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PERIOD VARIATIONS OF RT PERSEI

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ABSTRACT

RT Per has been known as a close binary of which the orbital period has unpredictably varied so far. Although there are no agreements with the working mechanism for the changes of the period, two interpretations have been suggested and waiting for to be tested: 1) light-time effects due to the unseen 3rd and 4th bodies (Panchatsaram 1981), 2) Abrupt period-changes due to internal variations of the system (e.g. mass transfer or mass loss) superimposing to the light-time effect by a 3rd body (Frieboes-Conde & Herczeg 1973). In the point of view that the former interprepation models could predict the behavior of the changes of the orbital period theoretically, we checked whether the recent observed times of minimum lights follow the predictions by the first model or not. We confirmed that the observed times of minimum lights have followed the variations calculated by the light-times effects due to the 3rd and 4th bodies suggested by Panchatsaram. In this paper a total of 626 times of minimum lights were reanalyzed in terms of the light-time effects by the 3rd and 4th bodies. We concluded that the eclipsing pair in SV Cam system moves in an elliptic orbit about center of mass of the triple system with a period of about $42.^y2$, while the mass center of the triplet is in light-time orbit about the center of mass of the quadruple system with a period of 120^y. The mean masses deduced for the 3rd and 4th bodies were $0.89m_\odot$ and $0.82m_\odot$, respectively.

1. INTRODUCTION

RT Per (BD+46°740, Sp=dF2, $m_v = 10.5 + 1.4$) is a semi-detached eclipsing binary, a Roche lobe-filling secondary orbiting a hot dF2 primary every 0.8494 days. Although this star has been frequently studied since the discovery of its variability by Ceraski (1904), its properties were less known except quasi-cyclic changes of period and variations of its asymmetrical light curves. In addition we are not in general agreements with the causes producing both variations. The history of study for RT Per are summarized in Mancuso *et al.* (1977) and Panchatsaram (1981). Although many interpretations on the period changes of RT Per, the subject of this paper, have been suggested as the times of minimum lights observed have been accumulated, two working hypothesis among them are generally accepted as possible explanations for the period-variations of the system and waiting for testing which mechanism is correct: One is that irregular changes of the period have been superimposed over the cyclic variation due to light-time effect induced by an unseen third-body in RT Per system which we call this explanation as IC (Irregular changes plus Cyclic variations) hereafter (Frieboes-Conde & Herczeg 1973, Ahnert 1974). The other, firstly proposed by Panchatsaram (1981), is that the orbital period has been varied by the light-time effects caused by a third and fourth bodies (Hereafter call as TF(Third and Fourth)). Both suggestions for the explanations of the period changes of RT Per system are based on the recognition that the observd $(O-C)$ residuals of the system could not be reproduced by a single cause such as the light-time effect due to one more distant component and thus there are at least two or more mechanisms. Anyway two interpretations (IC and TF) proposed so far are to be tested. But IC mechanism can not predict the future behavior of the $(O-C)$ curve. On the other hand TF can estimate the $(O-C)$ value theoretically. In this sense IC may be thought as a passive mechanism and TF an active one. In the point of view that TF may be judged with the recent times of minimum lights for RT Per and because no such attempts were made after Panchatsaram's study, it is desirable to reinvestigate the period change of the system with the recent times of minimum lights. In this paper, we collected and analyzed all the times of minima available to us, found that the $(O-C)$ excursions of the recent timings for the system have followed the same trends with those predicted by the TF model, and improved the orbital elements for the light-time orbits of the third and fourth bodies suggested in the TF model using a differential corrections method.

2. THE EQUATIONS FOR THE LIGHT-TIME EFFECTS DUE TO ADDITIONAL BODIES

2.1 The light-time effects due to distant third and fourth bodies

Suppose a quadruple stellar system in which each masses of four member stars are m_1, m_2, m_3, and m_4, respectively. Two stars $(m_1,\ m_2)$ among the system constitute an eclipsing pair and the rest $(m_3,\ m_4)$ revolve around the center of mass of the quadruple system in a distant. Then, if we assume that gravitational interactions between the third and fourth bodies be neglected, the eclipse timings of an eclipsing pair perturbed by distant third and fouth bodies can be calculated with the following light-time ephemeris:

$$C' = C + \tau_3 + \tau_4 \tag{1}$$

where τ_3 is the light-time, produced by the third body (m_3), of the center of mass of the eclipsing pair relative to the center of mass of the triple stars $(m_1,\ m_2,\ m_3)$, τ_4 is the light-time, produced by the fourth body (m_4), of the center of mass of the triple stars$(m_1,\ m_2,\ m_3)$

relative to the center of mass of the quadraple stars ($m_1,\ m_2,\ m_3,\ and\ m_4$), and C is the usual light elements represented as:

$$C = T_o + PE \tag{2}$$

where $T_o,\ P$, and E are the initial epoch, orbital period of the eclipsing pair, and the number of revolution-cycles of the binary, respectively. Then, the time residual of the observed time of minimum lights (O) minus calculated one (C) at any epoch reflect the light time and is expressed as :

$$O - C = \sum_i \tau_i, \quad (i = 3,\ 4) \tag{3}$$

The light times τ_i due to a third body ($i = 3$) and a fourth body ($i = 4$) in equation (1), according to Irwin (1952), can be obtained by dividing the light traveling distance (z) refered to the center of the light-time elliptical orbits by the light speed (c). Namely,

