SUPPLEMENTARY ANALYSES OF ECONOMIC $\overline{X}$ CHART MODEL*

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ABSTRACT

With the increasing interest of reducing process variation, statistical process control has served the pivotal tool in most industrial quality programs. In this study, system analyses have been performed associated with a cost incorporated version of a process control, a quadratic loss-based $\overline{X}$ control chart model. Specifically, two issues, the capital/research investments for improvement of a system and the precision of a parameter estimation, have been addressed and discussed.

Through the analysis of experimental results, we show that process variability is seen to be one of the most important sources of loss and quality improvement efforts should be directed to reduce this variability. We further derive the results that, even if the optimal designs may be sensitive, the model appears to be robust with regard to misspecification of parameters. The approach and discussion taken in this study provide a meaningful guide for proper process control. We conclude this study with providing general comments.

Key Words : Economic Design of Control Charts, Taguchi, Variability Loss, Orthogonal Arrays, Investment Decision, Parameter Estimation

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I. INTRODUCTION

Statistical process control has served the pivotal tool in most industrial quality programs for improving quality of manufactured products. Duncan[3] has presented a cost incorporated version of a process control, the economic design of control charts (EDCC), and many researches have been performed to extend Duncan's model[1,4,6,7,8,10]. Recently, the significance of process variation receives wide attention primarily due to Taguchi[9]'s definition of quality loss. He further emphasizes that such loss can be reduced only through the reduction of product variation. Jeon[5] has presented an alternate designing approach which incorporates Taguchi's quadratic loss concepts into EDCC framework and thus accounts for a loss due to this variability explicitly. The major significance of his study lies not only in controlling a process in an optimal state but in providing impetus for continual process improvement. Remember that, even if the product variability loss may implicitly be derived, a direct and explicit quantification of this loss is not possible from the conventional model. He has also classified the system cost into various cost components including process (product) variability loss and performed extensive sensitivity analyses using statistical experimental design.

Industry is often faced with deciding whether or not to undertake a capital investment to improve system performance. Such investments might be made for a major change in the existing system, like the purchase of a new process, or for partial replacement of some component(s) within the system. A research investment which might yield a system cost reduction may also be a relevant example. Our concern is to determine if an investment proposal is cost effective or how much can be saved from it. Information that can act as an economic justification of the investment is needed in decision making, especially for management. We know that management commitment serves as the sole driving force in solving many system problems.

Another issue that is important but has not been fully addressed is to examine the precision to which a parameter needs to be estimated. The mathematical models developed require exact values of the various parameters in order to produce an optimal decision rule. In field exercises, however, precise measurements of parameters are generally difficult (or, at least, require significant efforts) to obtain, yet a misleading policy may be generated if imprecise measurements are incorporated into the model. Therefore, an examination of the impact of measurement errors on the optimal decision may tell us whether or not precise estimates are needed. Conceptually, if a slight misspecification of a parameter produces a policy to be adopted but that is not close to a true optimal, then that parameter should be estimated precisely. On the other hand, rough guesses may suffice for other parameters that produce designs that are close to optimal, even if the errors in the estimates are large.

In a strict sense, these issues are not independent but rather interrelated. Significant efforts would be required for an explicit analysis of each, and different approaches may be taken depending upon the situation. The major purposes of this study are thus to empirically generate data for i) the capital and research investments for improvement of a system, and ii) the precision of a parameter estimation within the EDCC model. We will first briefly review the quadratic loss based economic $\bar{X}$ chart model in section II. Then, in section III and IV, discussion associated with above issues will be given with examples. Finally, in section V, we conclude this study with providing general comments.

II. QUADRATIC LOSS BASED EDCC MODEL

In this section, the quadratic loss based EDCC model presented by Jeon[5] will be briefly reviewed. (Detailed discussion of basic concepts of EDCC may be referred to Duncan[3].)

