The Journal of Engineering Geology, Vol. 5, No. 2, September., 1995, pp. 215–231

Seismic Moment Tensor and Its Inversion: An Overview
지진모멘트 Tensor와 전환 : 개요

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요약/Abstract

지진 모멘트 tensor와 진원함수가 물리학자 방식으로 소개된다. 1970년도 이후에 개발된 지진모멘트 텐사와 진원 함수는 현대지진학에서 매우 중요한 역활을 하여 왔다. 현대 지진학 언어로 지진 진원 기술은 지하인공폭발, 광산 암석 붕괴, 저수지 유도지진과 같은 인공지진은 물론 고유 지진의 물리현상을 연구하게 하여 왔다. 더우기 고전 지진학에서 중요하지 않았고 고전 지진학에 포함되어 있지 않았던 새로운 개념, 디지털 지진학 관측은 더욱 중요성을 띄게 되었다. 지진학의 기초, 특히 지진진원의 물리학을 응용물리학의 일부분으로 다룰 때가 왔다고 본다.

The key concepts of seismic moment tensor are introduced in a 'physicist–oriented' style. The theory and application of seismic moment tensor which have been developed since the 1970s have become one of the most important branches in modern seismology. The description of earthquake sources in the modern seismology have led to much deeper understanding of the physics of indigenous earthquakes as well as various kinds of artificial seismic events, such as underground explosions, mining rockbursts, and reservoir induced tremors. Furthermore, with the development of digital seismological observation, some concepts, which were not included in 'classical' seismology, or not so important in 'classical' seismology, has become more and more important. It seems that it has been the time to have a new look at the fundamentals of seismology as a branch of applied physics, especially the part dealing with the physics of earthquake sources. Also in this field it may be important to clarify some fundamental concepts which, unexpectedly, have caused confusions even among professionals.

Introduction

In recent years, studies on the physics of earthquake sources have developed in many aspects, among which one of the most important advances might be the physical description of a seismic source in the language of seismic moment tensor. The concept of seismic moment tensor can be traced back to as early as the 1970s (Gilbert, 1970, 1973; Backus and Mulcahy, 1976, 1977; Backus, 1977a, b). Since then the moment tensor approach has been undertaken using the data ranging from normal modes (*e.g.*, Gilbert and Dziewonski, 1975) to surface waves (*e.g.*, McCowan, 1976) and body waves (*e.g.*, Fitch et al., 1980), from earthquakes (*e.g.*, Dziewonski, et al., 1981) to underground explosions (*e.g.*, Stump and Johnson, 1977, 1981) and mining tremors (*e.g.*, Sato and Fujii, 1989). At present, moment tensor inversion of earthquakes has become one of the most important analysis on routine basis. The literatures in this field is huge in number and will not be completely listed here. Various reviews and textbooks in different languages have covered these concepts at different levels (*e.g.*, Aki and Richards, 1980; Doorbos, 1981; Chen et al., 1992). However, it seems that a brief introduction to these concepts in a 'physicist—oriented' style is still necessary. Comparing to the endeavours to develop the theoretical models, the systematic description and discussion of the existing concepts is by no means unimportant. In physics there are so many examples that people would rather accept the 'simplified version' of the theories, among which one can have a long list including Maxwell's electromagnetic theory, theory of relativity, and quantum mechanics. In this review, attempts have been made to help the physicists who are not quite familiar with seismology to understand the physical significance of the concepts describing the properties of a seismic source. Also it is hoped that such a glimpse at the concepts of seismic moment tensor may be of help to the seismologists working in the field of observation, data interpretation, seismic zonation and earthquake forecasting.

First Principle in Modern Seismology

Homogeneous elastic Earth in the view of seismic waves

Up to now, most of the information about the seismic source come from the disturbances radiated from the seismic source and recorded by seismographs. The understanding of the nature of seismic source depends upon the capability of the interpretation of seismograms.

The order of the velocity of seismic wave propagation within the Earth is $10^0-10^1 km/s$, for example, the P—wave velocity is typically about $5km/s$ in the crust. If the predominant frequency under consideration is $1Hz$(in seismology, for example, the body wave magnitude m_b is measured using the body wave signals with period of 1s, and the surface wave magnitude M_s is measured using the surface wave signals with period of 20s), the predominant wavelength will be some kilometers. For seismic waves with these wavelengths, the inhomogeneities within the Earth are smoothed out, and the Earth can be regarded as a kind of homogeneous medium. As a modification to this simple model, one may consider a medium consisting different blocks (for $3-D$ cases) or different layers (for $1-D$ cases), with each block or layer being homogeneous.

The typical time duration for the radiation of seismic waves from the source is $10^{-1}-10^2$s, for example, the time for a magnitude 7 earthquake to complete its rupture process is typically some seconds. Within this time duration, 'seen' by seismic waves, the predominant process within the Earth is the radiation and propagation of ' accoustic' waves, and the Earth may be treated as a kind of elastic medium. (One of the characteristics distiguishing seismic waves from other kinds of accoustic waves is that in most cases the seismic waves have both the volumic component and the shear component, coupling with each other to generate a variety of seismological phenomena associated with wave propagation in the Earth, which forms both the difficulty and the pleasure of theoretical seismologists.) As a modification to this simple model, one may consider a small imaginary part in the wave velocity, *i.e.*, to represent the velocity of seismic wave c as $Re(c)+iIm(c)$, where the second term on the right hand side describes the intrinsic and apparent inelastic effects of the medium.

Representation theorem

Within a homogeneous elastic medium, the motion at position $\vec{r}$ and time t can be represented by the discontinuities of displacement and stress occurring at the boundary and the body forces acting on the medium. In physics, such a relation is equivalent to the fact that the solution to the equation of motion in dynamic elasticity is determined by the boundary conditions and initial conditions as well as the external forces. In such a way, the representation theorem can be formulated as

$$u_i(\vec{r},t)=\int_{-\infty}^{\infty}dt'\int_V f_i G_{ij}dV' - \int_{-\infty}^{\infty}dt'\oint_S u_i c_{jkpq}\frac{\partial G_{ip}}{\partial x'_q}\nu_k dS' + \int_{-\infty}^{\infty}dt'\oint_S G_{ij}c_{jkpq}\frac{\partial u_p}{\partial x'_q}\nu_k dS' \qquad (1)$$