$$\tau_i = \frac{z_i}{c} = \frac{K_i}{\sqrt{1 - e_i^2 \cos^2 \omega_i}}[(1 - e_i^2)(1 + e_i \cos \nu_i) \sin(\nu_i + \omega_i) + e_i \sin \omega_i],\ (i = 3,\ 4) \tag{4}$$

where $e_i,\ \omega_i$, and ν_i are eccentricity, longitude of periastron, and true anomaly at an arbitrary time of the light-time orbit of the center of mass of the eclipsing pair relative to the center of mass of the triple stars ($i = 3$) and of the triple stars relative to the center of mass of the quadruple stars ($i = 4$), respectively. The semi-amplitude of the light-time in days, K_i, is denoted by

$$K_i = \frac{a_i \sin i_i \sqrt{1 - e_i^2 \cos^2 \omega_i}}{c} \quad (i = 3,\ 4) \tag{5}$$

where a_i and i_i are semi-major axis in km and inclination, respectively. And thus the light speed c is $2.590 \times 10^{10} km/days$. The eccentric, true, mean anomalies (denoted by $E_i,\ \nu_i,\ M_i$, respectively) and time t are inter-related by the well-known formula:

$$\tan \frac{1}{2} E = \left[\frac{1 - e_i}{1 + e_i}\right]^{\frac{1}{2}} \tan \frac{1}{2} \nu_i, \tag{6}$$

$$n_i(t_i - T_i) = M_i = E_i - e_i \sin E_i, \tag{7}$$

$$n_i = \frac{2\pi}{P_i'}, \quad (i = 3,\ 4) \tag{8}$$

where $n_i,\ T_i$, and P_i' are the mean speed, time of peri-astron passage, and orbital period for the corresponding orbit. If the orbital elements of a light-time orbit are given, then Kepler's equation (7) via (8) are solved for E_i over the orbital period P_i' using a Newton-Rapson algorithm. Consequent calculations using equations (5) $\sim$ (3) yield a theoretical curve of light-times from which eclipse timings of an eclipsing pair have suffered during the period.

2.2 Differential corrections Method

To obtain new light-time orbits from the observed minima, a least-squares method by means of differential correctrions because of the non-linearities of the light-time equations as given in Eqs. (3) $\sim$ (5) was used. Expressing Equation (1) by the Taylor expansion for twelve unknowns and neglecting the terms higher than 2nd derivatives, we have the equations of condition for twelve elements to be determined as follows:

$$\begin{aligned}\Delta C' = {} & \frac{\partial C'}{\partial(a\sin i_i)}\Delta(a_i\sin i_i) + \frac{\partial C'}{\partial\omega_i}\Delta\omega_i + \frac{\partial C'}{\partial e_i}\Delta e_i + \frac{\partial C'}{\partial n_i}\Delta n_i + \\ & \frac{\partial C'}{\partial T_i}\Delta T_i + \frac{\partial C'}{\partial T_o}\Delta T_o + \frac{\partial C'}{\partial P}\Delta P, \quad (i = 3,\ 4)\end{aligned} \tag{9}$$

where the partial derivatives of the light-time ephemeris with respect to the unknowns were given by Irwin (1959) as:

$$\begin{aligned}\frac{\partial C'}{\partial(a_i\sin i_i)} &= \frac{\tau_i}{(a_i\sin i_i)}, \\ \frac{\partial C'}{\partial\omega_i} &= \frac{a_i sini_i}{c}\left[\frac{(1-e_i^2)}{(1+e_i\cos\nu_i)}\cos(\nu_i+\omega_i) + e_i\cos\omega_i\right], \\ \frac{\partial C'}{\partial e_i} &= \frac{a_i\sin i_i\cos(\nu_i+\omega_i)\sin\nu_i}{c(1+e_i\cos\nu_i)}, \\ \frac{\partial C'}{\partial n_i} &= \frac{a_i\sin i_i(t-T_i)}{c\sqrt{(1-e_i^2)}}\left[\cos(\nu_i+\omega_i) + e_i\cos\omega_i\right], \\ \frac{\partial C'}{\partial T_i} &= \frac{-n_i a_i\sin i_i}{c\sqrt{(1-e_i^2)}}\left[\cos(\nu_i+\omega_i) + e_i\cos\omega_i\right], \\ \frac{\partial C'}{\partial T_o} &= 1, \\ \frac{\partial C'}{\partial P} &= E,\end{aligned} \tag{10}$$

where the last two equations are different in sign from those of Irwin's paper in which they might result from typing errors. If the term $\Delta C'$ of the left hand side in Equation (9) is approximated by

