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NORMAL HYPERSURFACE IMMERSED IN A PRODUCT OF TWO SPHERES

SHIN, YONG HO

0. Introduction

Yano[1] studied the differential geometry of $S^n \times S^n$ and proved that the (f, g, u, v, λ)-structure is naturally induced on $S^n \times S^n$ as a submanifold of codimension 2 of a $(2n+2)$-dimensional Euclidean space or a real hypersurface of $(2n+1)$-dimensional unit sphere $S^{2n+1}(1)$. S.-S.Eum, U-H.Ki and Y.H.Kim [2] researched partially real hypersurfaces of $S^n \times S^n$ by using the concept of k-invariance. The purpose of the present paper is devoted to study some intrinsic characters of hypersurfaces immersed in $S^n \times S^n$ and characterize global properties of them by using some intergrable condition. In section 1, we recall the intrinsic properties of $S^n(1/\sqrt{2}) \times S^n(1/\sqrt{2})$ and obtain some algebraic relationships and structure equations of hypersurfaces of $S^n(1/\sqrt{2}) \times S^n(1/\sqrt{2})$. In section 2, we define an integrable condition for the induced strucrure on a hypersurface of $S^n \times S^n$ which is called to be normal, and look into an intrinsic character of a normal k-antiholomorphic hypersurface of $S^n \times S^n$.

1. Structure equations of hypersurfaces of $S^n(1/\sqrt{2}) \times S^n(1/\sqrt{2})$

Let M be a hypersurface immersed isometrically in $S^n(1/\sqrt{2}) \times S^n(1/\sqrt{2})$ and suppose that M is covered by the system of coordinate neighborhoods $\{\bar{V}; \bar{x}^a\}$, where here and in the sequel, the indices $a, b, c, d, \cdots$ run over the range $\{1, 2, \cdots, 2n-1\}$. From the (f, g, u, v, λ)-structure defined on $S^n \times S^n$, we obtain the so-called $(f, g, u, v, w, \lambda, \mu, \nu)$-structure given by[2],

$$f_b^e f_e^a = -\delta_b^a + u_b u^a + v_b v^a + w_b w^a, \tag{1.1}$$

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$$
(1.2)\qquad
\begin{aligned}
f_e^a u^e &= -\lambda v^a + \mu w^a,\\
f_e^a v^e &= \lambda u^a + \nu w^a,\\
f_e^a w^e &= -\mu u^a - \nu v^a
\end{aligned}
$$

or, equivalently

(1.3)

$$
\begin{aligned}
&u_e f_a^e = \lambda v_a - \mu w_a, \quad v_e f_a^e = -\lambda u_a - \nu w_a, \quad w_e f_a^e = \mu u_a + \nu v_a,\\
&u_e u^e = 1 - \lambda^2 - \mu^2, \quad u_e v^e = -\mu\nu, \quad u_e w^e = -\lambda\nu,\\
&v_e v^e = 1 - \lambda^2 - \nu^2, \quad v_e w^e = \lambda\mu,\\
&w_e w^e = 1 - \mu^2 - \nu^2
\end{aligned}
$$

where u_a, v_a and w_a are 1-forms associated with u^a, v^a and w^a respectively given by $u_a = u^b g_{ba}$, $v_a = v^b g_{ba}$ and $w_a = w^b g_{ba}$. By putting $f_{ba} = f_b^c g_{ca}$, f_{cb} is skew-symmetric.

(1.4) $k_c^e k_e^a = \delta_c^a - k_c k^a$,

(1.5) $k_c^e k_e = -\alpha k_c$,

(1.6) $k_e k^e = 1 - \alpha^2$.

(1.7) $k_c^e f_e^a + f_c^e k_e^a = k_c w^a - w_c k^a$,

(1.8) $k_c^e w_e + f_c^e k_e = -\alpha w_c$.

(1.9) $k_c^e u_e = -v_c - \mu k_c, \quad k_c^e v_e = -u_c - \nu k_c$,

(1.10) $k_e u^e = -\nu - \alpha\mu, \quad k_e v^e = -\mu - \alpha\nu$.

(1.11) $\nabla_d l_{cb} - \nabla_c l_{db} = k_d k_{cb} - k_c k_{db}$.

