⊙ 硏究論文

Optimization of the Constrained Machining Parameters Problem by the SUMT

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SUMT를 이용한 구속절삭조건의 문제에 관한 최적화

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요 약

생산스케쥴링의 기준이 되는 정보는 각 부품의 생산시간 이나 생산비용인데, 이들은 고정된 값으로 취급되는 경우가 많다. 그러나, 실제적으로는 가공조건에 의하여 그 값들이 변화한다. 가공조건은 절삭깊이(depth of cut), 이송속도(feed rate), 및 절삭속도(cutting speed)로 구성되어 있다. 본 연구에서는 주어진 조건에 있어서 가공의 최적절삭조건을 최적이론의 하나인 페널티 함수이론(SUMT)을 이용하여 결정하는 알고리즘을 개발하였다. 알고리즘에서 목표함수(objective function)로는 4개의 비용성분으로 구성된 생산비용을 채택했고, 제한함수(constraint functions)로는 표면거칠기, 파워 소비, 등을 고려했다. 개발된 알고리즘 프로그램의 유용성을 증명하기 위해 선반가공의 예를 실행하여 그 결과를 예시하였다.

1. Introduction

The determination of the optimum machining conditions in metal cutting is an important aspect in an economic manufacturing process. The main objective in general will be either to minimize the cost of production or to minimize the machining time. In practice there will be constraints on all the machining operations which put restrictions on the choice of the cutting parameters. These constraints may originate from various considerations like the tool life, the machine tool dynamics, and the required surface finish. All these requirements should be considered in finding a solution to the machining problem, which may be either minimization of the production cost or machining time.

Several workers have attempted to solve the

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problem of determining the optimum machining conditions by making some simplified mathematical models. Wu and Ermer [1] found the optimum machining parameters for the turning operation under the assumption that the empirical parameters in the tool - life eguation are Known. Barrow [2] showed that the indices involved in the extended taylor's equation for tool life could be treated as deterministic for a wide range of cutting conditions. Iwata, et al. [3] proposed an an analytical method applying a chance - constrained programming concept to determine the optimum cutting conditions considering the probabilistic nature of the objective function and constraints. Chang, et al. [4] developed a mathematical model for milling operations and identified the five primary control variables.

Many mathematical programming methods, such as geometric programming and goal programming, etc., have been reported in the literature. Ermer [5] solved the constrained machining economics problem by using geometric programming. Hati, et al. [6] applied the mathematical programming techniques to determine the optimum cutting parameters of machining operations. Philipson [7] discussed goal programming technique to resolve some multiple criteria machining problems. Iwata, et al. [8] dealt with the problem of optimizing the number of passes by applying the concept of dynamic programming and stochastic programming.

In this paper, the mathematical programming technique Seguential Unconstrained Minimigation Jechnigues(SUMT) is applied to find the cutting parameters which will achieve the objective stated earlier under a specified constraint set for machining operations. In the second section, some basic machining calculations (i.e., production cost, feed, speed, machining time, etc.) and some constraints are addressed. In the third section, the proposed method, SUMT, is described with the generalized object function and constraints of the cost model. In the fourth section, a typical example is demonstrated to obtain the cost - minimizing cutting conditions in a turning operation.

2. Basic model of the production cost

Mathematical models of the production cost and the constraints in machining the workpiece are given in the following.

2.1 Mathematical model of production cost

The production cost model, which describes the average unit cost to produce a workpiece by means of a simple, rough turning operation, is the sum of four costs : the machining cost, the tool cost, the tool changing cost, and the handling cost.

(a) Machining cost $= C_{o\,tm}$

where C_o is the cost of operating time, $/min, and t_m is the time to machine a workpiece, min.

(b) Tool cost $= C_t \frac{t_m}{T}$

where C_t is the tool cost per cutting edge, $/edge, T is the tool life, min/edge, and t_m/T is the number of tool edges required per workpiece.

(c) Tool changing cost $= C_o t_c \frac{t_m}{T}$

where tc is the tool changing time, min/edge.

(d) Handling cost $= C_o t_h$

where th is the handling time, min/piece.

Hence, the production cost per workpiece as the objective function is given by

$$C_u = C_o t_m + C_o t_h + \frac{t_m}{T} (C_o t_c + C_t) \quad (1)$$

The machining time(t_m) is the total amount of time it takes to finish a workpiece. For a constant depth of cut

$$t_m=\frac{\Pi DL}{12Vf} \tag{2}$$

where D is the diameter of workpiece, inch
L is the axial length of cut, inch
V is the cutting speed, sfm
f is the feed, ipr.