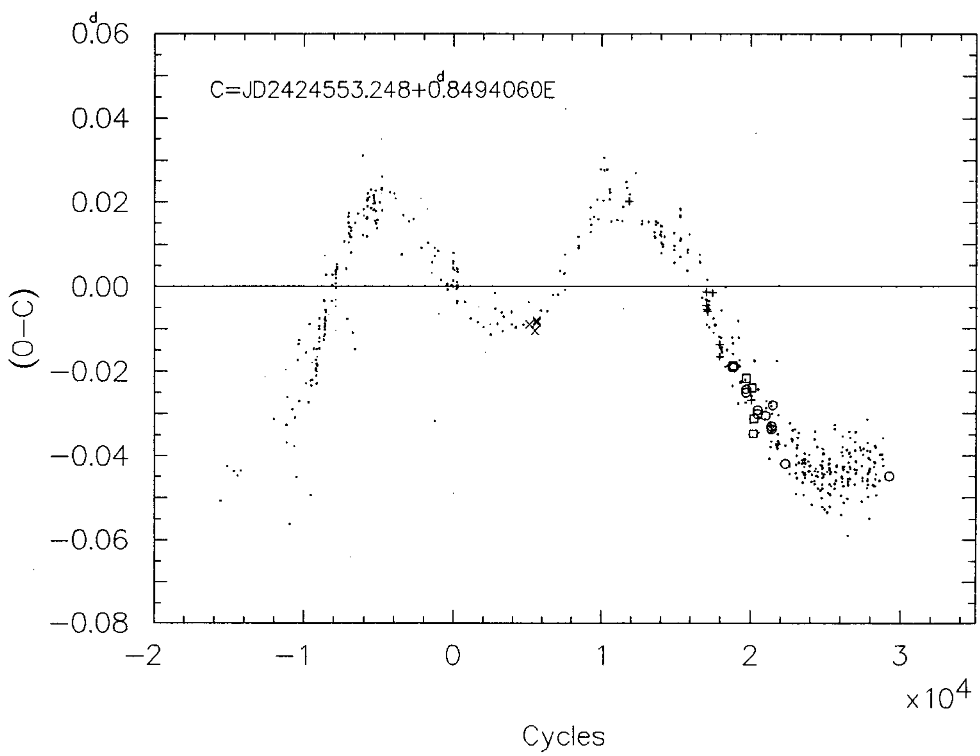

Figure 1. The $(O - C)$ diagram of RT Per constructed with ephemeris (12) given by Panchatsaram (1981). See the text for the explanations for symbols used in the figure.

$$\Delta C' = O - C'_o, \tag{11}$$

where O is the observed minima and C'_o is the one calculated with the initially estimated twelve elements, the difference $\Delta C'$ must be due to the inaccurately estimated twelve elements. Thus the problem is to obtain the corrections of the elements in Equation (9). To do this we used the Levenberg-Marquardt method (Press *et al.* 1989), a least-squares method for nonlinear equations such as Equation (9).

3. PERIOD STUDY

The published times of minima available to us have been collected from literatures. Most minima were listed in Mancuso & Milano (1975), Mancuso *et al.* (1977) and Szafraniec (1976). The recent timings after their study were listed in Table I. With all together 626 times of minima an $O - C$ diagram of RT Per is drawn in Figure 1, using the light elements given by Panchatsaram (1981) as :

$$C = JDHel\, 2424553.248 + 0^d.8494060E \tag{12}$$

In Figure 1 the small dots, plus signs, and open circles represent the visual, photographic, and photoelectric minima for primary eclipses, respectively. The crosses and squares denote photographic and photoelectric secondary minima, respectively. As seen from Figure 1, to be suprisingly, most minima are from visual ones with relatively well-defined scatter bands of about $\pm 0^d.01$. However the scatters for the photographic and photoelectric primary minima are within about $\pm 0^d.005$ and $\pm 0^d.001$, respectively. It seems to be reasonable that these scatters occurring in short time interval may be not real variations but due to the minima deduced from inaccurately or less observed data. Because these scatters could be a measure of observational errors for each minima, the errors of the visual, photographic, and photoelectric minima for primary eclipses were, according to their ammounts of scatter appeared in the $(O - C)$ diagram, assigned to $0^d.02$, $0^d.01$, and $0^d.002$, respectively, which were used as basic weights given to each minimum timings in our calculation. The secondary times of minimum lights were excluded in the calculation because they were determined from very shallow light curves.

Firstly, to see whether times of minima observed after his study follow the light-time orbits or not, the theoretical light-time curves with equations (3)-(8) using the orbital elements of the 3rd and 4th bodies determined by Panchatsaram (1981) were calculated and drawn as continuous curves in Figure 2 where the curve at the top represents the total contribution due to both the 3rd and 4th bodies, at the 2nd due to only the 3rd body, at the 3rd due to only the 4th body, and at the bottom differences of observed timings minus ones calculated with equation (1). As seen in Figure 2, the recent times of minima have been deviated from ones predicted by Panchatsaram. But such deviations implies that light-time interpretaion due to 3rd and 4th bodies suggesed by Panchatsaram is not incorrect but his orbital elements must be corrected. In this line, we iteratively applied the differential correction procedure described in section II to RT Per system where the orbital elements of Panchatsaram was used as initial values. Fortunately the calculations are quickly converged to yield the solutions as listed in Table 2. Light-time orbits corresponding to these solutions are plotted in Figure 3 where the expalnation for the Figure 3 is the same as Figure 1. In the Figure 3, the theorectical light-time curves we derived fit well to the behavior of the observed times of minimum lights.

Table 1. The observed times of minimum lights of RT Per.

