Prediction of Ultimate Scour Potentials in a Shallow Plunge Pool

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ABSTRACT : A plunge pool is often employed as an energy-dissipating device at the end of a spillway or a pipe culvert. A jet from spillways or pipes frequently generates a scour hole which threaten the stability of the hydraulic structure. Existing scour prediction formulas of plunge pool of spillways or pipe culverts give a wide range of scour depths, and it is, therefore, difficult to accurately predict those scour depths. In this study, a new experimental method and new scour prediction formulas under submerged circular jet for large bed materials with shallow tailwater depths were developed. A major variable, which was not used in previous scour prediction equations, was the ratio of jet size to bed material size. In this study, jet momentum acting on a bed particle and jet diffustion theory were employed to derive scour prediction formulas. Four theoretical formulas were suggested for the two regions of jet diffusion, i.e., the region of flow establishment and the region of established flow. The semi-theoretically developed scour prediction formulas showed close agreement with laboratory experiments performed on movable bed made of large spherical particles.

1. Introduction

A plunge basin is often employed as an energy dissipating device at the end of a spillway or pipe culvert. The jet from the pipe culvert or spillway frequently generates scour in the plunge pool and this scour creates risk to the structural stability. Failure to provide enough plunge pool depth can result in a serious sedimentation problem in the channel downstream from the plunge pool. Due to the complexity of the scour mechanism, most previous studies on scour in the plunge pool were performed experimentally. In past research the sizes of bed materials were small compared with the size of the jet. Because the size of bed material of plunge pool is usually as large as the thickness of the jet, new scour prediction formulas for large particles with shallow tailwater need to be developed. Therefore, studies on the effects of the parameters governing scour and on the developments of new experimental scour prediction formulas were carried out to aid the design of noncohesive flat bed

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under submerged circular jet issued from spill ways and pipe culverts.

2. Dimensional Analysis and Experiments

The experiments of scour under submerged circular jet were performed in a rectangular flume 3ft (W) x 2ft(H) x 23ft(L). The cohesionless bed material was simulated with spheres and hemispheres 2.6 inches in diameter. These hemispheres and spheres were fixed on a movable plate. The arrangement of hemispheres and spheres are shown in Fig. 1. A cavity was left in the layer of spheres to allow the placement of a test sphere whose diameter is 2.6 inches and specific gravity ranges from 1.145 to 3.614. Incipient motion under given characteristics of jet was observed by finding a specific gravity of test sphere which was just removed from the cavity in ten seconds.

The adopted significant variables were nozzle diameter(D_n), average flow velocity at nozzle (U_o), angle of jet impact on tailwater surface (θ), width of plunge pool (B), tailwater depth (H), density of water (ρ), kinematic viscosity of water(ν), diameter of bed material(D_s) and the submerged sphere weight($W_s = (\rho_s - \rho) g \pi D_s^3/6$).

The submerged sphere weight at critical condition can be expressed as follows:

$$W_s = f_1(D_n, D_s, U_o, H, B, X, \rho, \nu, \theta) \tag{1}$$

By selecting D_n, ρ and U_o as the repeating variables, and manipulation of jet momentum, Son et al. (1993) derived Eq. (2):

$$(\rho_s - \rho) g D_s/(\rho U_o^2) = f_2(D_s/D_n, H/D_n, X/D_n, B/D_n, 1/R_e, \theta) \tag{2}$$

From the results of experiments, Son et al. (1993) could simplify Eq. (2) as follows.