The generalized tool-life equation for the tool work pair is given by :

$$VT^n f^m \phi x=C_c \tag{3}$$

From which

$$T=(C_c f^{-m} \phi^{-x} V^{-1})^{\frac{1}{n}} \tag{4}$$

Substituting for t_m which is shown in equation (2) and T which is shown in equation (4) into Equation (1) results in the following :

$$C_u = C_o t_h + \frac{C_o \Pi DL}{12}\left[V^{-1}F^{-1}+\frac{V_n^{\frac{1}{n}-1} f^{\frac{m}{n}-1} t_t}{\Phi^{\frac{-1}{n}} C_c^{\frac{1}{n}}}\right] + \frac{C_t DL}{12\Phi^{\frac{-x}{n}}}\left[\frac{f^{\frac{m}{n}+1} V^{\frac{1}{n}-1}}{C_c^{\frac{1}{n}}}\right] = C_o t_h + \frac{C_o \Pi DL}{12f}V^{-1} + V^{\frac{1}{n}-1} f^{\frac{m}{n}}\left[\frac{C_o \Pi DL t_t}{12\Phi^{\frac{-x}{n}} C_c^{\frac{1}{n}}}+\frac{C_t \Pi DL}{12\Phi^{\frac{x}{n}} C_c^{\frac{1}{n}} f}\right] \tag{5}$$

For a given Φ, L, D, t_t, C_o, C_t, t_h and C_c, the equation (5) can be written in the form :

$$C_u = K_1 + \frac{K_2}{f}V^{-1} + \frac{K_3}{f}V^{\frac{1}{n}-1} f^{\frac{m}{n}} \tag{6}$$

where,

$$K_1 = C_o t_h$$

$$K_2 = \frac{C_o \Phi DL}{12}$$

$$K_3 = \frac{C_o \Pi DL t_t}{12\Phi^{\frac{-x}{n}} C_c^{\frac{1}{n}}} + \frac{C_t \Pi DL}{12\Phi^{\frac{x}{n}} C_c^{\frac{1}{n}} f}$$

2.2 Mathematical approach of Constraints.

For the optimization of the objective functions considered, the factors that impose restrictions on the cutting parameters need to be considered. The restrictions come from various considerations like the required surface finish, the availability of power, the force developed, and the minimum tool life. The dominant restricting parameter assumed here is surface finish while the other constrains equations include availability of speeds, feeds, and power.

(a) Surface finish

Equations for surface finish have been represented as only experimental forms. These equations are suitable in optimization problems. Oslen [9] expressed surface finish experimentally as a function of feed and cutting speed. According to his paper, the expressions for surface finish are represented as follows :

$$R_a=5.1\times 10^9 f^{1.54}\ \mu\ in.\ f>0.03\ ipr \tag{8}$$

$$R_a=1.36\times 10^8 f^{1.004}\ V^{-1.52}\ \mu\ in.\quad for\ 75\leq V\leq 750\ and\ f<0.03\ ipr \tag{9}$$

$$Ra=7.34\times 10^4 f^{1.54}\ \mu\ in\ for\ f\leq 0.0\ ipr \tag{10}$$

The surface finish constraint is represented as

$$R_a\leq R_{max} \tag{11}$$

(B) Bounds on the cutting speed

The lower and upper bounds on the cutting speed are taken as

$$V_{min}\leq V\leq V_{max} \tag{12}$$

(b) Bounds on the feed.

The feed is restricted as

$$f_{min}\leq f\leq f_{max} \tag{13}$$

(d) Restriction on the cutting force

It is necessary to put a restriction on the force developed because a higher value of the force may produce an excessive deflection of the workpiece and require a large power for the cutting operation. The constraint on F_c can be expressed as

$$F_c \leq F_{\max} \tag{14}$$

(e) Constraint on power

The cutting power should not exceed the available power, and thus the constraint becomes

$$P_c \leq P_{\max} \tag{15}$$

(f) Bounds on the tool life.

The bounds on the tool life are taken as

$$t_{\min} \leq T \leq t_{\max} \tag{16}$$

3. Penalty function method

Penalty function methods transform the basic optimization problem into alternative formulations such that numerical solutions are sought by solving a sequence of unconstrained minimization problems. The optimization problem can be represented as follows :

minimize $f(x)$

subject to $g_j(x) < 0, j = 1,2,3,\cdots\cdots,m$ (17)

This problem is converted into an unconstrained minimization problem by constructing a function of the form

$$\Phi_k = \Phi(X, r_k) + r_k \ \Sigma G_j \ [g_j(x)] \tag{18}$$

where G_j is some function of the constraint g_j, and r_k is a positive constant known as the penalty parameter. The second term on the right side of equation (18) is called the penalty term. If the unconstrained minimization of the the ϕ - function is repeated for a seguence of values of the penalty parameter Yκ(κ=1,2…), the solution may by brought to coverage w that of the original problem stated in equation (17). This is the reason why the penalty function methods are known as SUMT. The penalty function formulations for inequality constrained problems can be divided into two categories, namely the interior and the exterior methods. Since the interior method is adapted for this work, only interior penalty function will be discussed on detail.