JD Hel(2440000+)	Cycles	$O-C$	Type	Me	Note
2398.388100	21009.0	0.00466	I	PE	Patkos (1975)
2425.571000	21041.0	0.00660	I	VI	Baldwin (1978)
2453.603000	21074.0	0.00824	I	VI	Baldwin (1978)
2632.814000	21285.0	-0.00520	I	VI	Baldwin (1978)
2678.690000	21339.0	0.00294	I	VI	Baldwin (1978)
2683.786000	21345.0	0.00251	I	VI	Baldwin (1978)
2683.787000	21345.0	0.00351	I	VI	Baldwin (1978)
2684.634000	21346.0	0.00110	I	VI	Mallama *et al.*(1977)
2684.641000	21346.0	0.00810	I	VI	Baldwin (1978)
2689.727000	21352.0	-0.00232	I	VI	Baldwin (1978)
2723.713000	21392.0	0.00748	I	VI	Baldwin (1978)
2727.954000	21397.0	0.00145	I	VI	Baldwin (1978)
2728.801000	21398.0	-0.00095	I	VI	Baldwin (1978)
2742.394400	21414.0	0.00197	I	PE	Patkos (1976)
2743.244500	21415.0	0.00267	I	PE	Patkos (1976)
2751.738000	21425.0	0.00212	I	VI	Baldwin (1978)
2769.579000	21446.0	0.00561	I	VI	Baldwin (1978)
2811.202000	21495.0	0.00777	I	PE	Kreiner *et al.*(1980)
2842.624000	21532.0	0.00179	I	VI	AAVSO (1993)
2842.625000	21532.0	0.00279	I	VI	Mallama *et al.*(1977)
2993.815000	21710.0	-0.00128	I	VI	AAVSO (1993)
3044.799000	21770.0	0.01843	I	VI	AAVSO (1993)
3055.821000	21783.0	-0.00184	I	VI	AAVSO (1993)
3079.604000	21811.0	-0.00217	I	VI	AAVSO (1993)
3096.592000	21831.0	-0.00227	I	VI	AAVSO (1993)
3096.593000	21831.0	-0.00127	I	VI	AAVSO (1993)
3101.692000	21837.0	0.00130	I	VI	AAVSO (1993)
3113.582000	21851.0	-0.00037	I	VI	AAVSO (1993)
3123.781000	21863.0	0.00577	I	VI	AAVSO (1993)
3130.570000	21871.0	-0.00047	I	VI	AAVSO (1993)
3175.588000	21924.0	-0.00093	I	VI	AAVSO (1993)
3516.195000	22325.0	-0.00529	I	PE	Sanwal & Chaubey (1981)
3519.597000	22329.0	-0.00091	I	VI	AAVSO (1993)
3755.727000	22607.0	-0.00547	I	VI	AAVSO (1993)
3778.659000	22634.0	-0.00741	I	VI	AAVSO (1993)
3784.617000	22641.0	0.00476	I	VI	AAVSO (1993)
3899.283000	22776.0	0.00110	I	VI	Mikulase (1982)
3927.313000	22809.0	0.00074	I	VI	BBSAG (1979a)
3932.412000	22815.0	0.00331	I	VI	BBSAG (1979a)

Table 1. (Continued)

JD Hel(2440000+)	Cycles	$O-C$	Type	Me	Note
4078.510000	22987.0	0.00366	I	VI	BBSAG (1979b)
4110.777000	23025.0	-0.00672	I	VI	AAVSO (1993)
4133.706000	23052.0	-0.01165	I	VI	AAVSO (1993)
4133.713000	23052.0	-0.00465	I	VI	AAVSO (1993)
4173.639000	23099.0	-0.00069	I	VI	Mikulase (1982)
4175.336000	23101.0	-0.00249	I	VI	BBSAG (1979c)
4190.620000	23119.0	-0.00778	I	VI	AAVSO (1993)
4203.369000	23134.0	0.00014	I	VI	BBSAG (1979c)
4218.660000	23152.0	0.00186	I	VI	AAVSO (1993)
4274.722000	23218.0	0.00313	I	VI	AAVSO (1993)
4291.703000	23238.0	-0.00397	I	VI	AAVSO (1993)
4472.627000	23451.0	-0.00321	I	VI	BBSAG (1980a)
4488.765000	23470.0	-0.00390	I	VI	AAVSO (1993)
4505.751000	23490.0	-0.00600	I	VI	AAVSO (1993)
4506.598000	23491.0	-0.00841	I	VI	BBSAG (1980b)
4564.363000	23559.0	-0.00294	I	VI	BBSAG (1980c)
4574.556000	23571.0	-0.00280	I	VI	Mikulase (1982)
4575.397000	23572.0	-0.01120	I	VI	BBSAG (1981a)
4575.402000	23572.0	-0.00620	I	VI	BBSAG (1981a)
4603.432000	23605.0	-0.00656	I	VI	Mikulase (1982)
4603.435000	23605.0	-0.00356	I	VI	BBSAG (1981a)
4607.681000	23610.0	-0.00459	I	VI	AAVSO (1993)
4629.764000	23636.0	-0.00612	I	VI	AAVSO (1993)
4632.317000	23639.0	-0.00133	I	VI	Mikulase (1982)
4637.412000	23645.0	-0.00276	I	VI	BBSAG (1981b)
4642.504000	23651.0	-0.00719	I	VI	BBSAG (1981b)
4649.300000	23659.0	-0.00643	I	VI	BBSAG (1981b)
4660.345000	23672.0	-0.00369	I	VI	BBSAG (1981b)
4846.363000	23891.0	-0.00537	I	VI	BBSAG (1981c)
4849.763000	23895.0	-0.00299	I	VI	AAVSO (1993)
4900.729000	23955.0	-0.00128	I	VI	AAVSO (1993)
4912.618000	23969.0	-0.00395	I	VI	AAVSO (1993)
4919.412000	23977.0	-0.00519	I	VI	BBSAG (1981d)
4958.486000	24023.0	-0.00381	I	VI	BBSAG (1982a)
5027.284000	24104.0	-0.00761	I	VI	BBSAG (1982b)
5032.375000	24110.0	-0.01304	I	VI	BBSAG (1982b)
5038.328000	24117.0	-0.00587	I	VI	BBSAG (1982b)
5200.563000	24308.0	-0.00721	I	VI	Mikulase (1985)
5200.573000	24308.0	0.00279	I	VI	Mikulase (1985)