(1.12) $\nabla_c f_b^a = -g_{cb} u^a + \delta_c^a u_b - k_{cb} v^a + k_c^a v_b - l_{cb} w^a + l_c^a w_b$,

(1.13) $\nabla_c u_b = \mu l_{cb} - \lambda k_{cb} + f_{cb}$,

(1.14) $\nabla_c v_b = k_c^e f_{eb} - k_c w_b + \nu l_{cb} + \lambda g_{cb}$,

(1.15) $\nabla_c w_b = -\mu g_{cb} - \nu k_{cb} + k_c v_b - l_{ce} f_b^e$,

(1.16) $\nabla_c \lambda = -2v_c, \nabla_c \mu = w_c - \lambda k_c - l_{ce} u^e, \nabla_c \nu = k_{ce} w^e - l_{ce} v^e$,

(1.17) $\nabla_c k_b^a = l_{cb} k^a + l_c^a k_b$,

(1.18) $\nabla_c k_b = -k_{ba} l_c^a + \alpha l_{cb}$,

(1.19) $\nabla_c \alpha = -2 l_{ce} k^e$.

From these structure equations, we can easily see that the 1-form k_c is the third fundamental tensor when M is considered as a submanifold of codimension 2 immersed in $S^{2n+1}(1)$.

Finally, we mention the following remark and theorems for later use.

REMARK 1 [3]. *If $\lambda^2+\mu^2+\nu^2=1$ on the hypertsurface M, we see that*

$$\mu=0,\ \nu=constant(\neq 0),\ v_c=0 \quad and \quad \alpha=0.$$

And if the function λ vanishes on some open set, then we have $v_c=0$ and $\mu=0$. Moreover, if the 1-form u_b is zero on an open set in M, then (1.13) implies $f_{cb}=0$, which contradicts $n>1$ as is shown above.

THEOREM 1.2 [3]. *Let M be a hypersurface of $S^n(1/\sqrt{2})\times S^n(1/\sqrt{2})$ with $(f,g,u,v,w,\lambda,\mu,\nu)$-structure satisfying $\lambda^2+\mu^2+\nu^2=1$. If M is a minimal hypersurface with $(f,g,u,v,w,\lambda,\mu,\nu)$-structure, then M is a Sasakian C-Eienstein manifold.*

THEOREM 1.3 [3]. *Let M be a hypersurface of $S^n(1/\sqrt{2})\times S^n(1/\sqrt{2})$ $(n>1)$ with $(f,g,u,v,w,\lambda,\mu,\nu)$-structure satisfying $\lambda^2+\mu^2+\nu^2=1$. If M is minimal, then M as a submanifold of codimension 3 of a $(2n+2)$-dimensional Euclidean space E^{2n+2} is an intersection of a complex cone with generator C and a $(2n+1)$-dimensional unit sphere $S^{2n+1}(1)$.*

2. Antiholomorphic hypersurfaces with normal $(f,g,u,v,w,\lambda,\mu,\nu$ structure

We now define a tensor field S of type (1,2) as follows :

$$S_{cb}^a=[f,f]_{cb}^a+(\nabla_c u_b-\nabla_b u_c)u^a+(\nabla_c v_b-\nabla_b v_c)v_a \\ +(\nabla_c w_b-\nabla_b w_c)w^a,$$

where $[f,f]_{cb}^a$ is the Nijenhuis tensor formed with f_c^a, that is,

$$[f,f]_{cb}^a=f_c^e\nabla_e f_b^a-f_b^e\nabla_e f_c^a-(\nabla_c f_b^e-\nabla_b f_c^e)f_e^a.$$

The $(f,g,u,v,w,\lambda,\mu,\nu)$-structure is said to be ***normal*** [4] if S_{cb}^a vanishes identically.

In ths section, we assume that the hypersurface M with $(f, g, u, v, w, \lambda, \mu, \nu)$-structure of $S^n(1/\sqrt{2}) \times S^n(1/\sqrt{2})$ is nomal. Then we have

$$f_c^e \nabla_e f_b^a - f_b^e \nabla_e f_c^a - (\nabla_c f_b^e - \nabla_b f_c^e) f_e^a + (\nabla_c u_b - \nabla_b u_c) u^a + (\nabla_c w_b - \nabla_b w_c) w^a = 0.$$

because of (1.7) and (1.14).

Substituting (1.12),(1.13) and (1.15) into the last equation, we find

$$T_{ac} w_b - T_{ab} w_c = (k_{ae} f_b^e + k_{be} f_c^e) v_c - (k_{ae} f_c^e + k_{ce} f_a^e) v_b - (k_{ce} f_b^e) v_a - (k_c v_b - k_b v_c) w_a, \tag{2.1}$$

where

$$T_{cb} = l_{ce} f_b^e + l_{be} f_c^e. \tag{2.2}$$

Contradicting a and b in (2.1), we get

$$T_{ce} w^e = (\theta + 2\lambda) v_c + \lambda \mu k_c - 2\nu k_{ce} w^e, \tag{2.3}$$

where we have used (1.2), (1.3), (1.9) and $\theta = k_e w^e$.

