Transient Forces on Pipe Bends by the Propagation of Pressure Wave

Woo, Hyoseop* · Papadakis, C.N.** · Kim, Won***

ABSTRACT : External forces acting on a pipe bend change when a transient pressure wave propagates through the bend. Analytical expressions are derived to compute the changes of these forces which depend mainly on static pressure rather than fluid momentum. This analysis reveals that the change of the vertical component of the force acting on a pipe bend with an angle larger than 90 ° may reverse in direction during the passage of a pressure wave through the bend.

1. Introduction

The steady state flow of a liquid through a pipe bend results in external forces which interact with the system. These forces can be estimated by applying the principle of linear momentum, according to which the sum of the resultant forces acting on a control volume is equal to the sum of the time rate of change of increase of linear momentum within the control volume plus the net efflux of linear momentum from the control volume. The linear-momentum equation can be expressed as:

$$\Sigma\bar{F} = \frac{\partial}{\partial t}\int_{cv} \rho\,\bar{V}\,d\hat{V} + \int_{cs} \rho\,\bar{V}(\bar{V}\cdot d\bar{A}) \tag{1}$$

where $\bar{F}$ is the external force acting on the system; $\bar{V}$ is the velocity vector; ρ is the fluid density; $\bar{A}$ is the vector representing the cross-sectional area of the control volume; $\hat{V}$ is the volume of the fluid in the control volume; and t is the time.

---

* Division Chief, Water Resources Engineering Division, Korea Institute of Construction Technology, Socho-gu Umyeon-dong 142, Seoul 137-140, Korea

** Dean and Geier Professor of Engrg. Education, College of Engrg., The University of Cincinnati, Cincinnati, Ohio 45221, USA.

*** Researcher, Water Resources Engineering Division, Korea Institute of Construction Technology, Socho-gu Umyeon-dong 142, Seoul 137-140, Korea

For steady state conditions, the components of the external forces in the x–and y–directions, F_{xo} and F_{yo}, can be obtained by applying the momentum equation to the control volume shown in Fig. 1 which encompasses the entire pipe bend of length L and radius of curvature R:

$$F_{xo}=A(p_1-p_2\cos\theta_o)+\rho V_0^2 A(1-\cos\theta_o) \tag{2}$$

$$F_{yo}=p_2 A\sin\theta_o+\rho V_0^2 A\sin\theta_o+W_o \tag{3}$$

where V_o is the steady–state velocity; p_1 and p_2, respectively, are the pressures upstream and downstream of the bend; θ_o is the angle of the pipe bend; and W_o is the weight of the fluid in the control volume of the bend.

2. Transient Forces

The case where a pressure wave propagates through a pipe bend is examined. The wave front is within the bend at a section identified by the angle θ in the direction of the wave propagation as shown in Fig. 2. Only propagation of wharf wave with a vertical wave front is considered in this study. Application of the linear–momentum Eq. (1) to the control volume of the bend $\hat{V}=\hat{V}_1+\hat{V}_2$ in the x–direction yields:

$$p_1A-(p_2+\Delta p)A\cos\theta_o-F_x=$$
$$\frac{\partial}{\partial t}\left[\int_{\hat{V}_1}\rho V_o\cos\theta_1 d\hat{V}+\int_{\hat{V}_2}\rho(V_o+\Delta V)\cos\theta_2 d\hat{V}\right]$$
$$-\rho V_0^2 A+\rho(V_o+\Delta V)^2 A\cos\theta_o \tag{4}$$

where Δp is the change in pressure wave propagates through a pipe caused by a change ΔV in flow velocity; and θ_1, θ_2 are the elementary angles in the control volume 1 and 2, respectively. Because of the small length of the bend and the negligible losses involved, Δp and ΔV can be assumed constant across the bend at a certain instant. The following expressions can be readily obtained from Fig. 2:

