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ON CHARACTERIZATIONS OF REAL HYPERSURFACES OF TYPE B IN A COMPLEX HYPERBOLIC SPACE

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1. Introduction

A complex n-dimensional Kaehlerian manifold of constant holomorphic sectional curvature c is called a complex space form, which is denoted by $M_n(c)$. A complete and simply connected complex space form consists of a complex projective space P_nC, a complex Euclidean space C^n or a complex hyperbolic space H_nC, according as $c > 0$, $c = 0$ or $c < 0$. The induced almost contact metric structure of a real hypersurface M of $M_n(c)$ is denoted by (ϕ, ξ, η, g).

There exist many studies about real hypersurfaces of $M_n(c)$. One of the first researches is the classification of homogeneous real hypersurfaces of a complex projective space P_nC by Takagi [18], who showed that these hypersurfaces of P_nC could be divided into six types which are said to be of type A_1, A_2, B, C, D, and E, and in [3] Cecil-Ryan and [10] Kimura proved that they are realized as the tubes of constant radius over Kahlerian submanifolds.

In differential geometry of real hypersurfaces it is very interesting to characterize homogeneous ones. There are many characterizations of homogeneous ones of type A_1 and A_2 since these two examples have a lot of beautiful geometric properties ([3],[4],[15],[16]). In particular Okumura [16] proved that a real hypersurface M of P_nC is locally congruent to one of homogeneous ones of type A_1 and A_2 if and only if the structure vector ξ is an infinitesimal isometry, that is, $\mathcal{L}_\xi g = 0$, where

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$\mathcal{L}$ is the Lie derivative. Also Maeda and Udagawa [13] showed another charaterization of type A_1 and A_2 in terms of $\mathcal{L}_\xi \phi = 0$. Motivated by this result, Ki,Kim and Lee [6] proved the fact that "M is of type A_1 or type A_2" is equivalent to "$\mathcal{L}_\xi A = 0$, where A is the shape operator of M". Thus it becomes to know that $\mathcal{L}_\xi g = 0, \mathcal{L}_\xi \phi = 0$ and $\mathcal{L}_\xi A = 0$ are equivalent to each other.

From this point of view, to obtain a characterization of another types which are different from the above homogeneous ones of type A_1 or A_2 Ki, Maeda and the second author [9] have studied a characterization of type B in P_nC in terms of $(\mathcal{L}_\xi A)^2 = -c^2\phi^2$ or $(\mathcal{L}_\xi S)^2 = -c^2\phi^2$, where $c\neq 0$ constant and S denotes the Ricci tensor of M.

Now we are introduced in these problems when $c < 0$, that is, $M_n(c)$ is the complex hyperbolic space H_nC, which admits the Bergman metric normalized so that the constant holomorphic sectional curvature $c = -4$. Berndt [1] showed recently that all real hypersurfaces with constant principal curvatures of a complex hyperbolic space H_nC are realized as the tubes of constant radius over certain submanifolds when the structure vector field ξ is principal. Namely, by using some results about focal sets Berndt proved the following

THEOREM A. *Let M be a connected real hypersurface of H_nC ($n\geq 2$). Then M has constant principal curvatures and ξ is principal if and only if M is locally congruent to one of the following*

(A_0) *a horosphere in H_nC,*

(A_1) *a geodesic hypersphere or a tube over a complex hyperplane $H_{n-1}C$.*

(A_2) *a tube over a totally geodesic submanifold H_kC for $k = 1, ..., n-2$.*

(B) *a tube over a totally real hyperbolic space H_nR.*

On the other hand, Montiel and Romero [15] gave a characterization of the above homogeneous ones of types A_0, A_1 and A_2 by the commutativity of the structure tensor ϕ and the second fundamental tensor A of M in H_nC. Also Chen, Montiel and Ludden [4] gave a characterization of types A_0, A_1 and A_2 by the length of the covariant derivative of the shape operator A of M in H_nC.