Consider that we are to design a control chart for a process which produces r items per hour whose measured quality characteristics during in control are normally distributed with mean μ and variance σ^2. The process often shifts to an out-of-control state due to the occurrence of an assignable cause and is now represented by shifted mean $\mu=\mu+\delta\sigma$ but unchanged variance σ^2 during this period. The time between the occurrence of assignable causes is assumed to be exponentially distributed with mean $1/\lambda$ hours. We take a sample of size n every h hours and plot its average value on the chart. If the plot lies outside of the predetermined control limits, $\mu\pm k\sigma/\sqrt{n}$, the process is suspected of being out of control. A search for the potential assignable cause is followed and, once found, the process is returned to the in-control state with correction of it. Our objective is to find the optimal values for sample size(n), control limit(k), and sampling interval(h), which simultaneously minimize the total cost. In this study, we assume that production continues during the search and repair periods, called the continuous process model.

The loss associated with an item from quality loss form proposed by Taguchi[9] is

$$L(x)=C(x-m)^2$$

where C is a constant to be determined and m is the target value. Incorporation of this loss into the EDCC framework yields the losses per item during in- and out-of- control periods as follows: (The process mean is presumed to be equal to m during in-control period.)

$$E[L]=\begin{cases} C\sigma^2, & \text{if } \mu=m \\ C(1+\delta^2)\sigma^2, & \text{if } \mu=\mu+\delta\sigma\neq m \end{cases}$$

The final form of the hourly expected total cost, ETC(n,k,h), and five cost components classified by Jeon are summarized in the Appendix with notation. For system analyses, he applied a fractional factorial design, $L_{36}(3)^{13}$ orthogonal array [9]. We will generally follow his design and results for analysis. First, the three levels of each parameter considered are given in Table 1.

Table 1 Input Parameters and Their Values

Factor	Low	Level Medium	High
σ	3	4	5
δ	1	1.5	2
λ	0.01	0.02	0.03
r	10	20	30
T_s	0.05	0.1	0.2
T_e	0.1	0.5	2
T_r	0.1	0.5	2
C	0.2	0.3	0.4
C_a	100	300	500
C_f	1	5	10
C_s	0.5	2	5
C_e	10	50	100
C_r	10	50	100

These 13 parameters are arranged at the $L_{36}(3)^{13}$ array columns in the order

σ, δ, λ, r, T_s, T_e, T_r, C, C_a, C_f, C_s, C_e, and C_r.

Then, the optimal solutions of the 36 input data sets are obtained and they are summarized in Table 2. The optimal designs are given in columns 2 through 4, and the five cost terms are shown with ETC in later columns. ETC* in the table represents the sum of five cost components.

III. REDUCTION OF COSTS

Efforts to improve quality (reduce cost) through capital investment are often proposed in quality control programs and generation of data that can measure the obtainable cost savings from such potential investments is desirable. The classification of cost components, Equations (A-2) through (A-6), and the results given in Table 2 may be useful for this data gathering.

