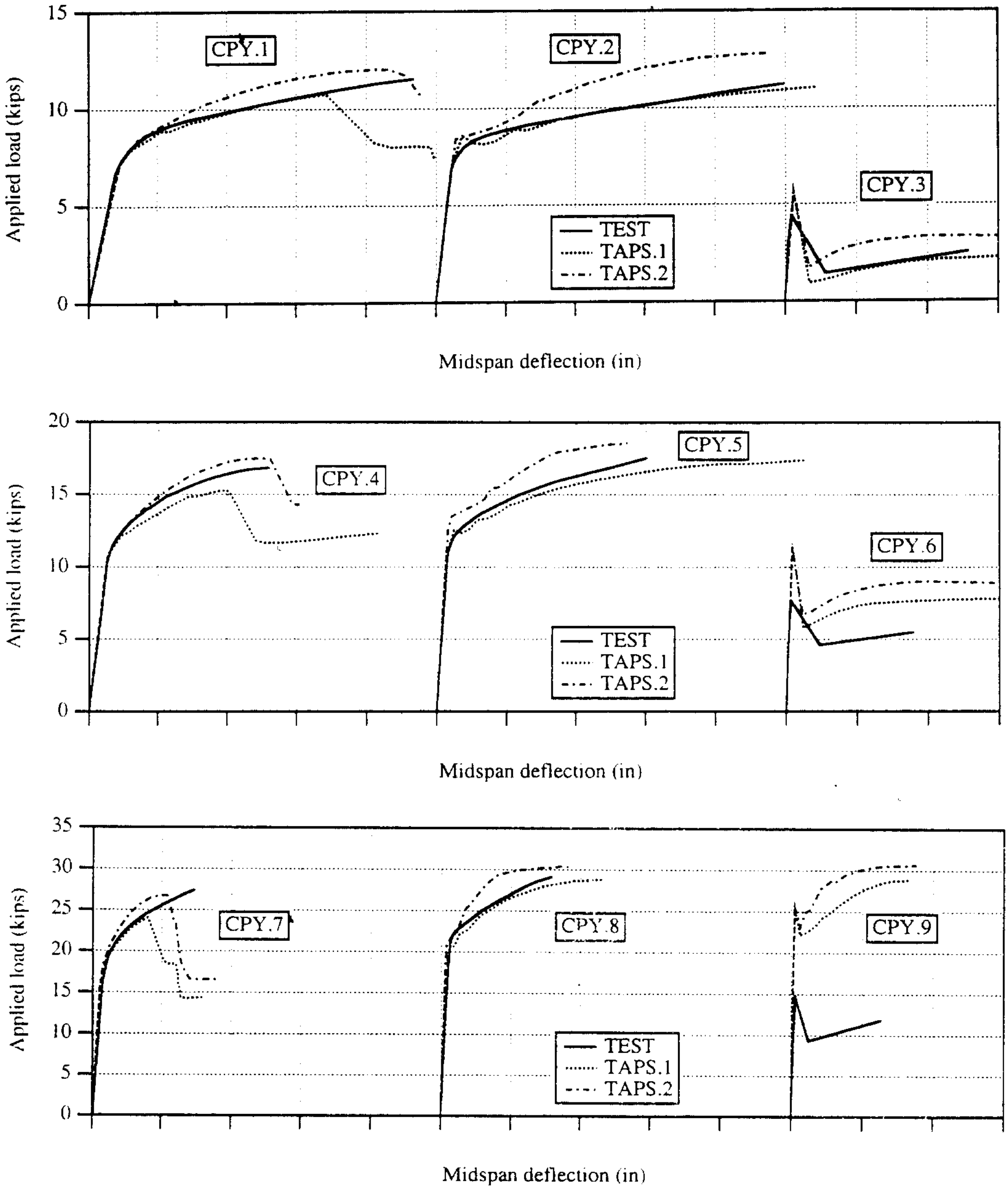

Fig. 1 Comparison of Tao and Du's tests (1 kip=4.448 KN, 1 in=25.4mm)
NOTE : TAPS.1=unbonded beam analysis, TAPS.2=bonded beam analysis