The function $G_{ij}=G_{ij}(\vec{r},t\,;\vec{r'},t')$, being referred to as the Green's function and represented by

$$\rho\frac{\partial^2}{\partial t^2}G_{ij}=\delta_{ij}\delta(\vec{x}-\vec{x'})\delta(t-t') + \frac{\partial}{\partial x_n}\left(c_{inkl}\frac{\partial}{\partial x_l}G_{kj}\right) \qquad (2)$$

$$G_{ij}=0, \quad \frac{\partial}{\partial t}G_{ij}=0 \qquad \text{as } t<t',\ \vec{r}\neq\vec{r'}$$

is the displacement at position $\vec{r}$ and time t along direction i caused by the unit impulse at position $\vec{r'}$ and time t′ along direction j. It is obvious that when dealing with a time-independent boundary condition,

$$G_{ij}(\vec{r},t\,;\vec{r},t')=G_{ij}(\vec{r},t-t'\,;\vec{r'},0)$$

In the above equations as well as the equations thereafter, the Einstein convention is adopted, *i.e.*, using repeated subscripts to represent summation, *e.g.*,

$$u_i v_i = u_1 v_1 + u_2 v_2 + u_3 v_3$$

Our discussion will start from the representation theorem. To some extent, the representation theorem shown in equation (1) is the 'first principle' in modern seismology. For example, if we consider a 'fault surface' Σ, the theorem will have the form of

$$u_i(\vec{r},t)=\int_{-\infty}^{\infty}dt'\int_V f_i G_{ij}dV' + \int_{-\infty}^{\infty}dt'\int_\Sigma [u_i]\, c_{jkpq}\frac{\partial G_{ip}}{\partial x'_q}\nu_k d\Sigma' - \int_{-\infty}^{\infty}dt'\int_\Sigma G_{ij}c_{jkpq}\left[\frac{\partial u_p}{\partial x'_q}\right]\nu_k d\Sigma'$$

where [] denotes the discontinuities along the surface Σ, *e.g.*,

$$[u_i]=u_i(\Sigma^+)-u_i(\Sigma^-)$$

In the view of seismological observation, this theorem pointed out that the recorded ground motion is determined by the properties of the source (represented by the terms f_i, $[u_i]$ and $[\frac{\partial u_i}{\partial x_j}]$) and the wave propagation (represented by the term G_{ij}). In other words, from the analysis of seismograms, it is possible to retrieve the information of the source and the medium. In the view of theoretical modelling, on the other hand, such a theorem provided a theoretical simplicity that one can model the source (f_i, $[u_i]$, and $[\frac{\partial u_i}{\partial x_j}]$) and the medium structure and wave propagation (G_{ij}), respectively, and then combine the results using equation (1). In this case, the representation theorem acts as the 'interface' between the study of earthquake sources and the study of seismic wave propagation and the structure of the Earth.

In fact, the range for the representation theorem to be valid is much wider than merely the homogeneous elastic medium. As long as the constitution law of the Earth can be represented by

$$\tau_{ij} = c_{ijpq} e_{pq} \quad (3)$$

where τ_{ij} is the stress, e_{ij} is the strain, and c_{ijpq} is the elastic moduli tensor, equation (1) will be valid. In this case, the situations of inhomogeneous anisotropic elastic medium can also be included.

Seismic Moment Tensor

In mathematics, there are two models which can lead to the concept of seismic moment tensor. As the first order approximation, these two ways lead to the same representation of the ' point source' seismic moment tensor which is refered to as the 'seismic moment tensor'.

Representation theorem for a 'small' seismic source

Suppose that the ground motion is caused only by body force f_i which includes both the real forces and the 'equivalent forces'. In this case, the ground motion $u_i(\vec{r}, t)$ can be represented by the first term in (1)

$$u_i(\vec{r}, t) = \int_{-\infty}^{\infty} dt' \int_V f_i G_{ij} dV \quad (4)$$

Seen in the scale of the Erath, the seismic source is 'small'. For example, the size of the source of a magnitude 7 earthquake is typically tens of kilometers. Comparing to the regions which could receive the disturbances from the earthquake, the source is really a small region.

In this case, the representation (4) can be expanded around a reference position $\vec{r_0}$:

$$u_i(\vec{r}, t) = \int_{-\infty}^{\infty} dt' \int_{V_0} G_{ij}(\vec{r}, t-t' ; \vec{r'}, 0) f_j (\vec{r'}, t')\, dV' = \int_{-\infty}^{\infty} dt' \int_{V_0} \{G_{ij}(\vec{r}, t-t' ; \overrightarrow{r_0'}, 0) + (x_k' - x_k^0) \frac{\partial G_{ij}(\vec{r}, t-t' ; \overrightarrow{r_0'}, 0)}{\partial x_k^0}$$
$$+ \frac{1}{2!}(x_{k_1}' - x_{k_1}^0)(x_{k_2}' - x_{k_2}^0) \frac{\partial^2 G_{ij}(\vec{r}, t-t' ; \overrightarrow{r_0'}, 0)}{\partial x_{k_1}^0 \partial x_{k_2}^0} + \cdots$$
$$+ \frac{1}{n!}(x_{k_1}' - x_{k_1}^0)(x_{k_2}' - x_{k_2}^0) \cdots (x_{k_n}' - x_{k_n}^0) \frac{\partial^n G_{ij}(\vec{r}, t-t' ; \overrightarrow{r_0'}, 0)}{\partial x_{k_1}^0 \partial x_{k_2}^0 \cdots \partial x_{k_n}^0} + \cdots\} f_j(\vec{r'}, t') dV'$$

As $G_{ij}(\vec{r}, t-t' ; \overrightarrow{r_0'}, 0)$ and its partial derivatives do not include $\vec{r'}$, one has

$$u_i(\vec{r}, t) = \int_{-\infty}^{\infty} dt' \, G_{ij}(\vec{r}, t-t' ; \overrightarrow{r_0'}, 0) \int_{V_0} f_j (\vec{r'}, t') dV' + \int_{-\infty}^{\infty} dt' \frac{\partial G_{ij}(\vec{r}, t-t' ; \overrightarrow{r_0'}, 0)}{\partial x_k^0} \int_{V_0} (x_k' - x_k^0) f_j(\vec{r'}, t') dV' + \int_{-\infty}^{\infty} dt' \frac{1}{2!} \frac{\partial^2 G_{ij}(\vec{r}, t-t' ; \overrightarrow{r_0'}, 0)}{\partial x_{k_1}^0 \partial x_{k_2}^0} \int_{V_0} (x_{k_1}' - x_{k_1}^0)(x_{k_2}' - x_{k_2}^0) f_j(\vec{r'}, t')\, dV' + \cdots$$
$$+ \int_{-\infty}^{\infty} dt' \frac{1}{n!} \frac{\partial^n G_{ij}(\vec{r}, t-t' ; \overrightarrow{r_0'}, 0)}{\partial x_{k_1}^0 \cdots \partial x_{k_n}^0} \int_{V_0} (x_{k_1}' - x_{k_1}^0) \cdots (x_{k_n}' - x_{k_n}^0) f_j(\vec{r'}, t') dt' + \cdots$$

The first term on the right hand side is the zeroth-order moment, representing the body force acting within volume V_0:

$$F_j(\vec{r_0}, t') = \int_{V_0} f_j(\vec{r'}, t') dV' \quad (5)$$

The second term on the right hand side is the first-order moment, being referred to as the moment tensor:

$$M_{jk}(\vec{r_0}, t') = \int_{V_0} (x_k' - x_k^0) f_j(\vec{r'}, t') dV' \quad (6)$$