In interior penalty function methods, a new function(Φ - function) is constructed by augmenting a penalty term to the objective function. The penalty term is chosen such that its value will be small at points away from the constraint boundaries and will tend to infinity as the constraint boundaries are approached. Hence the value of the Φ - function also "blows up" as the constraint boundaries are approached. Thus, once the unconstrained minimization of $\Phi(X, r_k)$ is started from any feasible point X_1, the subsequent points generated will always lie within the feasible domain since the constraint boundaries act as barriers during the minimization process.

The Φ - function defined originally is

$$\Phi(X, r_k) = f(X) - rk \ \Sigma \frac{1}{g_j(X)} \tag{19}$$

It can be seen that the value of the function o will always be greater than f since $g_j(X)$ is negative for all feasible points X. If any constraint $g_j(X)$ is satisfied critically (with an equality sign), the value of o tends to infinity. It is to be noted that the penalty term in equation (19) is not defined if X is infeasible. This introduces serious shortcomings. Since this equation does not allow any constraint to be violated, it requires a feasible starting point for the search toward the optimum point. However, in many engineering problems, it may not be very difficulty in finding a point satisfying all the constraints, $g_j(X) < 0$, at the expense of large values of the objective function, f(X).

In this study, Phi - function can be obtained

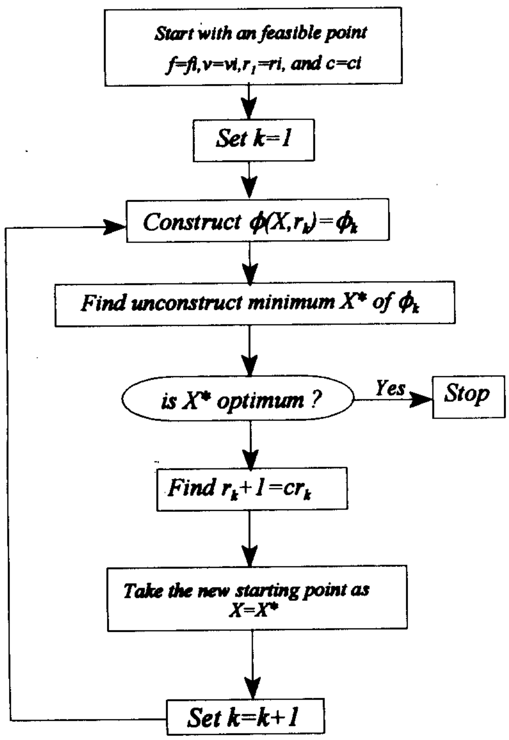

Fig. 1 Flow chart for penalty function method

by substituting the production cost Cu shown in equation (6) and constraints shown in equations (11) and (14) into equation (19). Thus Phi - function can be represented as follows :

$$\Phi(X)=K_1+\frac{K_2}{f}V^{-1}+\frac{K_3}{f}V_n^{\frac{1}{n}-1}f^{\frac{m}{n}}$$

$$-r_k\sum\left[\frac{1}{R_a-R_{\max}}+\frac{1}{P_c-P_{\max}}\right] \qquad (20)$$

The solution procedure of above penalty function method is coupled with the nelder and mead method of unconstrained minimization and coggins method of one dimensional search. The flow chart of the algorithm is shown in Fig. 1.

4. Numerical Experiment

In order to illustrate the algorithm, the optimization problem for the turning operation is used. The numerical data and results of the processing algorithm will be addressed.

4.1 numerical data

The numerical data of equation (20) are taken as follows :

Bhn=195

Φ=60degree

D=6in.

L=8in.

C_o=\$0.1/min.

C_t=\$0.5/cutting edge

th=2.0min.

tc=0.5min.

d=depth of cut=0.2in.

n=0.25

m=0.29

x=0.35

Substituting of the above values reduces equation (6) as follows :

$$C_u=0.20+1.26f^{-1}V^{-1}+1.799\times10^{8}V^{3}f^{0.16} \quad (21)$$

The restriction parameters for the above conditions are

(1) available speeds and feeds :

0.001≤f≤0.009 *ipr*

18 *rpm*≤*N*≤540 *rpm*

(2) surface finish :

Based on values about speeds and feeds, a surface finish constraint among equations (8), (9), and (10) is chosen as follows :

$$R_a=1.36\times10^{8}\times f^{1.004}V^{-1.52} \qquad (22)$$

Let Rmax=100 microin. The restricting equation for surface finish is

$$1.36\times10^{4}f^{1.004}V^{\ 1.52}\leq100 \qquad (23)$$

(3) power :