Now in this paper we want to investigate real hypersurfaces of type B in H_nC in terms of Lie derivatives of M. Until now there are few results about characterizations of homogeneous real hypersurfaces of type B in H_nC. The main purpose of this paper is to characterize this manifold in the class of real hypersurfaces on which the structure vector ξ is principal. We have the following :

THEOREM 1. *Let M be a real hypersurface of H_nC ($n\geq 2$). Suppose that ξ is a principal curvature vector and M satisfies $(\mathcal{L}_\xi A)^2 = c^2\phi^2$, where c is nonzero locally constant. Then M is locally congruent to a homogeneous real hypersurface of type B, which lies on a tube over a totally real hyperbolic space H_nR.*

THEOREM 2. *Let M be a real hypersurface with constant mean curvature in H_nC ($n\geq 2$). Suppose that ξ is a principal curvature vector and M satisfies $(\mathcal{L}_\xi S)^2 = c^2\phi^2$, where S is the Ricci tensor of M and c is nonzero locally constant. Then M is locally congruent to a homogeneous real hypersurface of type B, which lies on a tube over a totally real hyperbolic space H_nR.*

1. Preliminaries

Let M be an orientable real hypersurface of H_nC and let N be a unit normal vector field on M. The Riemannian connections $\tilde{\nabla}$ in H_nC and ∇ in M are related by the following formulas for any vector fields X and Y on M:

$$\tilde{\nabla}_X Y = \nabla_X Y + g(AX, Y)N, \tag{1.1}$$

$$\tilde{\nabla}_X N = -AX, \tag{1.2}$$

where g denotes the Riemannian metric of M induced from the Bergman metric G of H_nC and A is the shape operator of M in H_nC. An eigenvector X of the shape operator A is called a principal curvature vector. Also an eigenvalue λ of A is called a principal curvature. In what follows, we denote by V_λ the eigenspace of A associated with eigenvalue λ. It is known that M has an almost contact metric structure induced from the complex structure J on H_nC, that is, we define a tensor field ϕ of type (1,1), a vector field ξ and a 1-form η on M by

$g(\phi X, Y) = G(JX, Y)$ and $g(\xi, X) = \eta(X) = G(JX, N)$. Then we have

$$(1.3) \qquad \phi^2 X = -X + \eta(X)\xi, \quad g(\xi,\xi) = 1, \quad \phi\xi = 0.$$

It follows from (1.1) that

$$(1.4) \qquad (\nabla_X \phi)Y = \eta(Y)AX - g(AX, Y)\xi,$$

$$(1.5) \qquad \nabla_X \xi = \phi AX.$$

Let $\tilde{R}$ and R be the curvature tensors of H_nC and M, respectively. Since the curvature tensor $\tilde{R}$ has a nice form, we have the following Gauss and Codazzi equations:

(1.6)
$$\begin{aligned} &g(R(X,Y)Z,W) \\ =& -g(Y,Z)g(X,W) + g(X,Z)g(Y,W) - g(\phi Y, Z)g(\phi X, W) \\ &+ g(\phi X, Z)g(\phi Y, W) + 2g(\phi X, Y)g(\phi Z, W) \\ &+ g(AY, Z)g(AX, W) - g(AX, Z)g(AY, W), \end{aligned}$$

$$(1.7) \quad (\nabla_X A)Y - (\nabla_Y A)X = -\eta(X)\phi Y + \eta(Y)\phi X + 2g(\phi X, Y)\xi.$$

From (1.3),(1.5), (1.6) and (1.7) we get

$$(1.8) \qquad SX = -(2n+1)X + 3\eta(X)\xi + hAX - A^2X,$$

$$(1.9) \qquad \begin{aligned} (\nabla_X S)Y =& 3\{g(\phi AX, Y) + \eta(Y)\phi AX\} + (Xh)AY \\ &+ (hI - A)(\nabla_X A)Y - (\nabla_X A)AY. \end{aligned}$$

where $h = traceA$, S is the Ricci tensor of type (1,1) on M and I is the identity map.