$$(\rho_s - \rho) g D_s/(\rho U_o^2) = f_3(D_s/D_n, H/D_s, \theta) \tag{3}$$

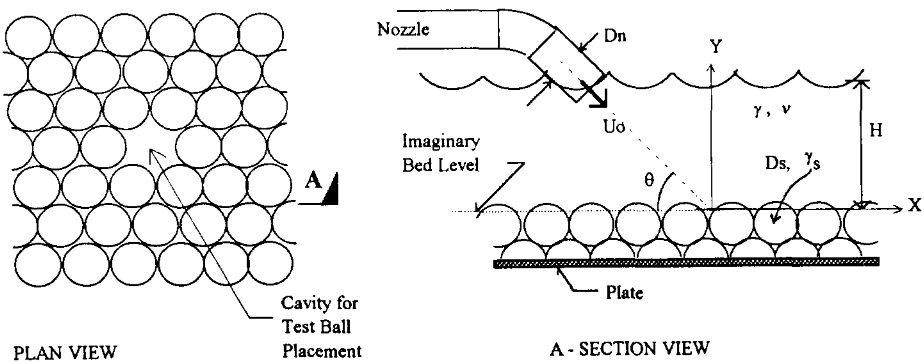

Fig.1.Spheres Alignment of Idealized Stream Bed

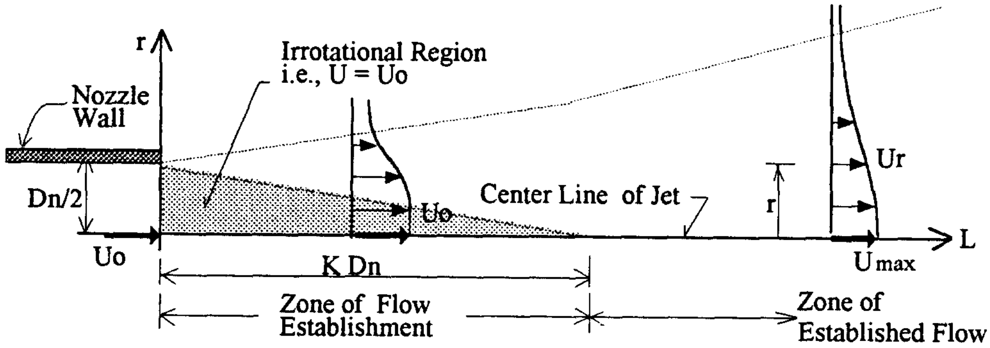

Fig. 2. Definition of Flow Region and Velocity Distribution of Jet Diffusion

The dependent variable of Eq. (3) is the ratio of the submerged sphere weight($W_s=(\rho_s-\rho)g\pi D_s^3/6$) to the momentum of jet($Mos=\rho\pi D_s^2U_o^2/4$), which has uniform velocity Uo, acting on a projected area of sphere normal to a jet direction. Therefore, the dependent variable of Eq. (3) can be expressed as W_s/Mos and will be called transformed scour potential. W_s/Mos is also the inverse of the squared densimetric Froude number, which was used as a scour prediction parameter by Rajaratnam and Beltaos (1977), Rajaratnam (1981, 1982), and Maynord et al. (1989).

3. Theory Development

Albertson et al. (1950) defined two flow regimes in jet diffusion area from a submerged circular jet as shown in Fig. 2. Velocity distribution and maximum jet velocity were expressed in different ways for different regions of flow, i.e., establishment region and established region. The border of these two flow regions is distance(L) far from the nozzle along the jet axis. L is often expressed as the multiplication of K and the nozzle diameter(D_n). K is sometimes called the diffusion constant. The maximum velocity decreases as jet travels downward in the region of established flow but the velocity distributions are geometrically similar.

Rao et al. (1965) found that 95 percent of jet energy at a nozzle exit was dissipated at a distance of 48.5 nozzle diameters from the exit. Hartung and Hausler (1973) described that the jet energy would be diminished rapidly when jet travel length was more than 20 times the jet diameter.

The velocity Umax at distance L from the nozzle exit and velocity U_r at radial distance r from the center line of jet were expressed as Eqs.(4)–(6) (Albertson et al., 1950).

for the zone of the established flow ($L\geq K\ D_n$):

$$U_{max}/U_o=\frac{1}{2C_1}\ \frac{D_n}{L}=K\frac{D_n}{L} \tag{4}$$

$$U_r/U_{max}=\frac{1}{2C_1}\ \frac{D_n}{L}\ \exp\left(-\frac{1}{C_1^2}\left(\frac{r}{L}\right)^2\right)=K\frac{D_n}{L}\ \exp\left(-2\left(K\frac{r}{L}\right)^2\right) \tag{5}$$

for the zone of the flow establishment($L \leq K\ D_n$):

$$\log_{10}\frac{U_r}{U_o} = \frac{\log_{10}e}{2C_1^2}\left(C_1 + \frac{r - D_n/2}{L}\right)^2 \tag{6}$$

where $C_1 = 0.5/K$.