Table 1. Design Parameters for TDs

name	f'_c (ksi)	A_{ps} (in^2)	f_{se} (ksi)	ρ_s	f_y (ksi)	ω_{ps}	ω_e	ω_e
TD.1	4.437	0.091	139.2	0.0038	35.72	0.0017	0.0045	0.0913
TD.2	4.437	0.152	131.1	0.0038	62.35	0.0028	0.0045	0.1449
TD.3	4.437	0.243	118.9	0.0053	62.35	0.0045	0.0067	0.2136
TD.4	4.437	0.091	126.0	0.0038	62.35	0.0017	0.0045	0.1101
TD.5	4.437	0.122	117.5	0.0069	58.00	0.0022	0.0088	0.1733
TD.6	4.437	0/243	123.8	0.0100	58.00	0.0045	0.0131	0.2959
TD.7	4.437	0.061	128.3	0.0069	58.00	0.0011	0.0088	0.1466
TD.8	4.800	0.091	129.6	0.0100	58.00	0.0017	0.0131	0.2037
TD.9	4.800	0.243	133.4	0.0180	58.00	0.0045	0.0228	0.3964

Note : $\omega_e=\rho_s\frac{f_y}{f'_c}+\rho_{ps}\frac{f_{se}}{f'_c}$, 1 in=25.4mm, 1 ksi=6.89 MPa

The beams with the highest reinforcing indices showed almost identical responses even if the tendons were assumed bonded. Thus, they were not included in the figures. Since relatively large amounts of bonded steels were provided, they acted more like the bonded beams. The minimum requirement for the bonded steel of ACI Code is 0.002 while actual amounts supplied were 3 to 9 times greater than the ACI minimum requirement. For the others slight increases of strength were predicted while deflections were twice as great as those of the bonded beams at the same levels of applied load. But strengths of the bonded beam analysis never exceeded experimental strengths, significantly. It was noted that the strain hardening of the unstressed steels affected the beam strengths. All the analytical solutions, however, were obtained without considering the strain hardening because such information was not available. However, Tao and Du mentioned in their paper that the strain hardening affected the beam strengths.

6.2 Cooke, Park, and Yong's slabs (CPYs)

The 1977 ACI Building Code was considered to overestimate the unbonded tendon stress at flexural failure for typical slabs. It has been pointed out that the span-depth ratio was a major source of the overestimation. In order to determine the effect of span-depth ratio, thus, Cooke, Park, and Yong[10] tested twelve prestessed concrete one-way slabs among which nine slabs were post-tensoned with unbonded tendons. The remaining three slabs were designed to be identical to the unbonded slabs except that the tendons were bonded. The test program was intended to investigate the effect of the amounts of prestressing steels as well as the span-depth ratio. However, no bonded unstressed steels were provided for the slabs although they were required by the 1977 ACI Code. These tests highlighted the effects of span-depth ratio in the 1983 ACI Code. In this study, the nine slabs with unbonded tendons were analyzed and compared with the test results. The test specimens were divided into three groups depending on the span-depth ratio of 40, 30, and 20. Three slabs in each group were designed with the prestressing steel indices of 0.25, 0.125, and 0.0025. The span-depth ratios were varied with different span lengths while keeping the slab depth constant as 7.06 in. The prestressing indices were adjusted to desired values by varying the slab widths. Table 2 shows design parameters.

The comparisons of load-deflection behavior are shown in Fig. 2. It was found that the load-deflection behavior was sensitive to the prestressing index experimentally and analytically. The slabs with medium prestressing steel indices (CPY.2, CPY.5, and CPY.8) showed the most ductile behavior. However, the slabs with the least level of prestressing indices (CPY.3, CPY.6, and CPY.9) failed suddenly after one or two large cracks were formed. The program TAPS can capture the phenomenon but the strengths were overestimated especially for CPY.9. As the span-depth ratio decreases, the analytical peak strength tends to be greater than the test strength. The unbonded tendon is much less efficient than the bonded tendon in spreading out locally concentrated cracking strains. Since the prestressing indices were very low, either one or two cracks might be considered enough to make the slabs failed at the lower loads than their theoretical strengths. When the prestressing indices were high as in CPY.1, CPY.4, and CPY.7, the earlier decrease of strength