Table 2 Optimal Solutions

TC	n*	k*	h*	L_0	L_1	Sampling	False Alarm	Search and Repair	ETC*
1	12	2.42	8.91	16.84	2.32	0.79	0.16	0.19	20.29
2	6	2.54	2.86	89.78	20.23	5.94	1.06	1.12	118.12
3	3	2.48	1.29	257.52	212.40	19.40	4.29	2.83	496.44
4	10	2.10	24.30	22.74	8.52	2.47	0.55	0.93	35.20
5	4	2.50	0.97	114.86	42.70	3.09	1.14	0.36	162.15
6	4	2.74	1.19	145.05	24.75	10.91	1.47	1.74	183.92
7	8	1.82	4.84	95.99	24.02	4.34	1.20	1.96	127.51
8	5	2.18	6.61	26.50	17.87	5.30	0.99	0.50	51.16
9	3	3.12	0.74	147.06	14.68	3.38	1.19	0.59	166.90
10	7	1.86	6.96	57.55	28.90	5.17	1.95	1.44	95.01
11	8	3.02	2.88	91.69	14.02	3.13	0.41	1.05	110.30
12	3	2.20	2.36	71.21	18.94	6.78	1.09	0.38	98.41
13	7	2.94	2.78	32.22	12.28	3.06	0.51	2.69	50.75
14	4	2.40	3.87	61.03	14.87	4.65	0.40	1.43	82.37
15	7	1.92	4.21	200.10	49.81	8.55	3.33	1.07	262.85
16	7	2.68	4.00	47.05	22.60	6.01	0.76	1.57	77.97
17	2	2.06	1.60	188.13	19.37	6.86	2.39	0.98	217.72
18	11	2.52	5.04	42.64	14.72	2.08	0.57	2.56	62.56
19	5	2.56	3.34	76.64	14.17	3.29	0.87	1.42	96.39
20	4	2.70	3.17	60.67	16.63	7.89	1.00	1.14	87.34
21	10	2.06	3.62	84.95	30.11	4.15	0.88	2.55	122.62
22	3	1.66	4.87	49.36	15.08	5.14	1.73	1.83	73.14
23	5	3.18	1.17	43.48	22.62	2.98	0.34	4.08	73.49
24	11	2.44	5.13	186.93	26.15	5.26	1.30	0.56	220.20
25	4	2.92	3.84	15.80	11.01	2.34	0.38	2.63	32.17
26	4	1.32	4.98	83.07	25.87	5.02	3.01	5.19	122.16
27	8	2.82	2.02	292.44	24.58	6.91	0.69	1.07	325.69
28	5	2.92	2.12	69.36	13.20	5.90	0.47	3.85	92.78
29	8	2.36	3.60	81.11	29.79	4.73	2.03	2.79	120.44
30	4	1.86	5.74	69.49	17.92	4.35	0.99	1.39	94.14
31	4	2.60	1.30	72.97	40.16	5.38	0.63	2.97	122.11
32	10	2.18	10.34	57.46	13.07	2.90	0.72	1.35	75.51
33	5	2.46	3.38	93.76	20.29	7.69	1.86	3.75	127.35
34	3	2.42	5.35	33.73	11.35	3.74	0.79	1.03	50.64
35	16	2.76	3.77	130.29	27.41	4.78	0.67	2.71	165.87
36	4	2.18	1.64	93.68	20.54	5.50	1.64	5.62	126.98

Before discussion about data generating, we first examine the system based on the cost components and Figure 1 displays the relative significance between them. The percentages shown in the figure are obtained by taking arithmetic averages from the results obtained in Table 2.