It can be assumed that the scour potential is a function of the jet forces acting on a sphere and jet impact angle. Because the skin friction is negligible (Hoerner, 1958), drag force consists of form drag force only. The form drag force is assumed to be proportional to the stagnation pressure over the sphere because both are functions of jet velocity against the sphere. The form drag force on a sphere is proportional to the pressure force on the projected area of sphere normal to flow direction. Experiments showed that a sphere subject to the maximum scour potential was very close to the center line of the circular jet intersect the plunge pool bed. Therefore, form drag force on a sphere at distance L from nozzle exit can be expressed as Eq. (7).

$$F = C_{d1}\int_0^{d/2} \rho\ \frac{U_r^2}{2} 2\pi r\ dr \tag{7}$$

Introduce Eqs. (4) and (5) into Eq. (7) and integrate Eq. (7).

$$\frac{F}{C_{d1}} = \frac{\pi\rho U_o^2 D_n^2}{8}\{1 - \exp(-(Kd/K)^2)\} \tag{8}$$

Therefore, drag force on a sphere in the zone of established flow can be written as Eq. (9).

$$F = C_{d1}(D_n/D_s)^2\left(1 - \exp(-(KD_s/L)^2)\right)\frac{\pi\rho^2 U_o^2 D_s}{8} \tag{9}$$

Meanwhile, dimensionless scour potential can be obtained by dividing the jet force acting on a sphere by Mos.

$$\max.(W_s/M_{os}) = K_1 F/M_{os} \tag{10}$$

K_1 in Eq. (10) is a function of D_n/D_s and the angle of jet impact(θ). Introducing Eq. (9) into Eq. (10) yields the equation of maximum W_s/M_{os}.

$$\max.(W_s/M_{os}) = K_1 C_{d1} (D_n/D_s)^2 \{1-\exp(-(K\,D_s/L)^2)\} \tag{11}$$

Eq. (11) is applicable only for the region of the established flow and K1 Cd1 can be decided from experiments.

The velocity distribution for the region of flow establishment was expressed as Eq. (6) which described the velocity distribution of the non-irrotational region. The form drag force acting on a sphere for the region of flow establishment can be expressed as:

$$F = C_{d3} \int_0^{D_s/2} \rho \frac{U_r^2}{2} 2\pi r\, dr = C_{d3} \left\{ \int_0^{r_{1mt}} \rho \frac{U_0^2}{2} 2\pi r\, dr + \int_{r_{1mt}}^{D_s/2} \rho \frac{U_r^2}{2} 2\pi r\, dr \right\} \tag{12}$$

where r_{1mt} is the radius of irrotational flow region.

To develop a dimensionless equation, transformation of radius r is necessary. Let

$$r = k\, D_s \tag{13}$$

Then, $dr = D_s\, dk$ and $k = 1/2$ when $r = D_s/2$.

Introducing Eqs. (13) and (5) into Eq. (12) yields:

$$F/C_{d3} = \int_0^{k_{1mt}} \frac{\pi\rho U_0^2}{2} 2k\, D_s^2\, dk + \int_{k_{1mt}}^{1/2} \pi\rho U_r^2 k\, D_s^2\, dk$$

$$= \frac{\pi\rho U_0^2\, D_s^2}{8} \left\{ 4k_{1mt}^2 + \int_{k_{1mt}}^{1/2} 10((-0.4343/C_1^2)(C_1 + \frac{kD_s - D_n/2}{L})^2)k\, dk \right\} \tag{14}$$

Introducing Eq. (14) into Eq. (10) yields Eq. (15).

$$\max.\ W_s/M_{os} = K_1 C_{d3} \left\{ 4k_{1mt}^2 + 8 \int_{k_{1mt}}^{1/2} 10^P\, k\, dk \right\} \tag{15}$$

where

$$P = ((-0.4343/C_1^2)\ (C_1 + \frac{KD_s - D_n/2}{L})^2) \tag{16}$$

Since no exact solution of Eq. (16) exists, the technique of numerical integration was applied to solve Eq. (16).