Table 1. (Continued)

JD Hel(2440000+)	Cycles	$O-C$	Type	Me	Note
5212.454000	24322.0	-0.00788	I	VI	BBSAG (1982c)
5212.457000	24322.0	-0.00488	I	VI	BBSAG (1982c)
5221.810000	24333.0	0.00467	I	VI	AAVSO (1993)
5263.422000	24382.0	-0.00417	I	VI	BBSAG (1983a)
5269.366000	24389.0	-0.00601	I	VI	BBSAG (1982d)
5342.415000	24475.0	-0.00583	I	VI	Mikulase (1985)
5342.422000	24475.0	0.00117	I	VI	BBSAG (1983a)
5388.280000	24529.0	-0.00869	I	VI	BBSAG (1983b)
5388.284000	24529.0	-0.00469	I	VI	BBSAG (1983b)
5399.325000	24542.0	-0.00596	I	VI	BBSAG (1983b)
5533.528000	24700.0	-0.00893	I	VI	BBSAG (1983c)
5533.534000	24700.0	-0.00293	I	VI	BBSAG (1983c)
5602.328000	24781.0	-0.01073	I	VI	BBSAG (1983d)
5613.375000	24794.0	-0.00599	I	VI	BBSAG (1983e)
5613.378000	24794.0	-0.00299	I	VI	BBSAG (1983e)
5619.322000	24801.0	-0.00482	I	VI	BBSAG (1983e)
5619.323000	24801.0	-0.00382	I	VI	BBSAG (1983e)
5619.328000	24801.0	0.00118	I	VI	BBSAG (1983e)
5622.717000	24805.0	-0.00744	I	VI	AAVSO (1993)
5636.313000	24821.0	-0.00192	I	VI	Mikulase (1985)
5641.403000	24827.0	-0.00835	I	VI	BBSAG (1983e)
5641.408000	24827.0	-0.00335	I	VI	BBSAG (1983e)
5646.495000	24833.0	-0.01278	I	VI	BBSAG (1983e)
5697.473000	24893.0	0.00092	I	VI	BBSAG (1984a)
5698.316000	24894.0	-0.00548	I	VI	BBSAG (1984a)
5670.291000	24861.0	-0.00012	I	VI	Mikulase (1985)
5724.653000	24925.0	-0.00003	I	VI	AAVSO (1993)
5743.335000	24947.0	-0.00494	I	VI	BBSAG (1984b)
5777.303000	24987.0	-0.01314	I	VI	BBSAG (1984b)
5911.518000	25145.0	-0.00411	I	VI	BBSAG (1984c)
5916.605000	25151.0	-0.01354	I	VI	BBSAG (1984c)
5940.393000	25179.0	-0.00888	I	VI	BBSAG (1984d)
5974.376000	25219.0	-0.00207	I	VI	BBSAG (1984d)
6028.737000	25283.0	-0.00299	I	VI	AAVSO (1993)
6028.738000	25283.0	-0.00199	I	VI	AAVSO (1993)
6028.739000	25283.0	-0.00099	I	VI	AAVSO (1993)
6029.586000	25284.0	-0.00339	I	VI	AAVSO (1993)
6034.679000	25290.0	-0.00682	I	VI	AAVSO (1993)
6054.213000	25313.0	-0.00913	I	VI	BBSAG (1984e)
6057.619000	25317.0	-0.00075	I	VI	AAVSO (1993)

Table 1. (Continued)