$$d\hat{V}\frac{d\theta}{\theta_o} \tag{5}$$

$$\theta_o=\frac{L}{R} \tag{6}$$

$$\frac{d\theta}{dt}=\frac{(a-V_o)}{R} \tag{7}$$

where a is the speed of propagation of a pressure wave. The first term of the right hand side of Eq. (4) which represents the time rate of change of increase of linear momentum within the control volume can then be expressed as:

$$\frac{\partial}{\partial t}\left[\int_0^{\Theta_o-\Theta} \rho V_o \cos\theta_1 \frac{AL}{\theta_o}\, d\theta_1\right]+\frac{\partial}{\partial t}\left[\int_{\Theta-\Theta_o}^{\Theta_o} \rho (V_o+\Delta V)\cos\theta_2 \frac{AL}{\theta_o}\, d\theta_2\right]$$

$$=\left[\rho \Delta V\frac{AL}{\theta_o}\cos(\theta_o-\theta)\right]\frac{d\theta}{dt}=\rho \Delta V(a-V_o)A\cos(\theta_o-\theta) \tag{8}$$

Then, the x-component of the external forces exerted on the pipe bend is obtained from Eq. (4):

$$F_x=p_1A-(p_2+\Delta p)A\cos\theta_0-\rho\Delta V(a-V_o)A\cos(\theta_o-\theta)$$
$$+\rho V_0^2A-\rho(V_2+\Delta V)^2A\cos\theta_o \tag{9}$$

Through a similar derivation the y-component of the external forces, F_y, can be obtained as:

$$F_y=(p_2+\Delta p)A\sin\theta_o+\rho\Delta V(a-V_o)A\sin(\theta_o-\theta)$$
$$+\rho(V_o+\Delta V)^2A\sin\theta_o+W \tag{10}$$

When a pressure wave propagates through a pipe at speed a, the change in pressure Δp at any section of the pipe caused by a change ΔV in flow velocity at the same pipe section can be expressed by Joukowsky's equation:

$$\Delta p=-\rho\Delta V(a+V_o+\Delta V) \tag{11}$$

The wave speed can be eliminated from Eqs. (9) and (10) by using Eq. (11). The change in the external forces caused by the pressure wave at the pipe bend can then be obtained by subtracting Eq. (2) from Eq. (9) and Eq. (3) from Eq. (10):

$$\Delta F_x=A[\Delta p+\rho\Delta V(2V_o+\Delta V)]\,[\cos(\theta_o-\theta)-\cos\theta_o] \tag{12}$$
$$\Delta F_x=A[\Delta p+\rho\Delta V(2V_o+\Delta V)]\,[\sin\theta_o-\sin(\theta_o-\theta)] \tag{13}$$

In obtaining these equations it was assumed that the weight of the control volume remains constant ($W=W_o$) despite minor changes in fluid density and pipe cross-sectional area and length caused by the pressure wave.

3. Case Study

A 0.9m diameter pipe forms a 90° bend on the vertical plane with a radius of curvature $R=1.05$ m. Water flows through the pipe with a velocity $V_o=4.45$ m/sec and it is assumed that head losses at the bend are negligible. The pressures upstream and downstream of the bend are $p_1=34.5$ kN/m^2 and $p_2=24.2$ kN/m^2, respectively. The steady state external forces exerted on the bend are equal to $F_{xo}=34.5$ kN and $F_{yo}=38.3$ kN, calculated using Eqs. (2) and (3).

At time $t=0$ the flow velocity at section 2 of the bend is reduced by 1.11m/sec by partially closing a valve downstream from the bend at an earlier time. For a speed of wave propagation of $a=1,000$ m/sec, the corresponding pressure rise at the bend is equal to $\Delta p=1,110$ kN/m^2. At that time $\theta=0$ and the change in the external forces is $\Delta F_x=\Delta F_y=0$. At time $t=0.00083$ sec the wave front reached the middle of the bend($\theta=45°$) and Eqs. (12) and (13) were used to obtain $\Delta F_x=495$ kN and $\Delta F_y=205$ kN. At time $t=0.00166$ sec the wave front reached section 1($\theta=\theta_o=90°$) and Eqs. (12) and (13) yield $F_x=F_y=700$ kN.