4. Derivation of Scour Prediction Formulas

Regressions were performed to find K_1C_{d3} of Eq. (11) and K_1C_{d3} of Eq. (16) with an assumption that these two coefficients are the functions of θ, D_n/D_s, H/D_s and Re. Optimum jet diffusion coefficient K were obtained by finding the jet diffusion coefficient K which gives best regression results. The optimum jet diffusion coefficient K equals 8.3 and 4.8 for Eqs. (11) and (16), respectively. Albertson et al. (1950) suggested jet diffusion coefficient K equals 6.2 and 4.5 for the jet flow established region and jet flow establishment region. Ackermann and Undan (1970) found that jet diffusion coefficient K varies from 4.5 to 7.9, depending upon the researchers.

Maximum transformed scour potential for the established flow region is:

$$\max.(W_s/M_{os}) = \{-0.0813\theta + 0.525(D_n/D_s) - 0.268(D_n/D_s)^2\}\ (D_n/D_s)^2\ \{1 - \exp(-(8.3D_s/L)^2)\} \quad (17)$$

Maximum transformed scour potential for the region of flow establishment is:

$$\max.(W_s/M_{os}) = K_1C_{d3}\left\{4\ k_{1mt}^2 + 8\int_{k_{1mt}}^{1/2} 10^p\, k\, dk\right\} \quad (18)$$

where

$$K_1C_{d3} = -0.871\theta + 0.735\theta^2 + 0.953D_n/D_s - 0.412(D_n/D_s)^2 - 0.00173(H/D_n)^2 \quad (19)$$

If the momentum per unit area is kept constant, the specific weight of sphere reaches the incipient motion will increase asymptotically as nozzle size is increased. As mentioned by Shields (1936), a limit of the required specific weight of the sphere exists which does not increase any more even if the size of jet is increased further. It means that a limit of D_n/D_s exists beyond which the parameter D_n/D_s does not affect scour potential any more. This limit of D_n/D_s will be called a critical (D_n/D_s) and will be expressed as $(D_n/D_s)_c$.

Therefore, scour potential prediction formulas for regions $D_n/D_s \geq (D_n/D_s)_c$ are to be developed. The value of $(D_n/D_s)_c$ of the established flow region could be found by calculating roots of the derivative of Eq. (17) with respect to D_n/D_s.

Beyond $(D_n/D_s)_c$, the scour potential at L/D_s will be the same as the scour potential at (L/D_s) $\{(D_n/D_s)_c/(D_n/D_s)\}$ for $(D_n/D_s)_c$ because the velocity along the centerline of jet varies linearly with L/D_n (Son, 1992). Therefore, max. W_s/M_{os} beyond critical D_n/D_s can be expressed as Eq. (20).

$$\max.W_s/M_{os} = \{-0.0813\theta + 0.525(D_n/D_s)_c - 0.268(D_n/D_s)_c^2$$

$$[1-\exp\{-(K(D_s/L)(D_n/S_s)/(D_n/D_s)_c^2\}] \quad (20)$$

As the same way to derive Eq. (20), scour potential equations were derived for the region of flow establishment and for the (D_n/D_s) greater than $(D_n/D_s)_c$. Max. W_s/M_{os} remains constant from nozzle exit unless the diameter of irrotational flow region is smaller than $D_s(D_n/D_s)_c$. Max. W_s/M_{os} for the remaining part of the irrotational region of flow establishment follows Eq. (18) with $D_n/D_s = (D_n/D_s)_c$. Eq. (21) is the mathematical expression of these descriptions (Son, 1992).