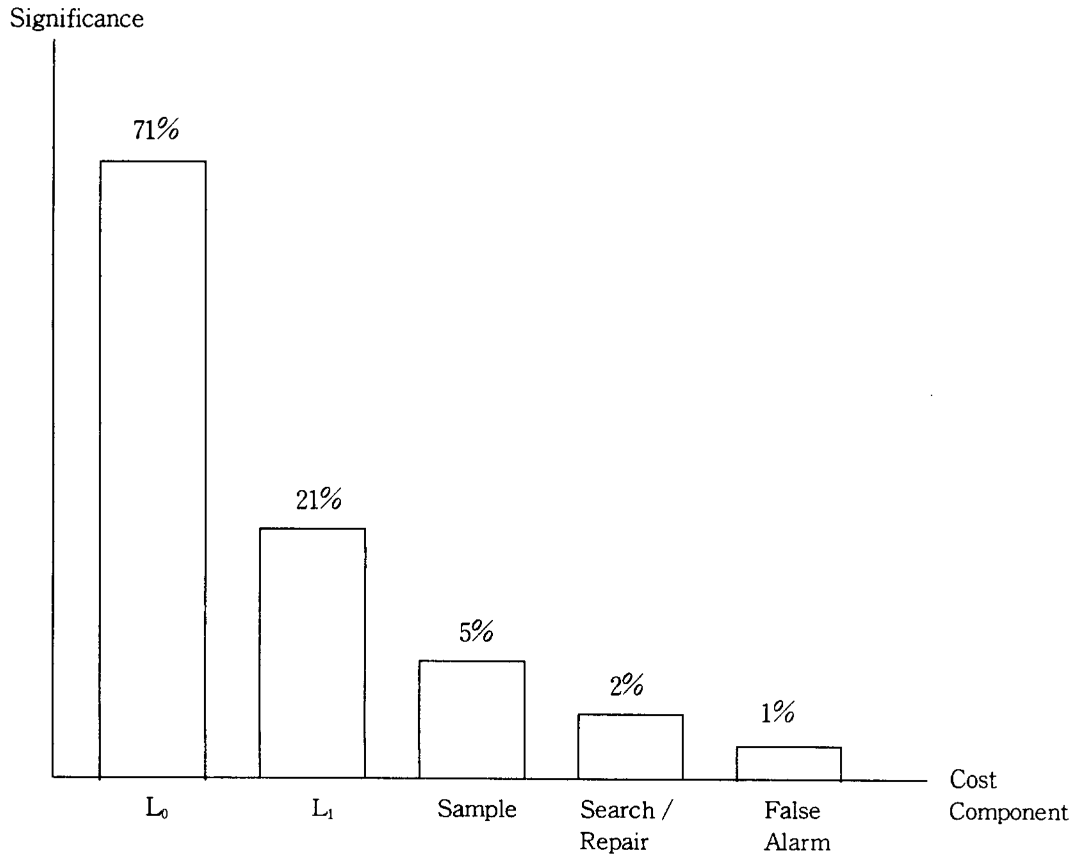

Figure 1 Comparative Display of Significance of the Cost Components

We see, from this figure, that the most significant portion, 90% or more, of the total cost is accounted for by L_0 and L_1 clearly indicating that the product variability loss is the most important source of waste and should be tightly controlled. This is the consistent result that the reduction of product variability should serve a major approach to improve quality and to reduce system cost in the long run. Therefore, a major effort in any quality control program should be directed toward reduction of product variability and its loss.

Next, the sampling cost accounts for roughly 5% of the total cost, and its magnitude is much smaller than those of the variability losses. This result may indicate that significant efforts or investments to reduce sampling cost would not be considered cost effective. Note, however, that to believe achievement of quality improvement through sampling alone is a definite misunderstanding. Although sampling may improve the quality of products by preventing bad items from being delivered to customers, the quality of products can be improved only when the process variance is reduced in a strict sense. That is, for a given process situation, the quality of products cannot be changed and the costs such as the variability loss are inevitable or not reducible regardless of sampling. This argument is, inpart, consistent with Deming's [2] admonishment to "cease dependence on inspection to achieve quality." The real meaning and role of sampling lie in the feedback mechanism, which continuously monitors the system status and generates timely signals against problems so that further unnecessary loss is prevented. Without sampling, more significant system loss may be incurred until an operator notices the process shift or the process is adjusted. This feedback mechanism is specifically important because more accurate information about process parameters--the process mean, the process variance, the mean shift amount, or even the time until the process shift--may be obtained or estimated by proper accumulation of the sampling results.

Although any quality control activity that reduces cost may be desirable, the search and repair cost component and that of false alarms call for special attention. The results described in the table and figure show that, although the cost component of search and repair is significantly affected by some parameters such as λ, C_e, and C_r, this component does not comprise a large portion of the total cost. Therefore, significant and urgent efforts may not be required to reduce this cost, unless frequent process shifts and large search and repair costs are expected. Similarly, the false alarm cost component is particularly dependent upon some parameters such as σ, λ, r, and C_s, but its portion of the total cost is smaller than that of any other cost component.

We now discuss about the quantification approach. Since a complicated and systematic approach is required for explicit analyses, a brief demonstration through numerical examples will be given. Suppose the current system status is represented by Table 3 as the parameter values given in the second column. Then some investment proposals: i) research performing a process capability study to reduce σ, ii) purchase of an automatic inspection device to reduce T_s, C_f, and C_s, and iii) research to improve T_e, T_r, C_e, and C_r. are considered. These proposals are also listed in separate columns of the table with appropriate new parameter values expected from each in brackets, [.]. The bottom half of the table shows the results and hourly cost saving obtainable if the new parameter sets are applied.

The results of this table show that a significant cost reduction (about 22%) is expected from the research investment improving σ only (proposal I). This is because both L_0 and L_1, two major system cost components, are specifically reduced through improvement of σ. Some cost savings are also expected from the other two proposals. Proposal II, purchase of an automatic inspection device, particularly reduces L_1, the sampling cost, and the false alarm cost through reductions in T_s,

C_f, and C_s. Finally, remarkable cost savings, more than 35% of L_1 and about 70% of the search and repair cost, are expected if proposal III is implemented. Nevertheless, total cost savings expected from proposals II and III are not so great as that of proposal I, since the cost component L_0 is not reduced much.