Similarly, the n-th order moment may be defined as

$$P_{k_1 k_2 \cdots k_n}(\vec{r_0}, t') = \int_{V_0} (x_{k_1}' - x_{k_1}^0)(x_{k_2}' - x_{k_2}^0) \cdots (x_{k_n}' - x_{k_n}^0) f_j(\vec{r'}, t') dV' \quad (7)$$

Thus

$$u_i(\vec{r}, t) = \int_{-\infty}^{\infty} dt' \{G_{ij}(\vec{r}, t-t' ; \overrightarrow{r_0'}, 0) F_j (\overrightarrow{r_0'}, t') + \frac{\partial}{\partial x_k^0} G_{ij}(\vec{r}, t-t' ; \overrightarrow{r_0'}, 0) M_{jk}$$

$$(\vec{r_0}', t') + \cdots + \frac{1}{n!}\frac{\partial^n}{\partial x_{k_1}{}^0 \cdots \partial x_{k_n}{}^0} G_{ij} (\vec{r}, t-t'\ ; \vec{r_0}' 0) P_{jk_1k_2\cdots k_n}(\vec{r'}_0, t') + \cdots\} \quad (8)$$

Seen in frequency domain, the contribution of the n-th order moment can be estimated by

$$|u_i(\omega)|_n \sim \frac{1}{n!}|G_{ij}|\,\omega^n \cdot 3^n \cdot L^n\,|f_j| = \frac{1}{n!}\left(\frac{6\pi L}{\lambda}\right)^n |G_{ij}|\ |f_j| \quad (9)$$

in which the factor ω^n comes from the partial derivative in the frequency domain, and the factor 3^n come from that k_1, k_2, ..., k_n all have the values of 1, 2, and 3, so $|u(\omega)|_n$ includes 3^n terms. As an approximate estimation, L is roughly the size of the source, contributed by the term $(x_{k_1}{}' - x_{k_1}{}^0)(x_{k_2}{}' - x_{k_2}{}^0)\cdots(x_{k_n}{}' - x_{k_n}{}^0)$. It is clear that when $\lambda = \frac{2\pi c}{\omega} \gg L$, *i.e.*, for the situation of a 'point source' in which the dimension of the source L is much less than the wavelength λ under consideration, only the first one or two terms play the predominant role.

For the situation of earthquake source as an internal source, the total force is to be zero

$$F_i(\vec{r_0}, t') = \int_{V_0} f_i(\vec{r'}, t')\, dV' = 0 \quad (10)$$

In this case, the point source representation may be written as

$$u_i(\vec{r}, t) = \int_{-\infty}^{\infty} \frac{\partial}{\partial x_k^0} G_{ij}(\vec{r}, t-t'\ ; \vec{r_0}', 0) M_{jk}(\vec{r_0}', t')\, dt' \quad (11)$$

Another way leading to the concept of moment tensor

On the other hand, if the ground motion may be considered as to be caused by a kind of 'stressless free strain' e_{ij}^T, and the total strain is represented by e_{ij}, the constitution law gives

$$\tau_{ij} = c_{ijkl}(e_{kl} - e_{kl}^T) \quad (12)$$

In this case, the equation of motion

$$\rho \frac{\partial^2}{\partial t^2} u_i = f_i + \frac{\partial \tau_{ij}}{\partial x_j} \quad (13)$$

with $f_i = 0$ may be written as

$$\rho \frac{\partial^2}{\partial t^2} u_i = f_i^T + \frac{\partial \tau_{ij}}{\partial x_j} \quad (14)$$

in which

$$f_i^T \equiv -\frac{\partial}{\partial x_j}(c_{ijkl} e_{kl}^T)$$

Letting the stress glut (also refered to as the moment density tensor) be

$$m_{ij} \equiv c_{ijkl} e_{kl}^T$$

and

$$f_i^T \equiv -\frac{\partial m_{ij}}{\partial x_j}$$

from equation (1) it comes that

$$u_i(\vec{r}, t) = \int_{-\infty}^{\infty} dt' \int_V f_j^T G_{ij}\, dV' - \int_{-\infty}^{\infty} dt' \oint_S (-G_{ij} m_{jk}) \nu_k\, dS' = -\int_{-\infty}^{\infty} dt' \int_V G_{ij} \frac{\partial}{\partial x'_k} m_{jk}\, dV' - \int_{-\infty}^{\infty} dt' \oint_S (-G_{ij} m_{jk}) \nu_k\, dS' = \int_{-\infty}^{\infty} dt' \int_V \frac{\partial G_{ij}}{\partial x'_k} m_{jk}\, dV' \quad (16)$$

In the view of the conservation of angular momentum, it is clear that m_{jk} is a symmetric tensor.

As the dimension of the source is much smaller than the wavelength under consideration, the source degenerates to a 'point source'. In the way similar to the above section, or more simply, represent m_{jk} as

$$m_{jk}(\vec{r}, t) = M_{jk}(t)\,\delta(\vec{r} - \vec{r_0})$$

one has

$$u_i(\vec{r}, t) = \int_{-\infty}^{\infty} \frac{\partial}{\partial x_k^0} G_{ij}(\vec{r}, t\ ; \vec{r_0}', t') M_{jk}(t')\, dt'$$

being the same as equation (11). (17)

More complex sources

From the above discussions it can be seen that in the view of 'point source', we have two ways which can both lead to the concept of 'seismic moment tensor':

$$M_{jk}(\vec{r_0}, t') = \int_{V_0} (x_k' - x_k^0) f_j(\vec{r'}, t')\, dV'$$

$$M_{ij}(\vec{r_0}, t') = \int_{V_0} c_{ijkl}(\vec{r'}, t') e_{kl}^T(\vec{r'}, t')\, dV'$$

In both cases, the ground motion can be represented by

$$u_i(\vec{r},t)=\int_{-\infty}^{\infty}\frac{\partial}{\partial x^0_k}G_{ij}(\vec{r},t\,;\,\vec{r_0},t')M_{jk}(t')dt'$$

As the source deviates from a 'point source', *i.e.*, the structure of the source must be accounted for, there are two different approaches to deal with the situation: to consider the higher-order moments, or to consider the distribution of the moment density. It seems that most seismologists prefer the latter.

Even if the structure of the source must be taken into consideration, the concept of 'point source' can still be used in the way that the centroid of the moment release is taken, as long as $\frac{\partial G_{ij}}{\partial x_k}$ varies smoothly within the source region.