4. Discussions

Substituting the expression that $\Delta p \cong -\rho a \Delta V$ both into Eqs. (12) and (13) reduces the expressions in the first bracket of those equations to :

$$\rho \Delta V(-a+2V_o+\Delta V) \tag{14}$$

Since the speed of the wave propagation in water is about 1,000 m/s which is far greater than the value of$(2V_o+V)$ which normally is less than 10m/s, the momentum terms both are negligible compared to the static pressure terms in Eqs. (12) and (13). Therefore, changes in external forces primarily depend on static pressure rather than momentum and Eqs. (12) and (13) reduce to:

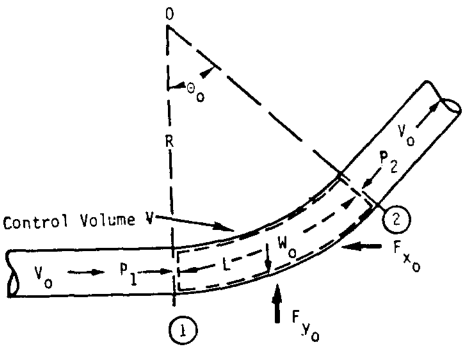

Fig. 1. Steady State Forces On A Pipe Bend

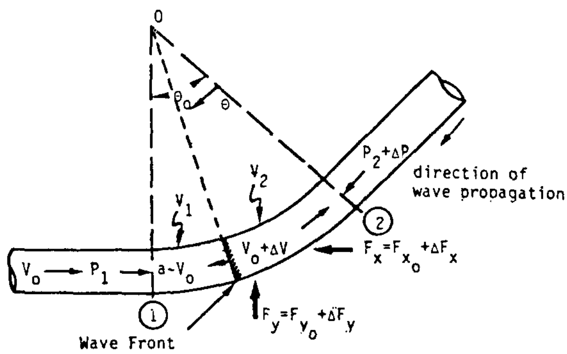

Fig. 2. Transient Forces Caused By A Pressure Wave Passing Through A Pipe Bend

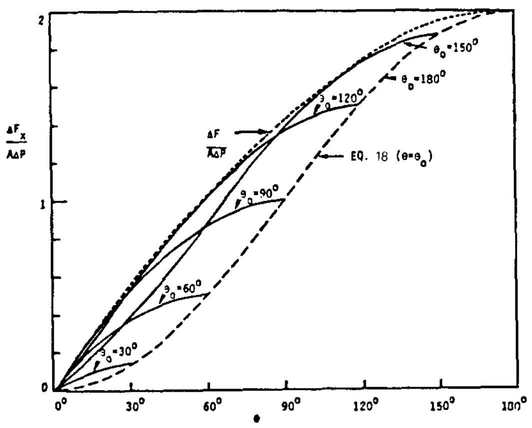

Fig. 3. Change in External Forces ΔF and ΔF versus the Wave Front Angle θ for Various Bend Angles θ_0.

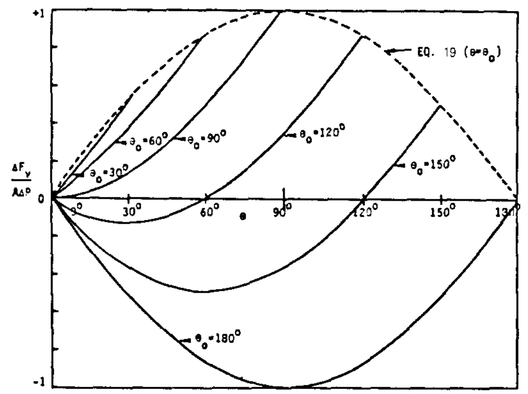

Fig. 4. Change in External Force ΔF_y versus the Wave Front Angle θ for Various Bend Angles θ_0.

$$\Delta F_x = A\,\Delta p[\cos(\theta_0-\theta)-\cos\theta_0] \tag{15}$$

$$\Delta F_y = A\,\Delta p[\cos\theta_0-\cos(\theta_0-\theta)] \tag{16}$$

The wave front angle θ identifies the position of the wave front within a pipe bend of curvature θ_0. Therefore, in all cases $\theta \leq \theta_0$.