Table 3 Investments and Expected Cost Savings

Factor	Current Status	Proposal I	Proposal II	Proposal III
σ	4	[3.5]	4	4
δ	1.5	1.5	1.5	1.5
λ	0.02	0.02	0.02	0.02
r	50	50	50	50
T_s	0.1	0.1	[0.05]	0.1
T_e	1	1	1	[0.3]
T_r	1	1	1	[0.3]
C	0.1	0.1	0.1	0.1
C_a	250	250	250	250
C_f	2	2	[0.5]	2
C_s	0.5	0.5	[0.1]	0.5
C_e	50	50	50	[15]
C_r	50	50	50	[15]
α	.0048	.0048	.0010	.0048
β	.1965	.1965	.1729	.1965
n	6	6	8	6
k	2.82	2.82	3.30	2.80
h	1.63	1.87	0.82	1.59
L_0	74.32	56.71	75.49	76.36
L_1	18.45	14.75	14.65	11.84
Sample	3.06	2.67	1.58	3.15
False	0.67	0.58	0.27	0.71
Search / Repair	1.86	1.85	1.89	0.57
ETC	98.37	76.56	93.88	92.63
Save /hr	– –	21.81	4.49	5.74

We see that explicit results from individual cost component changes can be obtained even when we cannot actually figure out the complicated interactions between parameters. That is, data not only justifying a given investment proposal but determining a priority order of implementation (when multiple proposals are available) are easily generated. Further, an appropriate pay-back period for a proposal may be obtained without any difficulty. More accurate and comprehensive investment decisions, however, should be made considering numerous other factors such as the time value of money, and the procedure may be more complicated in this situation.

IV. PARAMETER ESTIMATION

It is generally difficult to precisely evaluate the parameter values in field exercises and errors made in parameter estimates may produce suboptimal decisions. This concern calls for a study of how precisely a given parameter should be estimated before a systematic estimation procedure is taken. Our another task of this paper, therefore, is to briefly address an approach that can explicitly quantify the error that might be incurred in both the decision and the total cost.

Again, the results of the previously obtained provide some intuitive answers. Although this intuitive analysis may work in some cases, it is not comprehensive enough and a more systematic approach may be desirable. Let us define n_0, k_0, and h_0 to be the optimal design parameter values obtained from a model when a correct value(p_0) of a parameter, say p, is used. Then, $ETC[n_0,k_0,h_0:p_0]$ indicates the true optimal total cost. When the parameter value is incorrectly estimated(p_1), a suboptimal design will be produced and n_1, k_1, and h_1 are assumed to denote the resulting design. The actual cost incurred may then be obtained if the suboptimal design above is applied to the real system having the parameter value p_0. This cost is expressed by $ETC[n_1,k_1,h_1:p_0]$. The difference between this cost and the true optimal cost provides the actual cost error incurred from misspecifying parameter values. Our concern is to examine the errors incurred in both the design and cost.

We will also illustrate this approach using some examples. The same parameter values specified in the previous section are assumed for correct parameter values of a current system. Then, we postulate a scenario in which some parameters have been over-estimated by a certain degree from the assumed values. Tables 4 and 5 address situations where 20% and 50% over-estimates have been assumed for some parameters, respectively. Again, the values given in the second column show the correct parameter values specified and the generated optimal solutions. The considered parameters are then listed in later columns, and their over-estimated values are given in brackets. Finally, detailed results of the associated suboptimal solution for each case are described in the bottom part of each table. ETC10 in the table specifically implies the total cost $ETC[n_1,k_1,h_1:p_0]$ which is obtained by the suboptimal design to the actual system.