JD Hel(2440000+)	Cycles	$O-C$	Type	Me	Note
6062.704000	25323.0	-0.01218	I	VI	AAVSO (1993)
6065.260000	25326.0	-0.00440	I	VI	BBSAG (1984e)
6092.436000	25358.0	-0.00935	I	VI	BBSAG (1992)
6110.279000	25379.0	-0.00386	I	VI	BBSAG (1992)
6121.318000	25392.0	-0.00712	I	VI	BBSAG (1992)
6327.723000	25635.0	-0.00751	I	VI	AAVSO (1993)
6329.423000	25637.0	-0.00632	I	VI	Mikulase (1986)
6329.436000	25637.0	0.00668	I	VI	Mikulase (1986)
6329.437000	25637.0	0.00768	I	VI	Mikulase (1986)
6333.670000	25642.0	-0.00635	I	VI	AAVSO (1993)
6346.414000	25657.0	-0.00342	I	VI	BBSAG (1992)
6403.322000	25724.0	-0.00555	I	VI	BBSAG (1992)
6411.818000	25734.0	-0.00360	I	VI	AAVSO (1993)
6429.657000	25755.0	-0.00210	I	VI	AAVSO (1993)
6441.549000	25769.0	-0.00177	I	VI	AAVSO (1993)
6446.645000	25775.0	-0.00220	I	VI	AAVSO (1993)
6682.773000	26053.0	-0.00876	I	VI	AAVSO (1993)
6706.552000	26081.0	-0.01310	I	VI	BBSAG (1992)
6717.609000	26094.0	0.00164	I	VI	Mikulase (1988)
6723.549000	26101.0	-0.00419	I	VI	AAVSO (1993)
6734.591000	26114.0	-0.00446	I	VI	AAVSO (1993)
6747.332000	26129.0	-0.00453	I	VI	BBSAG (1987a)
6753.277000	26136.0	-0.00537	I	VI	BBSAG (1987a)
6768.570000	26154.0	-0.00165	I	VI	AAVSO (1993)
6769.422000	26155.0	0.00094	I	VI	Mikulase (1988)
6769.424000	26155.0	0.00294	I	VI	Mikulase (1988)
6770.276000	26156.0	0.00554	I	VI	BBSAG (1987b)
6779.612000	26167.0	-0.00192	I	VI	AAVSO (1993)
6820.383000	26215.0	-0.00235	I	VI	BBSAG (1987b)
6826.325000	26222.0	-0.00619	I	VI	Mikulase (1992)
6843.309000	26242.0	-0.01029	I	VI	BBSAG (1987b)
6858.607000	26260.0	-0.00157	I	VI	AAVSO (1993)
7011.504000	26440.0	0.00254	I	VI	BBSAG (1992)
7028.487000	26460.0	-0.00255	I	VI	Mikulase (1992)
7037.824000	26471.0	-0.00901	I	VI	AAVSO (1993)
7054.816000	26491.0	-0.00511	I	VI	AAVSO (1993)
7057.365000	26494.0	-0.00432	I	VI	BBSAG (1992)
7068.394000	26507.0	-0.01758	I	VI	BBSAG (1992)

Table 1. (Continued)

JD Hel(2440000+)	Cycles	$O-C$	Type	Me	Note
7083.696000	26525.0	-0.00487	I	VI	AAVSO (1993)
7085.394000	26527.0	-0.00568	I	VI	BBSAG (1992)
7111.731000	26558.0	-0.00023	I	VI	AAVSO (1993)
7112.578000	26559.0	-0.00264	I	VI	AAVSO (1993)
7153.354000	26607.0	0.00193	I	VI	BBSAG (1992)
7159.305000	26614.0	0.00709	I	VI	BBSAG (1986)
7169.493000	26626.0	0.00223	I	VI	BBSAG (1989a)
7170.339000	26627.0	-0.00117	I	VI	BBSAG (1989a)
7184.776000	26644.0	-0.00406	I	VI	AAVSO (1993)
7204.315000	26667.0	-0.00137	I	VI	BBSAG (1988b)
7209.411000	26673.0	-0.00180	I	VI	BBSAG (1988a)
7210.268000	26674.0	0.00580	I	VI	BBSAG (1988b)
7232.348000	26700.0	0.00127	I	VI	BBSAG (1988b)
7439.600000	26944.0	-0.00153	I	VI	AAVSO (1993)
7461.681000	26970.0	-0.00505	I	VI	AAVSO (1993)
7480.378000	26992.0	0.00504	I	VI	BBSAG (1989a)
7506.712000	27023.0	0.00749	I	VI	AAVSO (1993)
7525.389000	27045.0	-0.00242	I	VI	BBSAG (1989b)
7525.392000	27045.0	0.00058	I	VI	BBSAG (1989a)
7531.335000	27052.0	-0.00225	I	VI	BBSAG (1989b)
7540.681000	27063.0	0.00029	I	VI	AAVSO (1993)
7565.312000	27092.0	-0.00145	I	VI	BBSAG (1989b)
7744.542000	27303.0	0.00412	I	VI	BBSAG (1989c)
7810.795000	27381.0	0.00353	I	VI	AAVSO (1993)
7858.358000	27437.0	-0.00014	I	VI	BBSAG (1990a)
7863.446000	27443.0	-0.00857	I	VI	BBSAG (1990c)
7891.482000	27476.0	-0.00293	I	VI	BBSAG (1990b)
7897.427000	27483.0	-0.00377	I	VI	BBSAG (1990b)
7932.259000	27524.0	0.00263	I	VI	BBSAG (1990b)
7954.345000	27550.0	0.00410	I	VI	BBSAG (1990b)
8149.699000	27780.0	-0.00502	I	VI	AAVSO (1993)
8160.743000	27793.0	-0.00329	I	VI	AAVSO (1993)
8162.449000	27795.0	0.00390	I	VI	BBSAG (1990d)
8174.348000	27809.0	0.01124	I	VI	BBSAG (1990d)
8178.581000	27814.0	-0.00279	I	VI	AAVSO (1993)
8179.439000	27815.0	0.00581	I	VI	BBSAG (1990d)
8194.721000	27833.0	-0.00148	I	VI	AAVSO (1993)
8202.363000	27842.0	-0.00413	I	VI	BBSAG (1991a)
8211.709000	27853.0	-0.00158	I	VI	AAVSO (1993)

Table 1. (Continued)