Moment Tensors of Different Types of Earthquakes

Shear dislocation

In a homogeneous isotropic elastic medium, the moduli tensor can be represented as

$$c_{ijkl}=\lambda\delta_{ij}\delta_{kl}+\mu(\delta_{ik}\delta_{jl}+\delta_{il}\delta_{jk}) \tag{18}$$

and the 'point source' moment density tensor can be written as

$$m_{jk}(\vec{r},t)=(\lambda\delta_{jk}\varepsilon^T_{mm}+2\mu\varepsilon^T_{jk})\,V\delta(\vec{r}-\vec{r_0}) \tag{19}$$

If the source is a 'fault', *i.e.*, to consider the source as a cubic element with volume V, area A, and a small thickness h,

$$V=Ah$$

when $h\to 0$, one has

$$M_{jk}(t)=\{\lambda\delta_{jk}[u_m]\;\nu_m+\mu([u_j]\nu_k+[u_k]\;\nu_j)\}A$$

where ν_k is the normal of A, and $[u_i]$ represents the discontinuity of displacement, or the dislocation along the 'fault surface'. As a further approximation, taking the dislocation to be all the same along the surface, *i.e.*,

$$[u_i]=D(t)e_i$$

in which e_i denotes the direction of the fault motion, one has

$$M_{jk}(t)=\{\lambda\delta_{jk}\nu_m e_m+\mu(\nu_k e_j+\nu_j e_k)\}D(t)A \tag{20}$$

For a pure shear dislocation with $\vec{e}$ perpendicular to $\vec{\nu}$, or

$$\nu_m e_m=0$$

one has

$$M_{jk}(t)=M_0(t)(\nu_k e_j+\nu_j e_k) \tag{21}$$

in which

$$M_0(t)=\mu D(t)A \tag{22}$$

is the scalar seismic moment.

In the geographical coordinate system with x_1 pointing northwards, x_2 pointing eastwards and x_3 pointing downwards, using the strike ϕ_s (counting clockwise from the north) and dip δ (counting from the surface of the Earth) of the fault plane and the rake λ (counting counter-clockwise on the fault plane, from the direction of the strike) of the slip motion to describe the configuration of the shear dislocation, one has

$$\begin{aligned}
M_{11}&=-M_0(\sin\delta\cos\lambda\sin2\phi_s-\sin2\delta\sin\lambda\cos^2\phi_s) \quad (23)\\
M_{22}&=M_0(\sin\delta\cos\lambda\sin2\phi_s-\sin2\delta\sin\lambda\cos^2\phi_s)\\
M_{33}&=M_0\sin2\delta\sin\lambda\\
M_{12}&=M_0(\sin\delta\cos\lambda\cos2\phi_s+\tfrac{1}{2}\sin2\delta\sin\lambda\sin2\phi_s)=M_{21}\\
M_{13}&=-M_0(\cos\delta\cos\lambda\cos\phi_s+\cos2\delta\sin\lambda\sin\phi_s)=M_{31}\\
M_{23}&=-M_0(\cos\delta\cos\lambda\sin\phi_s-\cos2\delta\sin\lambda\cos\phi_2)=M_{32}
\end{aligned}$$

It is easy to see that the moment tensor of the shear dislocation satisfies

$$M_{11}+M_{22}+M_{33}=0 \tag{24}$$

and

$$\begin{vmatrix} M_{11} & M_{12} & M_{13}\\ M_{21} & M_{22} & M_{23}\\ M_{31} & M_{32} & M_{33}\end{vmatrix}=0 \tag{25}$$

In this way, the moment tensor of a shear dislocation has only four independent parameters, corresponding to the fact that the slip on the fault can be described by four independent parameters, ϕ_s, δ, λ, and M_0.

Volumic-change-type events: explosions and collapse

For volumic change, using

$$\varepsilon_{jk}^{T}=\frac{1}{3}\theta^{T}\delta_{jk}$$

in which

$$\theta^{T}=\varepsilon_{mm}^{T}$$

the moment tensor will be

$$M_{jk}=(\lambda+\frac{2}{3}\mu)\theta^{T}V\delta_{jk} \qquad (26)$$

or in another form,

$$M_{jk}=(\lambda+\frac{2}{3}\mu)\theta^{T}V\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$$

$$\equiv V\begin{bmatrix}\triangle P&0&0\\0&\triangle P&0\\0&0&\triangle P\end{bmatrix}$$

Inversion of the moment tensors of underground explosions shows that the predominate elements of the moment tensors are M_{11}, M_{22} and M_{33} (Stump and Johnson, 1981). More generally, the isotropic moment tensor can also be used to characterize many other kinds of volumic changes within the Earth, such as cavity collapse in mining regions, volumic changes caused by phase transition or other mechanisms associated with geothermal phenomena. It is also argued that deep and intermediate-depth earthquakes may be accompanied by volumic change. Up to now, however, the evidences are not so convincing.

Tensional component and the compensated linear vector dipole

Coming back to (20), if we consider the tensional dislocation, *i.e.*, ν_j is parallel to e_j, we will get the moment tensor of the tensional crack.

A good example of the tensional crack is spall. In underground explosions, the shock waves from the explosion are reflected by the surface of the Earth, the upward polarized shock wave and the downward polarized reflections cause the rock above the explosion to form tensional cracks paralell to the surface, being refered to as 'spall' (Day and McLaughlin, 1991). To obtain the moment tensor of spall, consider

$$\vec{e}=\vec{\nu}=(0,0,1)$$

In this case

$$M_{jk}=\begin{bmatrix}\lambda DA&0&0\\0&\lambda DA&0\\0&0&(\lambda+2\mu)DA\end{bmatrix} \qquad (27)$$

The tensional crack consists two parts: the volumic change and the deviatoric part. The deviatoric part

$$M_{jk}=\begin{bmatrix}-\frac{2}{3}\mu DA&0&0\\0&-\frac{2}{3}\mu DA&0\\0&0&\frac{4}{3}\mu DA\end{bmatrix} \qquad (28)$$

has the form of compensated linear vector dipole, abbreviated as CLVD (Knopoff and Randall, 1970).

Geometry of the Seismic Moment Tensor

Decomposition of seismic moment tensor

In the analysis of a seismic moment tensor, usually one can find the eigenvectors of the moment tensor, *i.e.*, to solve the equation

$$(M_{jk}-M\delta_{jk})a_k=0 \qquad (29)$$

in which M is the eigenvalue and a_k is the eigenvector, or in another form,

$$\begin{bmatrix}M_{11}&M_{12}&M_{13}\\M_{21}&M_{22}&M_{23}\\M_{31}&M_{32}&M_{33}\end{bmatrix}\begin{bmatrix}a_1\\a_2\\a_3\end{bmatrix}=M\begin{bmatrix}a_1\\a_2\\a_3\end{bmatrix}$$

which requires

$$\begin{vmatrix}M_{11}-M&M_{12}&M_{13}\\M_{21}&M_{22}-M&M_{23}\\M_{31}&M_{32}&M_{33}-M\end{vmatrix}=0 \qquad (30)$$

From (30) it may be seen that M_{jk} has three eigenvalues, namely M_1, M_2, and M_3 with $M_1 > M_2 > M_3$. The three eigenvectors corresponding to M_1, M_2, and M_3, respectively, are perpendicular to each other. In the principle coordinate system defined by the eigenvectors, a shear dislocation source has the form of

$$M_{jk} = M_0 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

representing a double couple. In this view of equivalence, a point shear dislocation can also be represented as a double couple, which is consistent with the conclusions obtained in seismology in the 1960s (Maruyama, 1963; Burridge and Knopoff, 1964), but in a more general way.