Table 4 Impact of Misspecification of a Parameter

[20% Over-Estimate]

Factor	Current Value	C	δ	σ	C_a	$C_e(C_r)$
σ	4	4	4	[4.8]	4	4
δ	1.5	1.5	[1.8]	1.5	1.5	1.5
λ	0.02	0.02	0.02	0.02	0.02	0.02
r	50	50	50	50	50	50
T_s	0.1	0.1	0.1	0.1	0.1	0.1
T_e	1	1	1	1	1	1
T_r	1	1	1	1	1	1
C	0.1	[0.12]	0.1	0.1	0.1	0.1
C_a	250	250	250	250	[300]	250
C_f	2	2	2	2	2	2
C_s	0.5	0.5	0.5	0.5	0.5	0.5
C_e	50	50	50	50	50	[60]
C_r	50	50	50	50	50	[60]
α	.0048	.0048	.0033	.0048	.0042	.0048
β	.1965	.1965	.1390	.1965	.2078	.1965
n	6	6	5	6	6	6
k	2.82	2.82	2.94	2.82	2.86	2.82
h	1.63	1.49	1.33	1.35	1.62	1.63
ETC_{10}	98.37*	98.40	98.68	98.50	98.38	98.37

* $ETC[n_0, k_0, h_0 : p_0]$ true optimum

From the tables, we see that the largest errors in the design parameter values are incurred when δ is estimated incorrectly; i.e. , as the estimation error δ of grows, the values of the design parameters move far off from the optimum. Therefore, careful and precise estimation of this parameter is needed to arrive at a correct decision. The opposite result is observed when C_e and C_r are misspecified. Even when they are over-estimated by 50%, no considerable errors are incurred in the design or the cost. Hence, the effects of these parameters are not so significant in the EDCC model and precise estimates of them may not be required. When other parameters are incorrectly estimated, some errors in either one or two of the design parameters are observed. In our examples, the sampling interval (h) is specifically affected when C, δ, or C_a is misspecified.

Table 5 Impact of Misspecification of a Parameter

[50% Over-Estimate]

Factor	Current Value	C	δ	σ	C_a	$C_e(C_r)$
σ	4	4	4	[6.0]	4	4
δ	1.5	1.5	[2.25]	1.5	1.5	1.5
λ	0.02	0.02	0.02	0.02	0.02	0.02
r	50	50	50	50	50	50
T_s	0.1	0.1	0.1	0.1	0.1	0.1
T_e	1	1	1	1	1	1
T_r	1	1	1	1	1	1
C	0.1	[0.15]	0.1	0.1	0.1	0.1
C_a	250	250	250	250	[375]	250
C_f	2	2	2	2	2	2
C_s	0.5	0.5	0.5	0.5	0.5	0.5
C_e	50	50	50	50	50	[75]
C_r	50	50	50	50	50	[75]
α	.0048	.0048	.0029	.0048	.0029	.0048
β	.1965	.1965	.1795	.1965	.1614	.1965
n	6	6	3	6	7	6
k	2.82	2.82	2.98	2.82	2.98	2.82
h	1.63	1.33	0.90	1.08	1.75	1.64
ETC_{10}	98.37*	98.52	101.06	99.01	98.45	98.37

* $ETC[n_0, k_0, h_0 : p_0]$ true optimum

Rather robust results of the sample size and control limits have been produced, however.

When we examine the total cost error incurred, an interesting phenomenon is observed. In contrast to the case of the design parameters, no significant errors in the total cost have been produced. Hence, errors made in the estimation of the other parameters particularly affect the design parameters but not the expected total cost. Therefore, the total cost is not considerably affected by measurement errors, indicating that the EDCC model appears to be robust to errors in parameter estimates. As seen, the approach taken in this section explicitly quantifies the impact of errors made in parameter estimates, even if only some limited examples are provided.

V. CONCLUSION

In this study, two issues concerning to the capital investments for quality improvement and the effects of errors in parameter estimates have been addressed. Our results specifically indicate that product variability reduction appears to be the major way to improve quality and reduce system cost. The results further indicate that, even if the optimal design may be rather sensitive, the model appears to be rather robust with regard to errors in parameter estimates. The results are dependent on the parameter value ranges specified and thus may be different in other cases. The approach and discussion given in this study, however, provide a meaningful guide for proper process control and management. More comprehensive analysis with proper decision criteria may be applied for more general conclusions and may form a good future research.