JD Hel(2440000+)	Cycles	$O-C$	Type	Me	Note
8212.553000	27854.0	-0.00698	I	VI	AAVSO (1993)
8245.689000	27893.0	0.00222	I	VI	AAVSO (1993)
8251.632000	27900.0	-0.00061	I	VI	AAVSO (1993)
8292.392000	27948.0	-0.01205	I	VI	BBSAG (1991a)
8292.403000	27948.0	-0.00105	I	VI	BBSAG (1991a)
8296.651000	27953.0	-0.00007	I	VI	AAVSO (1993)
8330.627000	27993.0	-0.00027	I	VI	AAVSO (1993)
8332.328000	27995.0	0.00192	I	VI	BBSAG (1991a)
8332.330000	27995.0	0.00392	I	VI	BBSAG (1991a)
8472.474000	28160.0	-0.00388	I	VI	BBSAG (1991b)
8538.732000	28238.0	0.00053	I	VI	AAVSO (1993)
8540.432000	28240.0	0.00172	I	VI	BBSAG (1991c)
8552.318000	28254.0	-0.00394	I	VI	BBSAG (1991c)
8601.589000	28312.0	0.00157	I	VI	AAVSO (1993)
8601.593000	28312.0	0.00557	I	VI	AAVSO (1993)
8619.421000	28333.0	-0.00393	I	VI	BBSAG (1992)
8620.272000	28334.0	-0.00234	I	VI	BBSAG (1992)
8625.376000	28340.0	0.00523	I	VI	BBSAG (1992)
8653.400000	28373.0	-0.00113	I	VI	BBSAG (1992)
8659.341000	28380.0	-0.00596	I	VI	BBSAG (1992)
8838.569000	28591.0	-0.00240	I	VI	BBSAG (1992)
8890.389000	28652.0	0.00391	I	VI	BBSAG (1992)
9003.360000	28785.0	0.00405	I	VI	BBSAG (1992)
9020.347000	28805.0	0.00296	I	VI	BBSAG (1992)
9043.286000	28832.0	0.00802	I	VI	BBSAG (1992)
9065.364000	28858.0	0.00150	I	VI	BBSAG (1992)
9397.479200	29249.0	-0.00062	I	PE	Cracow data

4. MASSES OF THE 3RD AND 4TH BODIES

According to the Panchatsaram (1981), the mass functions of the 3rd and 4th bodies are, respectively,

$$f(m)_3 = \frac{(a_3 \sin i_3)^3}{P_3^2} = \frac{m_{12} \sin^3 i_3}{\alpha_3(1+\alpha_3)^2} \tag{13}$$

$$f(m)_4 = \frac{(a_4 \sin i_4)^3}{P_4^2} = \frac{m_{123} \sin^3 i_4}{\alpha_4(1+\alpha_4)^2} \tag{14}$$

where $\alpha_3 = m_{12}/m_3$, $\alpha_4 = m_{123}/m_4, m_{12} = m_1 + m_2, m_{123} = m_1 + m_2 + m_3$. Rearranging Equations (13) and (14) for m_3 and m_4, then we have

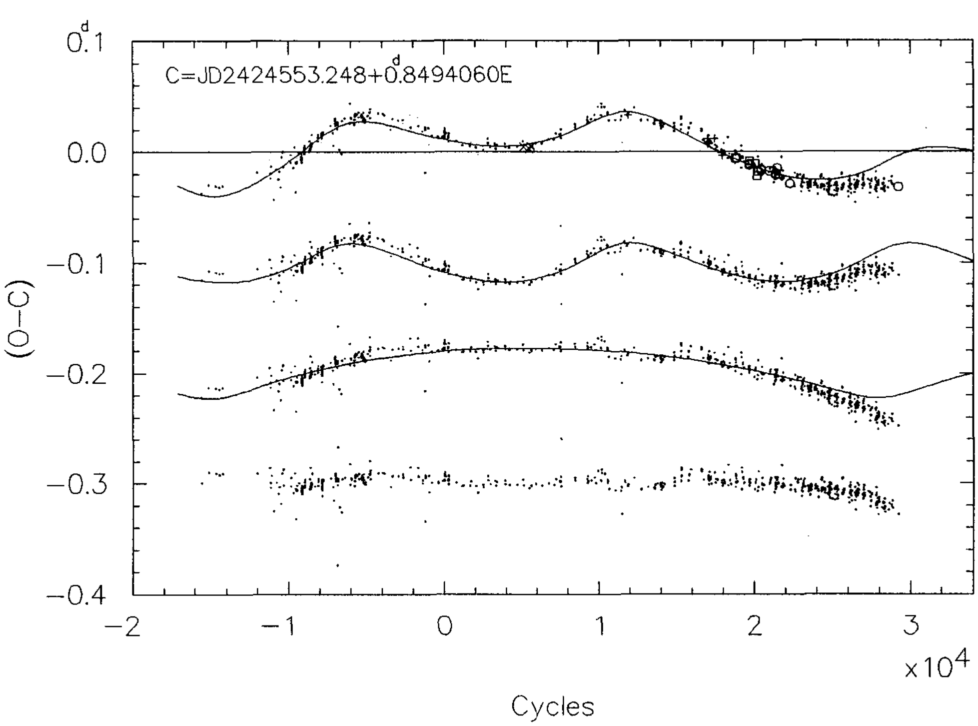

Figure 2. The $(O - C)$ diagram of RT Per constructed with ephemeris (12) given by Panchatsaram (1981). The curve at the top represents the total contribution due to both the 3rd and 4th bodies, at the 2nd due to only the 3rd body, at the 3rd due to only the 4th body, and at the bottom differences of observed timings minus ones calculated with equation (1).

$$C_3 m_3^3 - m_3^2 - 2m_{12}m_3 - m_{12}^2 = 0 \tag{15}$$

$$C_4 m_4^3 - m_4^2 - 2m_{123}m_4 - m_{123}^2 = 0 \tag{16}$$

where

$$C_3 = \frac{\sin i_3^3}{f(m)_3},\ C_4 = \frac{\sin i_4^3}{f(m)_4} \tag{17}$$

By solving Equations (15) and (16) for the $i_3 = i_4 = 90$ °, 60 °, and 30 °by a Newton-Rapson method, m_3 and m_4 corresponding to each inclinations are obtained. In the calculations $m_{12} = 2.15m_{\odot}$ for RT Per derived by Mancuso & Milano (1975) is used.