Generally, in the principle coordinate system, the moment tensor can be written in a diagonalized form

$$M_{jk} = \begin{bmatrix} M_1 & 0 & 0 \\ 0 & M_2 & 0 \\ 0 & 0 & M_3 \end{bmatrix}$$

Such a representation can be decomposed as

$$M_{jk} = \begin{bmatrix} M_1 & 0 & 0 \\ 0 & M_2 & 0 \\ 0 & 0 & M_3 \end{bmatrix}$$

$$= \frac{1}{3}(M_1 + M_2 + M_3) \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad (31)$$

$$+ \frac{1}{2}(M_1 - M_3) \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

$$+ \frac{1}{6}(2M_2 - M_1 - M_3) \begin{bmatrix} -1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

It is clear that the first term on the right hand side is the volumic change; the second term is a double couple; and the third term is a compensated linear vector dipole (CLVD).

The best double couple

In mathematics, the decomposition of a general seismic moment tensor is not unique. However, to have a good look at the properties of a seismic source, one has to seperate a general moment tensor into some components. In principle, such a decomposition is a problem of physics rather than mathematics.

For earthquakes occurred in the crust and upper mantle, one of the most important decomposition scheme is to consider the best double couple. Generally, a moment tensor can be written as the superposition of a symmetric volumic change component $\frac{1}{3} M_{mm}\delta_{jk}$ and a deviatoric component M_{jk}'. In this case, one may define a double couple which is nearest to the deviatoric part, *i.e.*, introducing a double couple $M_0(\nu_j e_k + \nu_k e_j)$, and letting the norm of

$$d_{jk} = M_{jk}' - M_0(\nu_j e_k + \nu_k e_j) \qquad (32)$$

to minimize. It can be proved that the double couple defined in such a way is just the double couple component in (31). The eigenvector associated with the largest eigenvalue points to the direction of the T axis of the best double couple, the eigenvector associated with the smallest eigenvalue points to the direction of P axis, while the other eigenvector is along the B axis.

Geometrical representation of a moment tensor

For a homogeneous elastic medium, the Green's function G_{ij} can be calculated analytically, and the seismic wave radiation can be represented by

$$u_i(\vec{r},t) = \frac{(15\gamma_i\gamma_j\gamma_k - 3\gamma_i\delta_{jk} - 3\gamma_j\delta_{ik} - 3\gamma_k\delta_{ij})}{4\pi\rho R^4} \int_{R/\alpha}^{R/\beta} \tau M_{jk}(t-\tau)\,d\tau + \frac{(6\gamma_i\gamma_j\gamma_k - \gamma_i\delta_{jk} - \gamma_j\delta_{ik} - \gamma_k\delta_{ij})}{4\pi\rho\alpha^2 R^2} M_{jk}(t-\frac{R}{\alpha}) - \frac{(6\gamma_i\gamma_j\gamma_k - \gamma_i\delta_{jk} - \gamma_j\delta_{ik} - 2\gamma_k\delta_{ij})}{4\pi\rho\beta^2 R^2} M_{jk}(t-\frac{R}{\beta}) + \frac{(\gamma_i\gamma_j\gamma_k)}{4\pi\rho\alpha^3 R} \frac{\partial}{\partial t} M_{jk}(t-\frac{R}{\alpha}) - \frac{(\gamma_i\gamma_j - \delta_{ij})\gamma_k}{4\pi\rho\beta^3 R} \frac{\partial}{\partial t} M_{jk}(t-\frac{R}{\beta}) \qquad (33)$$

in which $\vec{e}_R=\frac{\vec{R}}{R}=\frac{\vec{r}-\vec{r}_0}{|\vec{r}-\vec{r}_0|}=\gamma_i e_i$. The first three terms on the right hand side represent the 'near—field' radiation, while the last two terms represent the 'far—field' radiation. In previous literatures often the first term was called the 'near—field' term, and the two following terms were called the 'intermediate—field' terms. However, to some extent, such a naming is a misleading one, since there is no 'intermediate' range of distances in which the 'intermediate—field' terms dominate. In fact, they are small in 'far—field' and are often of comparable importance to the 'near—field' term for 'near—field' cases.

In the spherical coordinate system with its origin at the source, the far—field radiation pattern can be written into a simple form

$$u_P=\frac{1}{4\pi\rho\alpha^3R}\frac{\partial}{\partial t}P$$
$$u_{SV}=\frac{1}{4\pi\rho\beta^3R}\frac{\partial}{\partial t}SV \qquad (34)$$
$$u_{SH}=\frac{1}{4\pi\rho\beta^3R}\frac{\partial}{\partial t}SH$$

The radiation pattern factors are

$$P=\sin^2 i_\xi[\cos^2\phi M_{11}+\sin2\phi M_{12}+\sin^2\phi M_{22}-M_{33}] +2\sin i_\xi\cos i_\xi[\cos\phi M_{13}+\sin\phi M_{23}]+M_{33}$$
$$SV=\sin j_\xi\cos j_\xi[\cos^2\phi M_{11}+\sin2\phi M_{12}+\sin^2\phi M_{22}-M_{33}]+(1-2\sin^2 j_\xi)[\cos\phi M_{13}+\sin\phi M_{23}] \qquad (35)$$
$$SH=\sin j_\xi[\frac{1}{2}\sin2\phi(M_{22}-M_{11})+\cos2\phi M_{12}]+\cos j_\xi[\cos\phi M_{23}-\sin\phi M_{13}]$$

in which i_ξ and j_ξ are the take—off angles (counting from the vertical direction) of P and S waves, respectively, and ϕ is the azimuth (counting clockwise from the north) of the station.

Similar to 'classical' seismology, the nodals at which

$$\sin^2 i_\xi[\cos^2\phi M_{11}+\sin2\phi M_{12}+\sin^2\phi M_{22}-M_{33}] +2\sin i_\xi\cos i_\xi[\cos\phi M_{13}+\sin\phi M_{23}]+M_{33}=0 \qquad (36)$$

may be used as a representation of a general moment tensor.

To some extent, such a representation may be regarded as a kind of 'double—couple oriented'. When the predominant component of a moment tensor is a double couple, as for most of the earthquakes, such a representation is convenient. On the other hand, however, if the seismic source contains a large non—double—couple component, *e.g.*, for the case of underground explosions, such representation can not work well to characterize the properties of the seismic source. For example, underground explosions are often accompanied by a double couple caused by the strain release near to the explosion (see, *e.g.*, Wallace, 1983). In the geometrical representation, it is natural to hope that such representation would reflect the properties of both the explosion and the strain release. However, if the explosion component were overwhelming so that $P>0$ for all i_ξ, j_ξ and ϕ in (35), the characteristics of the strain release would not be reflected in such representation. In this case, a simple alternative scheme is to use the geometrical representation for these three components, respectively, and possibly, to let the radius of the 'source sphere' proportional to the logarithm of the scalar moment of each component to represent their relative importance.