Now let us introduce a lemma which was proved by Ki and the second author [7].

LEMMA B. *Let M be a real hypersurface of a complex hyperbolic space H_nC. Assume that the structure vector ξ of M is a principal curvature vector with the corresponding principal curvature α. Then α is locally constant on M.*

Now we suppose that the structure vector ξ is a principal curvature vector of the shape operator A of M, that is $A\xi = \alpha\xi$, where α is the principal curvature corresponding to ξ. Then the covariant derivative gives

$$(\nabla_X A)\xi = (X\alpha)\xi + \alpha\phi AX - A\phi AX,$$

where we have used the second formula of (1.2). Thus it follows that

$$g((\nabla_X A)Y, \xi) = (X\alpha)\eta(Y) + \alpha g(Y, \phi AX) - g(Y, A\phi AX),$$

for any tangent vector fields X, and Y on M. From this, using the equation of Codazzi (1.7) and the fact that $X\alpha = (\xi\alpha)\eta(X)$, we have

$$A\phi A = -\phi + \frac{\alpha}{2}(A\phi + \phi A). \tag{1.10}$$

From this and the equation of Codazzi (1.7) we also get the following

$$\nabla_\xi A = -\frac{\alpha}{2}(A\phi - \phi A). \tag{1.11}$$

2. Proof of Theorem 1

Let us suppose that the structure vector field ξ is principal with corresponding principal curvature α. Then by Lemma B it was known that α is locally constant on M. For any $X \in TM$ from (1.5) we have

(2.1)
$$\begin{aligned}
(\mathcal{L}_\xi A)X &= \mathcal{L}_\xi(AX) - A\mathcal{L}_\xi X \\
&= [\xi, AX] - A[\xi, X] \\
&= \nabla_\xi(AX) - \nabla_{AX}\xi - A(\nabla_\xi X - \nabla_X \xi) \\
&= (\nabla_X A)\xi - \phi X - \phi A^2 X + A\phi AX \\
&= \nabla_X(A\xi) - A(\nabla_X \xi) - \phi X - \phi A^2 X + A\phi AX \\
&= \alpha\phi AX - \phi X - \phi A^2 X,
\end{aligned}$$

where we have used the equation of Codazzi (1.7) to the fourth equality. Hence we have $(\mathcal{L}_\xi A)\xi = 0$, from which $(\mathcal{L}_\xi A)^2\xi = c^2\phi^2\xi$ holds for any c. Now let us consider any X in TM orthogonal to ξ such that $AX = \lambda X$. Substituting in (2.1), we get

$$(\mathcal{L}_\xi A)X = (-\lambda^2 + \alpha\lambda - 1)\phi X.$$

From this and (2.1) we have

$$(\mathcal{L}_\xi A)^2 X = (-\lambda^2 + \alpha\lambda - 1)(\alpha\phi A\phi X - \phi^2 X - \phi A^2\phi X). \tag{2.2}$$

Thus by Lemma B we can consider the following two cases

Case I. $\alpha^2 = 4$

Now we consider for the case $\alpha = 2$. Then for any X in $\xi^\perp$ such that $AX = \lambda X$ (1.10) gives

$$(\lambda - 1)A\phi X = (\lambda - 1)\phi X.$$

Then let us consider an open set $M_0 = \{x \in M | \lambda(x) \neq 1\}$. Thus $A\phi X = \phi X$ on M_0. From this and (2.2) we know $(\mathcal{L}_\xi A)^2 X = 0$, which makes a contradiction to the hypothesis of our Theorem 1. Thus the set M_0 should be empty. Then $\lambda = 1$ on M. Accordingly, by Theorem A M is locally congruent to a horosphere. But in this case its principal curvatures $\alpha = 2, \lambda = 1$ also satisfies a quadaratic equation of $\lambda^2 - \alpha\lambda + 1 = 0$ so that the right side of (2.2) vanishes, which makes a contradiction.

Similarly, for the case $\alpha = -2$ we also get the same conclusion. Accordingly, the Case I can not occur.