$$\max.W_s/M_{os} = \text{Eq.}(18) \mid \text{at}(D_n/D_s) = (D_n/D_s)_c \text{ and nozzle exit if } L/D_n \leq D_x$$
$$\max.W_s/M_{os} = \text{Eq.}(18) \mid \text{at}(D_n/D_s) = (D_n/D_s)_c \text{ and } L = (L - D_n D_x) \text{ if } L/D_n > D_x$$
$$\text{where } D_x = K(1-(D_n/D_s)_c/(D_n/D_s)) \quad (21)$$

The experimental ranges and the application ranges for the developed equations are summarized in Tables 1 and 2. The jet travel length of this research reaches up to 13.5 times the jet diameter.

Table 1. Experimental Ranges of This Research

Description	Experimental Range
Jet impact angle on water surface	28.5 °–45.0 °
Ratio of jet diameter to bed material diameter	0.78–1.55
Ratio of jet diameter to tailwater depth	1.2–6.5

Table 2. Summary of Application Ranges for the Developed Equations

Application Range	Establishment Range	Established Range
$(D_n/D_s) \leq (D_n/D_s)_c$	Eq.(18)	Eq.(17)
$(D_n/D_s) \geq (D_n/D_s)_c$	Eq.(21)	Eq.(20)

5. Estimation of Representative Particle Size

The experiments for this research were carried out with uniform spheres to similate mixed bed materials of a plunge pool. To apply the results of this experiment to a field design, engineers have to know the effect of grain size distribution on the critical condition, i.e., quantitative relation of scour potential between uniform spheres and mixed bed material. Richardson et al. (1975) compared scours due to two different gradations of riprap having the same median diameter. They found that the larger size ingradation had a dominant effect in the determination of the representative grain size. Breusers and Raudikivi (1991) mentioned two criteria describing uniformity of grain, σ_{g1} and σ_{g2}.

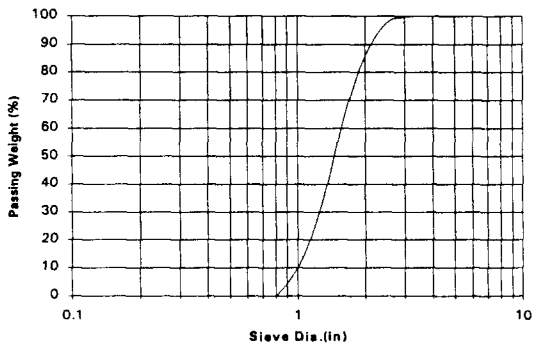

Fig. 3. Distribution of the Mixed Bed Material

$$\sigma_{g1} = (d_{84.1}/d_{15.9})^{0.5} \tag{22a}$$

$$\sigma_{g2} = (d_{84.1}/d_{50})^{0.8} = d_{50}/d_{15.9} \tag{22b}$$

To find a representative particle size for uniformly graded materal, experiments were performed with the bed material having uniformity index $\sigma_g = 1.343$.The distribution is shown in Fig. 3.

The experiments with the above mentioned bed materials were performed as follows.

1) Measure the jet characteristics when a critical condition of the distributed bed material occurs under given boundary conditions.
2) Estimate the uniform sphere diameter under the critical condition of step 2 with the scour prediction formulas developed in this research.
3) Find the percentage (in weight) of bed material of which is finer than the uniform sphere diameter of the step 2.

Experiments show the average of the representative bed material size is 0.159 ft and it corresponds to $d_{81.5}$ of the mixed bed material. The estimated representative bed material sizes are nearly constant and statistical analysis shows the representative bed material size is independent of the jet characteristics, and tailwater depths. Therefore, $d_{81.5}$ is the representative grain size of the mixed bed material if the mixed bed material having the Fig. 5 distribution. It is worth to compare the representative bed material size of this research and Breusers and Raudikivi's (1991) suggestion. They suggested that the use of d_{50} is adequate for relatively narrow grain size distributions and d_{75} or d_{80} is more appropriate for roughness calculations.