Moment Tensor Inversion

If one can say that the concept of seismic moment tensor came from the advance of theoretical seismology, then the application of seismic moment tensor as a successful phenomenological description of earthquake sources are mainly due to the development of digital seismological observation and the development of computation in seismology.

Moment tensor inversion

Seen in frequency domain, equation (17) can be rewritten as

$$U_i(\vec{r},\omega)=\frac{\partial}{\partial x^0_k}G_{ij}(\vec{r},\omega;\vec{r_0}')\cdot M_{jk}(\vec{r'}_0,\omega) \qquad (37)$$

or more conveniently, as

$$U_i(\vec{r},\omega)=G_{ij,k}(\vec{r},\omega;\overrightarrow{r_0'})M_{jk}(\overrightarrow{r_0'},\omega)$$

in which the comma among subscripts represents the partial derivative.

As mentioned in equation (33), for the case of 'near−field', one has

$$u\sim\frac{1}{R^4}\int_{R/\beta}^{R/\alpha}\tau M_{jk}(t-\tau)d\tau+\frac{C_1}{R^2}M_{jk}(t-\frac{R}{\alpha})+\frac{C_2}{R^2}M_{jk}(t-\frac{R}{\beta})$$

For the case of 'far−field', on the other hand, one has

$$u\sim\frac{D_1}{R}\frac{\partial}{\partial t}M_{jk}(t-\frac{R}{\alpha})+\frac{D_2}{R}\frac{\partial}{\partial t}M_{jk}(t-\frac{R}{\beta})$$

In convention, there are two kinds of seismic moment tensors: the 'near−field' moment tensor M_{jk}, and the 'far−filed' moment tensor $\frac{\partial}{\partial t}M_{jk}$. The latter is also called the 'moment rate tensor'. Accordingly, the inversion problem can be represented by two forms. One is

$$U_i(\vec{r},\omega)=G_{ij,k}(\vec{r},\omega;\overrightarrow{r_0'})M_{jk}(\overrightarrow{r_9'}.\omega) \quad (38)$$

in which the Green's function G_{ij} represents the ground motion generated by an impulse point force (described by a Dirac−δ function). The other form is

$$U_i(\vec{r},\omega)=G^H_{ij,k}(\vec{r},\omega;\overrightarrow{r_0'})\frac{\partial}{\partial t}M_{jk}(\overrightarrow{r_0'}.\omega) \quad (39)$$

in which the Green's function $G^H_{ij,k}$ represents the ground motion generated by a step point force (described by a unit step function). Keeping in mind this difference, we will only use equation (38) and the corresponding expressions in the following discussions.

Noticing that the moment tensor is symmetric, (38) can be rewritten into a more simple form:

$$U_i(\vec{r},\omega)=g_{ij}(\vec{r},\omega;\overrightarrow{r_0'})M_j(\overrightarrow{r_0'}.\omega) \quad (40)$$

in which

$$M_j=\{M_{11},M_{12},M_{13},M_{22},M_{23},M_{33}\} \quad j=1,2,\cdots 6 \quad (41)$$

and g_{ij}, being a linear combination of the original Green's function, act as the corresponding 'coefficients'. In this case, the inverse problem is straightforward, for example, the least squares solution of this problem is

$$M_j=(g_{ij}^T g_{ij})^{-1}g_{ij}^T U_i$$

Obviously, the inversion can also be conducted in time domain.

Centroid moment tensor (CMT)

Generally, not only the six independent elements of the moment tensor M_j, but also the location $\vec{r_0}$ and time t' of the 'point source' as a centroid of the moment release need to be determined. For a distribution of moment density tensor, suppose that the Green's function varies soomthly within the source region, introducing

$$\Lambda_{ijk}=\int_{-\infty}^{\infty}dt\int_{V_0}(x_k-x_k^0)\frac{\partial}{\partial t}m_{ij}(\vec{r},t)dV' \quad (42)$$
$$H_{ij}=\int_{-\infty}^{\infty}dt\int_{V_0}(t-t^0)\frac{\partial}{\partial t}m_{ij}(\vec{r},t)dV$$

the centroid coordinate (x_k^0, t^0) may be defined as the values which make Λ_{ijk} and H_{ij} to minimize. The moment tensor is thus

$$M_{ij}=\int_{-\infty}^{\infty}dt\int_{V_0}\frac{\partial}{\partial t}m_{ij}(\vec{r},t)dV \quad (43)$$

As can be seen from (42) and (43), the inversion of centroid moment tensor is not a linear inversion. Actually such inversion is often carried out by iterative procedures.

Generally the position of the centroid and the 'hypocenter' determined by the arrivals of different phases are quite different. The latter can be regarded as the initial nucleation position of the rupture, while the former may be regarded as to be associated with the overall characteristics of the source process (see, e.g., Zobin, 1991). In practice, the definition of the centriod given in (42) and (43) has not obtained widespread acceptance. Despite that the basic idea is common that one needs to determine both the moment tensor and the coordinates of the 'centroid', in practical analysis, the understandings of the details of the 'centroid' are never clarified. This means that one has to be careful in using the results of CMTs from different sources, especially in discussing some phenomena associated with the centroids.

The estimation of CMTs for large and intermediate earthquakes has become an analysis on routine basis since the 1980s. At present, within approximately ten hours after an earthquake with $m_b \geq 5.5$, CMT is estimated by Harvard University using mantle waves and by NEIC, USGS using long−priod body waves from the global seismograph network. Results are sent via electronic mail to agencies and seismologists registered as the users of the information service system. Using seismic data from regional seismic networks, the regional CMT, sometimes called RCMT, can also be estimated. To meet the needs of emergency response such as earthquake rescue and tsunami warning, the 'quick CMT' estimation with a few data and maybe with lower but still acceptable quality can be made shortly after the earthquake.

Constraints in the inversion

Sometimes it may be useful to put some constraints on the solution. One of the most commonly used constraints is to assume that the source is a deviatoric source, i.e.,

$$M_1+M_4+M_6=M_{11}+M_{22}+M_{33}=0 \tag{44}$$

Another useful constraint is

$$M_{ij}(t)=M_{ij}S(t)$$

where $S(t)$ is refered to as the source time function. Still another constraint is to assume that the earthquake is a shear dislocation, *i.e.*,

$$M_1+M_4+M_6 \equiv M_{11}+M_{22}+M_{33}=0$$

$$\begin{vmatrix} M_1 & M_2 & M_3 \\ M_2 & M_4 & M_5 \\ M_3 & M_5 & M_6 \end{vmatrix} \equiv \begin{vmatrix} M_{11} & M_{12} & M_{13} \\ M_{21} & M_{22} & M_{23} \\ M_{31} & M_{32} & M_{33} \end{vmatrix} = 0 \tag{45}$$

which makes the inversion to be a nonlinear inversion.