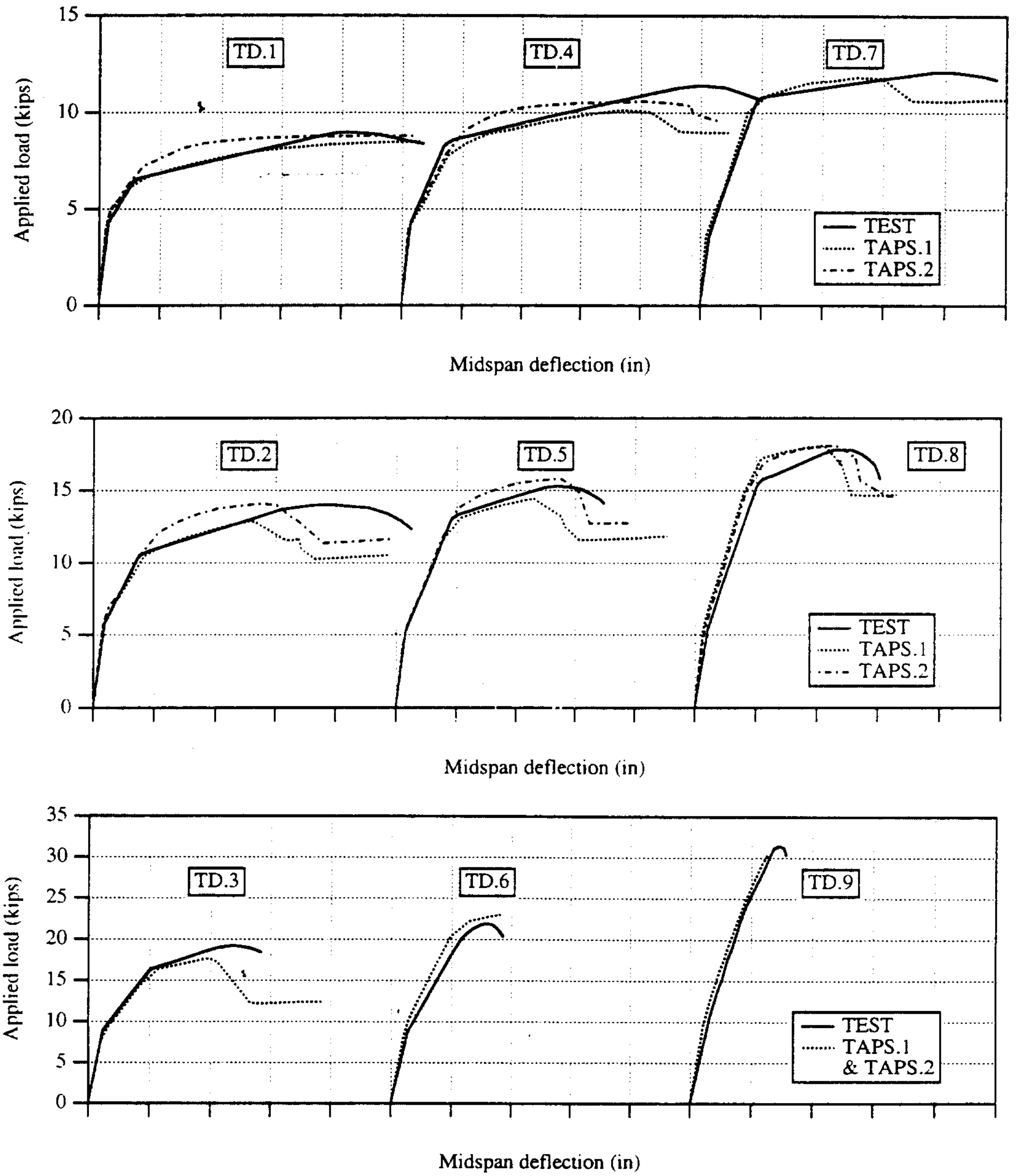

Fig. 2 Comparison of Cooke, Park, and Yong's test (1 kip=4.448 KN, 1 in=25.4mm)
NOTE : TAPS.1=unbonded beam analysis, TAPS.2=bonded beam analysis