Fig. 3 is a plot of Eq. (15) and shows the change of the horizontal external force ΔF_x versus the wave front angle θ for various bend angles θ_0 ranging from 0 ° to 180 °. Similarly, Fig. 4 is a plot of Eq. (16) and shows the change of the vertical external force Fy versus the wave front angle θ for various bend angles θ_0 varying from 0 ° to 180 °. Eqs. (15) and (16) were divided by A Δp to render dimensionless the ratio used in Figs. 3 and 4. Fig. 4 indicates that the change of the vertical component ΔF_y of the external force acting on bends of curvature larger than 90 ° , reverses in direction when a pressure wave front passes through the bend.

The total change in external force exerted on the bend is

$$\Delta F=\sqrt{F_x^2+\Delta F_y^2} \tag{17a}$$

or

$$\Delta F=24\Delta p(1-\cos\theta) \tag{17b}$$

which is independent of the bend angle θ_o and is plotted in Fig. 3 as a function of the wave front angle θ.

When the pressure wave passes the bend, i.e., the wave front reaches section 1 of Fig. 2, the angle $\theta=\theta_o$ and both ΔF_x and ΔF_y attain their maximum values:

$$\Delta F_x=A\ \Delta p(1-\cos\theta_o) \tag{18}$$

$$\Delta F_y=A\ \Delta p\ \sin\theta_o \tag{19}$$

Eqs. (18) and (19) are plotted in Figs. 3 and 4, respectively. As shown in Fig. 3, Eq. (18) is identical to Eq. (15) when the bend angle θ_o is equal to 180 °.

5. Conclusions

Change in the external forces acting on the,pipe bend, when a transient pressure wave propagates through the bend, is derived using the momentum equation. This change is expressed for the x-and y-directions as Eqs. (15) and (16), respectively.

Total change in the external forces is expressed as Eq. (17).

The maximum external forces occur when the wave front passes the bend completely. Then, they are expressed in x- and y-directions as Eqs. (18) and (19), respectively.

These results are valid on the assumption that there is no interaction between the fluid and pipe material when the wave propagates through the pipe bend and changes in the pipe material due to the change in the fluid pressu re are negligible. At present, these results can be used as a first approximation for the design of pipe bend.

Acknowledgement

This study was initiated by the first two authors when the first author was in the University of Cincinnati during 1986～1988 as Research Assistant Professor. The third author helped completing this study in Korea in 1994.

Notation

The following symbols are used in this paper:

A : cross-sectional area of pipe;

a : pressure wave propagation velocity;
F : external force acting on pipe bend;
F_x, F_y : x- and y-components of external force, respectively;
F_{xo}, F_{yo} : x- and y-components of steady state external force, respectively;
L : length of pipe bend;
p_1, p_2 : upstream and downstream pressures of pipe bend, respectively;
R : radius of curvature of pipe bend;
t : time;
V : flow velocity;
V_o : steady state flow velocity;
$\hat{V}$: control volume of pipe bend;
$\hat{V}_1$, $\hat{V}_2$: control volumes upstream and downstream of the wave front;
W_o : weight of liquid in control volume;
Δ : change in the variable;
θ : angle of the position of the wave front in pipe bend;
θ_o : angle of pipe bend;
θ_1, θ_2 : elementary angles in the control volumes 1 and 2; and
ρ : density of liquid.