These constraints are adequate for earthquakes in which shear dislocation (double couple) plays the predominant role. For some other kinds of earthquakes, however, the constraints should be adjusted according to the physics of the seismic events. For instance, when considering deep focus earthquakes and trying to retrive the non−double−couple components, the constraints given by (45) is absolutely not valid. Generally, applying a constraint reduces the number of unknown parameters and increases the stability of the inversion. The constraint itself is a kind of *a priori* knowledge about the physics of the earthquake source. On the other hand, however, applying a constraint will also lose some useful information. In the inversion, such benefit and cost have to be balanced.

Green's function

To some extent, the difficulty of moment tensor inversion does not lie in the inversion itself but lie in the Green's function. The calculation of the Green's function requires enough knowledge about the seismic wave propagation within real Earth medium, and the structure between the source and the receiver. With incomplete knowledge about the wave propagation and the structure, one might also have a very nice fitting between the observational seismograms and the theoretical ones, but such a good match is only apparent:the match between the observational seismograms and the theoretical ones are obtained at the cost that false information are introduced into the inversion result. To some extent, this is the manifestation of the phenomena of GIGO ('garbage in, garbage out') in computational sciences. This means that before the inversion, the 'calibration' of Green's function is of most importance.

In the situation that the Green's functions are not complete to reflect the structure of the Earth, the azimuth coverage of stations becomes more important. Sometimes to retrieve the geometry of the moment tensor, *e.g.*, to obtain the strike, dip and rake of the best double couple for strike−slip type earthquakes, a 'good' station azimuth coverage can compensate, to some extent, the incompleteness of the Green's functions calculated. The less complete the Green's functions, the more important the azimuth coverage would be.

Scalar seismic moment and magnitude

In 'calssical' seismology, the size of an earthquake is estimated by its magnitude, a quantity measured by the logarithm of the amplitude of the seismogram with certain period and corrected according to the distance from the source to the receiver and the site effect of the seismic station. The seismic moment tensor is also a measure of the size of an earthquake. Taking far-field P-wave for example, one has

$$| u^P | \sim \frac{M_0}{R} \frac{\partial}{\partial t} S(t)$$

in which M_0 is the scalar seismic moment and $S(t)$ is the source time function. Taking the logarithm of $| u^P |$, one has

$$m = \log | u^P | + C = \log M_0 + D$$

Here m can be regarded as some kind of magnitude. It is clear that the magnitude in traditional seismological observations reflect the orders of the scalar seismic moment.

As to the measure of the size of an earthquake, it should be noted that seismic moment tensor, or more exactly, the point source moment tensor, is a 'low−frequency' concept. What the seismic moment tensor tells is the overall properties of an earthquake. In the study of stress field and tectonic movements, moment tensor is a useful quantity. In engineering seismology, on the other hand, laying more emphasis on the high−frequency contents of the seismic radiation, the broadband radiated energy (Boatwright and Choy, 1986) is more often used.

Example of moment tensor analysis: mining induced seismicity

Mining induced tremor is a special class of seismic events which cannot be described by a pure double couple. Comparing to earthquakes, it has the advantage that often people are able to 'access' the seismic source to have a close look at the mechanism of the tremor. Different from earthquakes, in which double couple plays the predominant role, and underground explosions, in which isotropic component plays the predominant role, mining seismicity has more complex focal mechanisms and, sometimes, cannot be explained by a predominant component of the seismic moment tensor. In recent years, studies on the mechanism of mining seismicity has attracted much attention in seismology and engineering (e.g., Gibowicz, 1989), among which the moment tensor approach is of special interest. Also it may be interesting that comparing to the moment tensor studies of earthquakes and underground explosions, the moment tensor studies of mining seismicity has only been started. Several problems still need to be discussed, although some interesting results have been obtained in Poland, Czech Republic, South Africa, Japan and China (Gibowicz, 1995, personal communication).

The necessity to use moment tensors in the characterization of mining seismicity comes from the fact that mining seismicity has focal mechanism which seems more complex than that of earthquakes. For instance, it was reported that (Sileny, 1989) some mining induced tremors can not be explained by a pure double couple, instead, they can be explained by a pure double couple plus an isotropic component with the proportion of the isotropic component being up to as much as some tens of percent comparing to the double couple component. In contrast, the proportion of the non−double−couple component in earthquakes are reported to be only some percent comparing to the double couple component, and there are many controversies about the reliability of the evidences obtained from waveform inversion. Furthermore, what is important is that field investigations indicate that the existence of the non−shear−dislocation mechanism for mining tremors is reliable.

According to their mechanisms and seismic radiation patterns, the sources of mining seismicity can be classified into some categories (Hasegawa, et al., 1989):cavity collapse as a result of the rockburst in a mine ceiling or the falling of a large mass of rock loosened by mining; pillar burst due to a combination of convergent forces related to the stope face advancement and time-dependent inelastic after-effects;tensile failure of cap rock above a mine occuring near the middle of a wide excavation;edge dislocations and comminuted faults occured near the stope face. In general, the moment tensor of a mining induced tremor has three components:isotropic component, of which the positive value represents the explosion and/or gas burst and the negative value represents the collapse;double couple caused by shear dislocations; and compensated linear vector dipole caused by the tensile component. The inversion did find these three components, which are of comparable importance to each other in some cases (Gibowicz, 1995, personal communication).

Again the difficulty of seismic moment tensor inversion for mining seismicity lie in the calculation of Green's functions. Different from the situation of earthquakes, in mining regions, the shallow structure of the medium, in which the seismic waves are excited by the source and propagated with reflection, refraction, scattering, attenuation and possibly reverberation and/or tunnelling, is quite complex, especially for the seismic signals with high frequency contents recorded near to the source. To some extent, the moment tensor study of mining seismicity can be regarded as a challenge to seismologists. On the other hand, mining seismicity has provided seismologists with a quasi-controllable situation to test the methodology and algorithms developed mainly for earthquakes and have a clear look at their capabilities and limitations.

Source Time Function (STF)

Source Time Function

Usually the result of moment tensor inversion will lead to a time dependent moment tensor. The time dependent moment tensor can be generally refered to as the source time function. However, in the study of earthquake sources, more often it is assumed that

$$M_{ij}(t) = M_{ij}S(t) \tag{46}$$

In such a way the time dependent $S(t)$ is refered to as the 'source time function'. Similar to the convensions which have been noted in equations (38) and (39), there are two kinds of source time functions:the 'near-field' source time function and the 'far-field' one, the latter is the derivative of the former.

In seismology, the representation theorem is the 'interface' between the study of seismic source and the study of seismic wave propagation and the structure of the Earth. In the physics of earthquake sources, the source time function is the 'interface' between the phenomenological description and the dynamic modelling. The dynamics of earthquake sources is beyond the scope of this introduction. What we are going to pointed out is that with the development of seismological observation, it has been possible to reach broader frequency band, higher dynamic range, wider and denser station coverage, and higher quality of seismic data. The study of the complexity of earthquake sources has become one of the most active fields in seismology. In such studies the source time function as an indicator of the complex motions of earthquakes plays more and more important role.