3.58 f0.78 V0.91= "(HP)" eta (24)

where, Hp=power in horsepower

η=efficiency

Let (HP)η=5 hp. The restricting equation for power is :

$$3.58f^{0.78}V^{0.91} \leq 5 \tag{25}$$

4.2 Procedure of solving problem

The problem reduces to the minimizing of the objective function of C_u subject to restrictions listed above in (1), (2), and (3).

$$min\ C_u = 0.20 + 1.26f^{-1}V^{-1} = 1.799 \times 10^{-8}V^3f^{.16} \tag{26}$$

subject to

$$g_1(X) = 136.84 \times 10^6 f^{1.004} V_{1.52} - 100 \leq 0$$
$$g_2(X) = 3.58 f^{0.78} V^{0.91} - 5 \leq 0$$

The Φ - function is obtained by substituting above C_u, $g_1(X)$, and $g_2(X)$ into equation (20) and is given by

$$\Phi(X) = 0.20 + 1.26f^{-1}V^{-1} + 1.799 \times 10^{-8}V^3f^{0.1} - r_k \sum \left[\frac{1}{136.84 \times 10^6 f^{1.004} V^{-1.52} - 100} \frac{1}{3.58^{0.78} V^{0.91} - 5} \right] \tag{27}$$

Let starting feasible points, f=0.02, V=200.0, and r_1=10. The equations embodied in equation (23) are solved for determining the optimized parameters. The results of the program execution are shown in Table 1. With reference to the table, the following results have been obtained after 10 iterations carried out by a computer :

f=0.002465 ipr and V=248.240540 fpm

Table 1. Results of processing SUMT

Iteration	φ(x)	feed	speed 1
1	2.565900	0.002004	284.000061
2	2.565900	0.002004	284.000061
3	2.565900	0.002004	284.000061
4	2.483929	0.002301	251.711548
5	2.404464	0.002301	263.210571
6	2.385096	0.002398	253.360504
7	2.376772	0.002424	251.380676
8	2.368825	0.002450	249.400848
9	2.365742	0.002463	248.240540
10	2.364515	0.002465	248.240540
Final value	2.364515	0.002465	248.240540

5. Conclusions

An analytical procedure and a software program have been developed using SUMT to determine the optimum value of the cutting speed and the feed considering the production cost and the constraints inherent to machining processes. The suggested criterion for the determination of the optimum cutting conditions is production cost which can be obtained by the summation of four costs : (1) machining cost, (2)tool cost, (3) tool changing cost, and (4) handling cost. The establishment of accurate mathematical models of the surface finish and the cutting power is essential to the determination of the optimum cutting conditions. The more constraints, such as cutting force, chatter vibration etc., are included in the penalty function, the more accurate machining parameters can be obtained.

The effectiveness of the proposed algorithm was demonstrated by means of an example. If the penalty function method implements several starting points, we can determine more accurate optimization parameters with fewer iterations.

References

1) Wu, S. M., and Ermer, D. S., "Maximum profit as the criterion in the determination of the optimum cutting conditions," Journal of Engineering for Industry, Transaction of ASME, 1966, Vol. 88, pp. 435 - 442.

2) Barrow, G., "Tool - life equations and machining

economics," Proceedings of the Twelfth International Machine Tool Design and Research conference, 1971, Manchester, 15 – 17 September, pp. 481 – 493.

3) Iwata, K., Morutsu, Y., and Oba, F., "Optimization of cutting conditions for multi – pass operations considering probabilistic nature in machining processes," Journal of Engineering for Industry, Transaction of the ASME, 1977, Vol. 98, pp. 210 – 217.

4) Chang, T. C., Wysk, R. A, Davis, R. P, and Choi, B.,"Milling parameter optimization through a discrete variable transformation," International Journal of Production Research, 1982, Vol. 20, No. 4, pp. 507 – 516.

5) Ermer, D. S., "Optimization of the constrained machining economics problem by geometric programming," Journal of Engineering for Industry, Transaction of the ASME, 1971, Vol. 93, pp. 1067 – 1072.

6) Hati, S. K., and Rao, S. S., "Determination of optimum machining conditions – Deterministic and probabilistic approaches," Journal of Engineering for Industry, Transactions of the ASME, 1976, Vol. 99, pp. 354 – 359.

7) Philipson, R. H., and Ravindran, A., "Application of goal programming to machinability data optimization," Journal of Mechanical Design, Transaction of the ASME, 1978, Vol. 100, pp. 286 – 291.

8) Iwata, K., Morutsu, Y., and Oba, F., "Optimization of cutting conditions for multi – pass operations considering probability nature in machining processes", Journal of Engineering for Industry, Transaction of the ASME, 1977, Vol. 98, pp. 210 – 217.

9) Olsen, K. F., "Surface Roughness as a Function of Cutting Conditions When Turning Steel", Machine Tool and Production Trends, Engineering Proceedings, The Pennsylvania State University, 1965, pp. 149 – 160.