REFERENCE

1. Chiu, W. K. and Wetherill, G. B. (1974): A Simplified Scheme for the Economic Design of X-bar Charts, *Journal of Quality Technology*, vol. 6, no. 2, April, pp. 63-69.
2. Deming, A. J. (1982): *Quality Productivity and Competitive Position*, MIT Center for Advanced Engineering Study.
3. Duncan, A. J. (1956): The Economic Design of X-bar Charts Used to Maintain Current Control of a Process, *Journal of the American Statistical Association*, vol. 51, pp. 228-242.
4. Duncan, A. J. (1971): The Economic Design of X-bar Charts When There is a Multiplicity of Assignable Causes, *Journal of the American Statistical Association*, vol. 66, pp. 107-121.
5. Jeon, T. B. (1992): *Designing An Economic X-bar Chart Model Using A Quadratic Loss Structure*, '92 IE/MS Joint Meeting, May, Ulsan University.
6. Lorenzen, T. J. and Vance, L. C. (1986): The Economic Design of Control Charts: A Unified Approach, *Technometrics*, 28, pp. 3-10.
7. Montgomery, D. C. (1982): Economic Design of an X-bar Control Chart, *Journal of Quality Technology*, vol. 14,no. 1, pp. 40-43.
8. Panagos, M. R. , Heikes, R. G. , and Montgomery, D. C. (1985): Economic Design of X-bar Control Charts for Two Manufacturing Process Models, *Naval Research Logistics Quarterly*, vol. 32, pp. 631-646.
9. Taguchi, G. (1985): Introduction to Off-Quality Control, *Central Japanese Quality Control Association*, Tokyo, Japan. 10. von Collani, E. (1986): A Simple Procedure to Determine the Economic Design of an X-bar Control Chart, *Journal of Quality Technology*, vol. 18, no. 3, pp. 145-151.

APPENDIX

In this Appendix, we summarize the total hour cost of EDCC model and cost components given by Jeon[5]. First, the model and cost parameters used in this study are defined as follows:

δ : process mean shift amount during out-of-control period in terms of
r : production rate per hour
C_p : production cost per item
C_f : fixed sampling cost
C_s : variable sampling cost per item
C_a : false alarm cost
C_e : cost to search for an assignable cause
C_r : cost to repair an assignable cause
T_s : time to measure the quality characteristic for each item
T_e : time to search for an assignable cause, and
T_r : time to repair an assignable cause.

Next, the objective function, hourly expected total cost, is derived as follows:

$$\mathrm{ETC}(n, k, h) = r[C_p + C(1+\delta^2)\sigma^2] = \frac{C_p + nC_s}{h} + \frac{1}{E[T]}\left[\frac{C_a\alpha}{e^{\lambda h}-1} \frac{rC\delta^2\sigma^2}{\lambda} + C_e + C_r\right]$$

where $\alpha=2\Phi(-k)$ and $\Phi(.)$ denotes the standard normal CDF. Further, in this equation, T represents the cycle length and its expected value is given by

$$E[T] = \frac{h}{1-\beta} + \frac{h}{e^{\lambda h}-1} + nT_s + T_e + T_r$$

where $\beta=\Phi(k-\delta\sqrt{n})-\Phi(k-\delta\sqrt{n})$. Note that this equation is dependent upon the magnitude of process mean shift(δ) and the process variance(σ^2) which have not been explicitly shown in the conventional models.

Classification of this equation with elimination of the term rC_p results in five cost components as follows:

$$L_0 = \frac{rC\sigma^2}{\lambda E[T]}, \tag{A-1}$$

$$L_1 = rC(1+\delta^2)\sigma^2\left(1-\frac{1}{\lambda E[T]}\right), \tag{A-2}$$

$$\text{Sampling Cost} = \frac{C_f+nC_s}{h}, \tag{A-3}$$

$$\text{False Alarm Cost} = \frac{C_a\alpha}{E[T](e^{\lambda h}-1)}, \tag{A-4}$$

$$\text{Search / Repair Cost} = \frac{C_e+C_r}{E[T]}. \tag{A-5}$$

In ETC(n, k, h), rC_p is a constant and does not have any effect on the optimal decision. So we eliminate this term from consideration and the notation ETC is adopted to the sum of these cost terms.