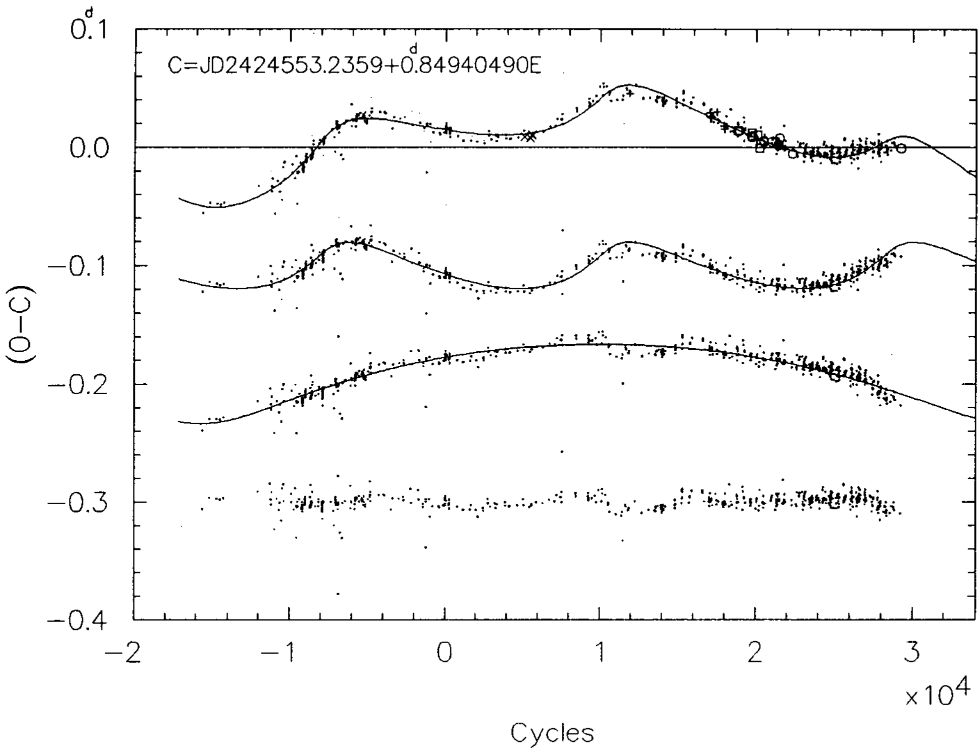

Figure 3. The $(O - C)$ diagram of U Cep constructed with the linear term of our light elements. Explanations of the curves are the same as Figure 2. The theorectical light-time curves we derived fits well to the behavior of the observed times of minimum lights.

And the arithmetic mean value of masses of m_{123} calculated for the three orbital inclnations is used in Equation (16). The results are listed in Table 2. The derived masses for m_3 and m_4 are about three times smaller than the mass of m_{12}, which are hardly detectable in visible lights.

5. DISCUSSION AND CONCLUSION

In this paper, we collected and analyzed all the times of minima available to us, found that the $(O - C)$ excursions of the recent timings for the system have followed the same trends with those predicted by the TF model, and improved the orbital elements for the light-time orbits of the third and fourth bodies suggested in the TF model using a differential corrections method. The period changes are supposed to occur on account of the fact that

Table 2. Light-time orbits of the 3rd and 4th bodies.

	Panchatsaram (1981)	Kim (This paper)
T_0	2424553.2480	2424553.2359(28)
P	0.8494060	0.84940490(14)
$a_{12} \sin i_{12}$(km)	.	5.42×10^8(1.01)
ω_{12} (°)	60.0	38.3(11.3)
e_{12}	0.3	0.47(0.64)
P_{12} (year)	41.86	42.22(0.46)
T_{12} (day)	2418607.0	2418033.5(248.1)
$a_{123} \sin i_{123}$ (km)	.	8.66×10^8(1.09)
ω_{123} (°)	280.0	273.1(7.3)
e_{123}	0.6	0.520(0.19)
P_{123} (day)	100.0	121.03(2.12)
T_{123} (day)	2412098.0	2411411.0(533.4)
K_{12} (day)	0.018	0.019456
K_{123} (day)	0.023	0.033413
$f(m)_3(m_\odot)$	0.017	0.027
$\bar{m}_3(m_\odot)$	0.74	0.89
$f(m)_4(m_\odot)$	0.006	0.013
$\bar{m}_4(m_\odot)$	0.59	0.82

the eclipsing pair includes two more additional companions. This eclipsing system moves in an elliptic orbit about mass center of the triple system with a period of about 42.2 yr, while the mass center of the triplet itself is in light-time orbit about the center of mass of the quadruple system with a period of 120 yr.

Therefore, 3rd and 4th bodies interpretation for the period variations of RT Per is possible, but must be confirmed by other observations (e. g., astrometric and/or spectroscopic ones as well as timings of minimum lights).

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