Table 2. Design parameters for CPYs

name	L (in)	B (in)	f'_c (ksi)	A_{ps} (in^2)	f_{se} (ksi)	L/d	ω_e
CPY.1	181.1	13.9	4.37	0.432	169	40	0.253
CPY.2	181.1	27.8	4.37	0.432	166	40	0.124
CPY.3	181.1	46.5	4.37	0.180	174	40	0.032
CPY.4	133.8	13.9	4.99	0.432	169	30	0.222
CPY.5	133.8	27.8	4.99	0.432	167	30	0.110
CPY.6	133.8	46.5	4.99	0.180	177	30	0.029
CPY.7	86.6	13.9	4.47	0.432	169	20	0.248
CPY.8	86.6	27.8	4.47	0.432	169	20	0.124
CPY.9	86.6	46.5	4.47	0.180	175	20	0.032

was predicted. However, the predicted load-deflections generally agreed well with the test results. Up to 80 to 90% of ultimate loads, the load-deflection responses were found to be almost identical regardless of tendons either bonded or unbonded. A slight increase of strength was expected when the slabs were assumed to have bonded tendons.

6. CONCLUSIONS

1. The hybrid-type element method is not restricted by the shape variations of an element. Thus the curvatures and axial deformations can be computed in a theoretically exact manner from the cross-sectional forces that are determined by the structural analysis.

2. Since no assumptions on the deformed shape are made with the hybrid-type element method, the element lenght should not be as short as that of the conventional finite element method but it may be long enough to model a member with an element if suitable integral schemes are provided to cover discontin uities and end effects in the curvature variations.

3. It is desirable that the prestressing effects are taken into account consistently with the analytical method of structure in such a way that all the prestressing effects are evaluated at the integration points.

4. The compatibility equation for the unbonded tendon is derived from the requirement that the total elongation of tendon is equal to the total deformational change of the concrete at the level of tendon along its length.

5. The strengths of the TDs were predicted close to the strengths that were computed with the assumption of bonded tendons. This was ascribed to the loading condition of two point loads on short single span and the relatively large amount of bonded steels.

6. The predicted load-deflection and load-stress increase curves for TDs and CPYs showed a good agreement with the test data.

7. The studies on TDs and CPYs proved that the stress in the unbonded tendon at flexural failure is affected by the ratio of unstressed bonded steels. The tendon stress at failure decreases as the steel ratio increases.

REFERENCES

1. Karsan, I.D., and Jirsa, James O., "Behavior of concrete Under Compressive Loadings," *Journal of Structural Divison, ASCE,* Vol. 95, No. ST12, Dec., 1969, pp.2543-2563.
2. Devalapura, R.K. and Tadros, M.K., "Stress-Strain Modeling of 270 ksi Low-Relaxation Prestressing Strands," *PCI Journal,* Vol.37, No.2, Mar.-Apr., 1992, pp.100-106.
3. Cofie, N.G., and Krawinkler, H., "Uniaxial Cyclic Stress-Strain behavior of Structural Steel," *Journal of Engineering Mechanics,* Vol. 111, No. 9, Sept., 1985, pp.1105-1120.
4. Backlund, J. "Large Deflection Analysis of Elasto-Plastic Beams and Frames," *Journal of Mechanical Science,* Vol. 18, 1976, pp.269-277.
5. Carol, I., and Murcia, J., "Nonlinear Time-Dependent Analysis of Planar Frame Using an 'Exact' Formulation- I. Theory," *Computer & Structures,* Vol. 33, No. 1, 1989, pp.79-87.
6. Carol, I., and Murcia, J., "Nonlinear Time-De-

pendent Analysis of Planar Frame Using an 'Exact' Formulation-Ⅱ. Computer Implementation for R. C. Structures and Examples," *Computer & Structures*, Vol. 33, No. 1, 1989, pp. 89-102.

7. Hernandez, H.D., and Gamble, W.L., *Time-Dependent Prestress Losses in Pretensioned Concrete Construction*, Structural Research Series No. 417, Civil engineering Studies, University of Illinois, Urbana, May, 1975.
8. Riks, E. "An Incremental Approach to the Solution of Snapping and Buckling Problems," *Int. J. Solids Struct*, 15, 1979, pp.529-551.
9. Tao, X., and Du, G. "Ultimate Stress of Unbonded Tendons in Partially Prestressed Concrete Beams," *PCI Journal*, Vol. 30, No. 6, Nov.-Dec., 1985, pp.72-91.
10. Cooke, N., Park, R., and Yong, P., "Flexural Strength of Prestressed Concrete Members with Unbonded Tendons," *PCI Journal*. Vol. 26, No. 6, Nov.-Dec., 1981, pp.52-80.

(접수일자 : 1995. 6. 19)