Case II. $\alpha^2 - 4 \neq 0$.

Then we have $2\lambda - \alpha \neq 0$. In fact, if we suppose $2\lambda - \alpha = 0$, then (1.10) gives $\alpha\lambda = 2$. Together with this fact we have $\alpha^2 - 4 = 0$, a contradiction. Thus from (1.10) it follows $A\phi X = \frac{\alpha\lambda - 2}{2\lambda - \alpha}\phi X$. Substituting this into (2.2), we have

$$(\mathcal{L}_\xi A)^2 X = (-\lambda^2 + \alpha\lambda - 1)\Big\{-\Big(\frac{\alpha\lambda + 2}{2\lambda - \alpha}\Big)^2 + \alpha\frac{\alpha\lambda + 2}{2\lambda - \alpha} - 1\Big\}\phi^2 X \tag{2.3}$$

for any X in $V_\lambda (= \{X : AX = \lambda X, X \perp \xi\})$. From this, together with the hypothesis, we become to know that λ is constant. Therefore by

Theorem A the manifold M satisfying the hypothesis of our Theorem 1 is locally congruent to one of types A_1, A_2 or B. Thus the rest of the proof is to check $(\mathcal{L}_\xi A)^2 = c^2\phi^2, c\neq 0$ for these homogeneous types one by one.

For a case where M is congruent to one of types A_1 or A_2 it was known that its principal curvatures are given by $\alpha = 2coth2r$, $\lambda = tanhr$, and $\mu = cothr$ with the multiplicities $1, 2k$, and $2(n-k-1)$ respectively. Then these principal curvatures $cothr, -tanhr$ are solutions for the quadratic equation $\lambda^2 - \alpha\lambda + 1 = 0$, because it was known that $\alpha = 2cot2hr = cothr + tanhr$(cf.[1],[2]). Accordingly, from (2.2) we know that each of them does not satisfy $(\mathcal{L}_\xi A)^2 = c^2\phi^2, c\neq 0$.

Next let us consider for a case where M is of type B. In this case its principal curvatures are given by $\alpha = 2tanh2r, \lambda_1 = cothr$ and $\lambda_2 = tanhr$ with the multiplicities $1, n-1$, and $n-1$ respectively. Let $x = cothr$. Then $\alpha = \frac{4x}{1+x^2}$. From this we can calculate the following

$$-\lambda_1^2 + \alpha\lambda_1 - 1 = -\frac{(x^2-1)^2}{1+x^2}$$

and

$$-\lambda_2^2 + \alpha\lambda_2 - 1 = -\frac{(x^2-1)^2}{x^2(1+x^2)}.$$

And hence a computation yields that $(\mathcal{L}_\xi A)^2 = c^2\phi^2$ holds, when $c^2 = \left\{\frac{(x^2-1)^2}{x(1+x^2)}\right\}^2$ $(\neq 0)$. So we can conclude that a real hypersurface of type B satisfies the assumption of Theorem 1. This completes the proof of Theorem 1.

REMARKS.

(1) Due to the proof of Theorem 1, we get

COROLLARY 1. *Let M be a real hypersurface of H_nC. Suppose that ξ is a principal curvature vector and M satisfies $(\nabla_\xi A)^2 = c^2\phi^2$, where c is nonzero locally constant. Then M is locally congruent to a homogeneous real hypersurface of type B which lies a tube over a totally real hyperbolic space H_nR.*

(2) Kimura and Maeda classified real hypersurfaces M satisfying $\nabla_\xi A = 0$ in P_nC (cf.[11]).