6. Comparisons and Application

Following two figures show comparisons of raw experimental measurements of maximum transformed scour potentials and the developed equations. Developed equations for the prediction of maximum transformed scour potential agree with experimental measurements.

Scour potential can be used to find the tailwater depth which just allowed incipient motion of

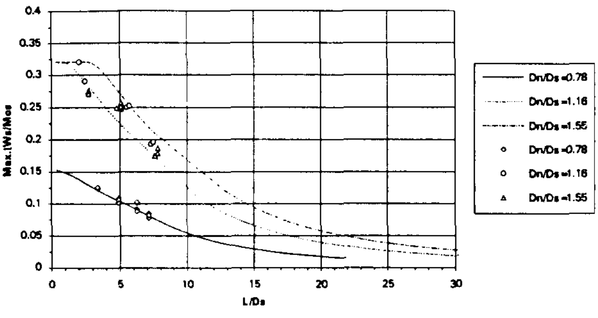

Fig. 4. Experimental Measurements and Regression Lines for $\theta=28.5^\circ$

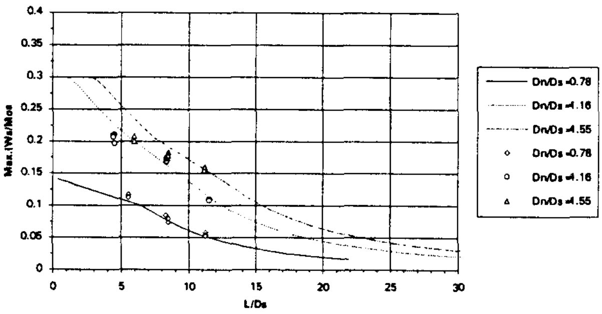

Fig.5. Experimental Measurements and Regression Lines for $\theta=45.0^\circ$

given bed material. The minimum size of bed material, which allows incipient motion only, for a given tailwater depth also can be found from the scour potential. Brief steps to find the minimum size of bed material for given jet velocity(U_o), diameter of jet(D_n), angle of jet impact(θ), and tailwater depth(H) are as follows.

1) Calculate the ratio of diameter of jet to size of bed material(D_n/D_s) and the ratio of jet travel length to size of bed material ($L/D_s=(H/D_s)/\sin\theta$) by guessing the size of bed material(D_s).
2) Calculate max. W_s/M_{os}. with the equations developed in this research.
3) From Eq. (23), find calculated minimum bed material size(D_{sc}).

$$D_{sc}=(\max.W_s/M_{os})\rho U_o^2/(\rho_s-\rho) \tag{23}$$

4) Repeat the above calculation procedure until the calculated size of bed material(D_{sc}) corresponds to the initially assumed size of bed material(D_s).

7. Conclusions

(1) New experimental method and a new dimensionless parameter governing scour, the ratio of the bed material diameter to the jet diameter, were found.

(2) New Albertson et al.'s (1950) jet diffusion coefficients K for the region of jet establishment and the jet established region were found 8.3 and 4.8, respectively.

(3) Semi-theoretical scour prediction formulas using jet diffusion momentum acting on a sediment particle and submerged jet diffusion were newly developed for two regions of jet diffusion, i.e., establishment region and established region. Each region of jet diffusion has two equations, and the application of these two equations depends upon the ratio of jet size to bed material size. The semi-theoretical scour prediction formulas showed close agreement with experiments.

(4) For the critical condition of particles, the representative grain size of narrowly distributed bed material was found to be $d_{81.5}$.

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Notations

The following symbols are used in this paper:

B = Plunge pool width;
D_n = Nozzle diameter;
D_s = Sphere diameter;
H = Tailwater depth;
L = Travel length of jet;
M_{os} = Momentum of jet acting on a projected area of sphere normal to jet direction;
Q = Flow rate of jet;
U_o = Velocity of jet at nozzle;
W_s = Submerged weight of sphere;
X = Axis along bed level;
Y = Axis normal to bed level;
θ = Agle of jet impact (radian);
ν = Kinematic viscosity of water;
ρ = Density of water; and
ρ_s = Density of bed material.