Constraints in the determination of STF

In the determination of source time functions, to put some constraints on the solution may also increase the stability of the inversion. The constraints are based on the understanding of the physics of the seismic sources. For instance, in considering an earthquake occurred in the crust, often it is assumed that the slip is irreversible, i.e., letting $S(t)$ increase with time, or in a more useful form,

$$\frac{d}{dt}S(t)\geq 0$$

which makes the inversion to be a nonlinear inversion. On the other hand, however, in considering underground explosions with large yields, such a constraint is not valid, because in underground explosions, it is possible that $\frac{d}{dt}S(t)<0$. For example, when the pressure on the elastic radius can be represented by (Mueller and Murphy, 1971;Murphy and Mueller, 1971)

$$P(t)=[P_0\exp(-rt)+P_2]H(t)$$

in which

$$r=r_0\frac{R_0}{R}$$

with R being the elastic radius, r_0 and R_0 being empirical constants, and

$$H(t)=\begin{cases}0 & t<0\\ 1 & t\geq 0\end{cases}$$

being the unit step function, the spectra of the source time function will be

$$s(\omega)=\pi\rho\alpha^4\left(\frac{P_0}{r+i\omega}+\frac{P_2}{i\omega}\right)\frac{i\omega R}{\mu}\frac{\exp(i\omega R/\alpha)}{\omega_0^2+i\omega_0\omega-[(\lambda+2\mu)/4\mu]\omega^2}$$

where

$$\omega_0=\frac{\alpha}{R}$$

In this case, $\frac{d}{dt}S(t)$ is not always positive.

As a kind of simplification, the constraints are also determined by the situation of the seismic records under consideration. For example, in the inversion of long period seismic data, say, the long period data with dominant period of 20s, because the dominant period is much longer than the duration of the STF, the source time function can thus be taken as an impulse. In case that some variations are to be considered, some regular forms with only a few parameters, *e.g.*, a trapezoid function with rising time t_1, flat time t_2 and falling time t_3, are enough in the modelling. More often triangle elements with different amplitudes are taken to model an arbitrary STF. Such simplification is a reflection of the resolution of the seismic data.

The empirical Green's function

Within a broader frequency band, to retrieve the source time function requires a 'good' Green's function. Such a requiement is not easy to reach. However, in practical data analysis, sometimes the Nature has provided seismologists with good 'calculation results'. The empirical methods in seismology (to assign some 'calculations' to the Earth) is an interesting approach and plays important role in the study of Earth structure and seismic sources.

Suppose that there happens to be a small earthquake with its location and focal mechanism quite similar to the large earthquake. In this case, the ground motion can be represented by

$$U_i^L(\vec{r},\omega)=M_{jk}^L G_{ij,k}^L(\vec{r},0\,;\,\omega)s^L(\omega)I(\omega) \qquad (47)$$

$$U_i^S(\vec{r},\omega)=M_{jk}^S G_{ij,k}^S(\vec{r},0\,;\,\omega)s^S(\omega)I(\omega)$$

in which L and s denote the large earthquake and the small one, respectively, $I(\omega)$ is the reponse spectra of the recording instrument, $s(\omega)$ is the spectra of the source time function. As the locations of the two earthquakes are near to each other, one has

$$G_{ij,k}^L\approx G_{ij,k}^S$$

The mechanisms of these two earthquakes being near to each other gives ·

$$M_{jk}^L\approx\frac{M_0^L}{M_0^S}M_{jk}^S$$

where M_0 is the scalar seismic moment. In this case

$$U_j^L(\vec{r},\omega)=U_j^S(\vec{r},\omega)\frac{M_0^L}{M_0^S}\frac{s^L(\omega)}{s^S(\omega)}$$

If the small earthquake is so little that its (near-field) source time function may be regarded as a unit step function

$$S^s(t)=H(t)=\begin{cases}0 & t<0\\ 1 & t\geq 0\end{cases}$$

then

$$s^s(\omega)=H(\omega)=\frac{1}{i\omega}$$

thus

$$U_i^L(\vec{r},\omega)=U_i^s(\vec{r},\omega)\frac{M_0^L}{M_0^s}\, i\omega s^L(\omega)$$

Back to time domain, we have

$$U_i^L(\vec{r},t)=\frac{M_0^L}{M_0^s}\,U_i^s(\vec{r},t)*\frac{d}{dt}S^L(t) \qquad (48)$$

Therefore, by deconvolution, it is possible to retrieve $\frac{d}{dt}S^L(t)$, i.e., the far-field source time function. In this process, the calculation of the Green's function is not needed. And actually the seismic record of the small earthquake is called the 'empirical Green's function' (Harzell, 1978; Mueller, 1985).

It should be noted that the 'STF' here has a clear difference from the STFs mentioned above. Here the 'STF' calculated by deconvolution is only the apparent STF seen at a seismic station rather than the real STF for the source. Therefore, the 'STF' obtained here is also called pseudo-STF or apparent STF.

Due to the complexity of earthquake sources and the finite propagation speed of seismic waves, it is possible that the pseudo-STFs retrieved from different seismic stations are different. In physics, this phenomena can be understood by Doppler effect. This effect is often used to study the rupture process of earthquakes. In mathematics, the inversion of rupture process using pseudo-STFs at a number of different stations leads to the problem of tomographical imagining of seismic sources(Ruff, 1987). In spite of the uncertainties in such inversion, the result can at least provide some clues leading to the understanding of the nature of earthquake source process.

Discussion

In this brief introduction we have outlined the key concepts of seismic moment tensor theory. These concepts have been proved to be a useful tool in the analysis of seismic source. In this introduction we also tried to clarify some fundamental concepts which, unexpectedly, sometimes caused confusion even among professionals. One of the methods dealing with the fundamentals is to rearrange the concepts according to their logical order rather than their chronicle order. In fact such a method is quite common in physics: the appearence of Newton's law today is quite different from that in Newton's time.

In the theoretical discussions inevitably a lot of mathematics has been cited. However, from the very beginning, seismic moment tensor is a physical concept rather than a mathematical one. As can be seen in the discussion, in the theoretical modelling, several simplifications are taken. It is a problem of physics rather than mathematics to investigate on which conditions such simplifications are valid and how well such simplifications fit the real situation of earthquake sources. In this point of view, to have a better understanding of these concepts, it is better to go to a computer to process some real seismic data.

Acknowledgements

This work was sponsored by the STEPI of Korea and the Joint Foundation of Seismological Sciences of China. The research was also financed in part by the Ministry of Education, Korea (project number BSRI-95-5420). One of the authors (W. Z. L.) thanks the Seismological Institute of Hanyang University for its support and help during his stay as a visiting research professor.

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