3. Proof of Theorem 2

From the assumption we can put $A\xi = \alpha\xi$. Now for any $X \in TM$ let us calculate the following

$$(3.1) \qquad \begin{aligned}(\mathcal{L}_\xi S)X &= \mathcal{L}_\xi(SX) - S(\mathcal{L}_\xi X)\\ &= [\xi, SX] - S[\xi, X]\\ &= \nabla_\xi(SX) - \nabla_{SX}\xi - S(\nabla_\xi X - \nabla_X \xi)\\ &= (\nabla_\xi S)X - \phi ASX + S\phi AX.\end{aligned}$$

Since ξ is an eigenvector of S and α is constant, we find that $(\mathcal{L}_\xi S)\xi = \mathcal{L}_\xi(S\xi) - S(\mathcal{L}_\xi \xi) = 0$, from which $(\mathcal{L}_\xi S)^2\xi = c^2\phi^2\xi$ holds for any c.

Now let us consider any X in TM orthogonal to ξ such that $AX = \lambda X$. Then Equation (1.8) yields the following:

$$(3.2) \qquad \begin{aligned}\phi ASX &= \lambda(2n + 1 + h\lambda - \lambda^2)\phi X,\\ S\phi AX &= \lambda S\phi X\\ &= \lambda\{-(2n+1)\phi X + hA\phi X - A^2\phi X\}.\end{aligned}$$

Next we compute $(\nabla_\xi S)X$. Since we have assumed that the mean curvature of M is constant, from (1.9) for any $X \in V_\lambda = \{X \in TM : AX = \lambda X\}$ it follows that

$$(3.3) \qquad (\nabla_\xi S)X = (hI - A)(\nabla_\xi A)X - \lambda(\nabla_\xi A)X.$$

From (3.1),(3.2) and (3.4) we have

$$(3.4) \qquad \begin{aligned}(\mathcal{L}_\xi S)X = &-\frac{\alpha}{2}\{(hI - A) - \lambda\}(A\phi - \phi A)X\\ &- \lambda^2(h - \lambda)\phi X + \lambda\{hA - A^2\}\phi X.\end{aligned}$$

Now as in the proof of Theorem 1 we can consider the following two cases.

Case I. $\alpha^2 = 4$.

If we consider for a case $\alpha = 2$, we also get $A\phi X = \phi X$ on an open subset $M_0 = \{x \in M : \lambda(x) \neq 1\}$ for any X in $\xi^\perp$ such that $AX = \lambda X$.

That is, ϕX is a principal curvature vector with principal curvature $\mu = 1$ on M_0. Thus (3.4) implies

$$(\mathcal{L}_\xi S)X = P(\lambda)\phi X,$$

where $P(\lambda)$ denotes the polynomial $-\frac{\alpha}{2}(1-\lambda)(h-1-\lambda) - \lambda^2(h-\lambda) + \lambda(h-1)$. Then we know $P(1) = 0$. Thus

$$(\mathcal{L}_\xi S)^2 X = P(\lambda)(\mathcal{L}_\xi S)\phi X = P(\lambda)P(1)\phi^2 X = 0,$$

which makes a contradiction to the hypothesis. Thus the open set M_0 should be empty so that $\lambda = 1$ on M. Then M is locally congruent to a horosphere and its principal curvatures are given by $\alpha = 2, \lambda = 1$ with the multiplicities 1 and $2(n-1)$ respectively. Thus (3.4) reduces to

$$(\mathcal{L}_\xi S)X = -(2n-1)\phi X + (2n-1)\phi X = 0.$$

Therefore in this case we know that our manifold M does not satisfy the hypothesis of Theorem 2.

Case II. $\alpha^2 \neq 4$

In this case we also know $2\lambda \neq \alpha$ as in the proof of Theorem 1. Then by (2.10) we have the last formulas of (3.2) and (3.3) as follows:

$$S\phi AX = \lambda\Big\{-(2n+1) + h\cdot\frac{\alpha\lambda-2}{2\lambda-\alpha} - \Big(\frac{\alpha\lambda-2}{2\lambda-\alpha}\Big)^2\Big\}\phi X,$$

$$(\nabla_\xi S)X = \frac{\alpha}{2}\Big(\lambda - \frac{\alpha\lambda-2}{2\lambda-\alpha}\Big)\Big(h - \lambda - \frac{\alpha\lambda-2}{2\lambda-\alpha}\Big)\phi X.$$

Substituting these into (3.1), we get

$$(\mathcal{L}_\xi S)X = \Big(\frac{\alpha}{2} - \lambda\Big)\Big(\lambda - \frac{\alpha\lambda-2}{2\lambda-\alpha}\Big)\Big(h - \lambda - \frac{\alpha\lambda-2}{2\lambda-\alpha}\Big)\phi X.$$

for any $X \in V_\lambda$ so that

$$(3.5)\qquad \begin{aligned}(\mathcal{L}_\xi S)^2 X = &\Big(\frac{\alpha}{2} - \lambda\Big)\Big(\frac{\alpha}{2} - \frac{\alpha\lambda-2}{2\lambda-\alpha}\Big)\Big(\lambda - \frac{\alpha\lambda-2}{2\lambda-\alpha}\Big)\cdot\\ &\Big(\frac{\alpha\lambda-2}{2\lambda-\alpha} - \lambda\Big)\Big(h - \lambda - \frac{\alpha\lambda-2}{2\lambda-\alpha}\Big)^2\phi^2 X.\end{aligned}$$

Thus from the assumption of Theorem 2 we know that (3.2) implies λ is constant. Accordingly, by Theorem A the manifold M satisfying the hypothesis of Theorem 2 is locally congruent to a homogeneous one of types A_1,A_2,or B.

First let us consider for a case where M is of type A_1,and A_2. Then its principal curvatures are given by $\alpha = 2coth2r, \lambda = cothr$, and $\mu = tanhr$ with the multiplicities $1, 2k$, and $2(n-k-1)$ respectively. From this we can calculate the following

$$\begin{aligned}\frac{\alpha\lambda-2}{2\lambda-\alpha} &= \frac{2coth2rcothr-2}{2cothr-(cothr+tanhr)}\\ &= \frac{coth^2r-1}{cothr-tanhr} = cothr.\end{aligned}$$

This means $\lambda - \frac{\alpha\lambda-2}{2\lambda-\alpha} = 0$. From this, together with (3.5) we know that our manifold M does not satisfy the hypothesis of Theorem 2.

Next it remains only to find a nonzero constant c for a case where M is of type B. Then making use of the method given in the proof of Theorem 1 we can calculate the right hand side of (3.5) as follows:

$$\begin{aligned}\frac{\alpha}{2}-\lambda_1 &= -\frac{x(1-x^2)}{1+x^2},\\ \frac{\alpha}{2}-\lambda_2 &= \frac{x^2-1}{x(1+x^2)},\\ \lambda_1-\lambda_2 &= \frac{x^2-1}{x}, \quad \text{and}\\ h-\lambda_1-\lambda_2 &= \frac{4x}{1+x^2}+\frac{(n-2)(x^2+1)}{x}.\end{aligned}$$

Summing up these formulas, we know that $(\mathcal{L}_\xi S)^2 = c^2\phi^2$ holds, when $c^2 = \left(\frac{1-x^2}{1+x^2}\right)^2\left(\frac{x^2-1}{x}\right)^2\left\{\frac{4x}{1+x^2}+\frac{(n-2)(x^2+1)}{x}\right\}^2$.

REMARKS.

(3) By virtue of the proof of Theorem 2, we have

COROLLARY 2. *Let M be a real hypersurface with constant mean curvature in H_nC. Suppose that ξ is a principal curvature vector* *and M satisfies $(\nabla_\xi S)^2 = c^2\phi^2$, where S is the Ricci tensor of M and c is nonzero locally constant. Then M is locally congruent to a homogeneous real hypersurface of type B which lies a tube over a totally real hyperbolic space H_nR.*

(4) Ki ([5]) proved that $H_nC(n\geq 3)$ does not admit a real hypersurface M with parallel Ricci tensor.

(5) Maeda classified real hypersurfaces M satisfying $\nabla_\xi S = 0$(in P_nC) under the condition that ξ is a principal curvature vector (cf.[12])

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