If we transvect (2.1) with w^b and use (1.2),(1.3) and (2.3) we obtain

$$\begin{aligned}(1 - \mu^2 - \nu^2) T_{ac} &= \{(\theta + 2\lambda) v_a + \lambda \mu k_a - 2\nu k_{ae} w^e\} w_c \\ &- k_{ae}(\mu u^e + \nu v^e) v_c + k_{ce}(\mu u^e + \nu v^e) + k_{be} w^b f_a^e v_c \\ &+ k_{be} w^b f_c^e v_a - \lambda \mu (k_{ae} f_c^e + k_{ce} f_a^e) - (\lambda \mu k_c w_a - \theta v_c w_a),\end{aligned}$$

or, taking account of (1.9), we get

(2.4)

$$\begin{aligned}&(1 - \mu^2 - \nu^2) T_{ac} + \lambda \mu (k_{ae} f_c^e + k_{ce} f_b^e) - k_{be} w^b (f_a^e v_c + f_c^e v_a) \\ &= \theta (v_a w_c + v_c w_a) + 2\lambda v_a w_c + \lambda \mu (k_a w_c - k_c w_a) - 2\nu (k_{ae} w^e) w_c) \\ &+ \nu (u_a v_c - u_c v_a) + (\mu^2 + \nu^2)(k_a v_c - k_c v_a).\end{aligned}$$

Taking the skew-symmetric part of this with respect to a and c we get

(2.5)
$$\nu\{w_c(k_{be}w^e) - w_b(k_{ce}w^e)\} = \lambda(v_b w_c - v_c w_b) + \lambda\mu(k_b w_c - k_c w_b) + \nu(u_b v_c - u_c v_b) + (\mu^2+\nu^2)(k_b v_c - k_c v_b).$$

On the other side, transvecting (1.8) with w^b and considering (1.2),(1.3) and (1.10), we have

$$k_{cb}w^c w^b + \alpha + 2\mu\nu = 0. \tag{2.6}$$

Transvection of w^c to (2.5) gives

(2.7)
$$\nu(1-\mu^2-\nu^2)k_{be}w^e = \lambda\mu\nu u_b + \lambda\mu k_b + \{\lambda(1-\mu^2) - \theta(\mu^2+\nu^2)\}v_b - \{\alpha\nu + 2\mu\nu^2 + \lambda^2\mu + \lambda\mu\theta\}w_b,$$

where we have used (1.3) and (2.6).

If we transvect (2.7) with u^b and make use of (1.3), (1.9), it means

$$\mu\nu\theta(1+\lambda^2) = \lambda(\alpha+\mu\nu)\mu^2 - \nu^2). \tag{2.8}$$

Applying also (2.7) with v^b and k^b successively, we obtain respectively

$$\theta(\lambda^2\nu^2 - \mu^2) = \lambda(-1+\lambda^2+2\mu^2+2\nu^2+2\alpha\mu\nu-\nu^4+\mu^2\nu^2), \tag{2.9}$$

$$\mu\theta(\lambda^2+\nu^2-\mu^2+\lambda\theta) = \lambda(\mu^3-\alpha\nu-\mu\nu^2-\mu\alpha^2) \tag{2.10}$$

with the aid of (1.3),(1.5) and (1.9).

Combining (2.8) and (2.9), we can easily verify that

$$\alpha(\lambda^2\nu^4+\mu^4+\mu^2\nu^2+\lambda^2\mu^2\nu^2) + \mu\nu(-1+2\mu^2+2\nu^2+\lambda^4+2\mu^2\lambda^2+2\lambda^2\nu^2) = 0. \tag{2.11}$$

First of all, we prove

LEMMA 2.1. *Let M be a hypersurface with normal $(f, g, u, v, w, \lambda, \mu, \nu)$-structure of $S^n(1/\sqrt{2})\times S^n(1/\sqrt{2})$. Then the function α^2-1 is non-zero almost everywhere.*

Proof. If there exists an open interior M_α in $\{p \in M \mid \alpha^2(p) = 1\}$, then from (1.6) we see that $k^c = 0$ on M_α, which together with (1.8) gives $k_{ce}w^e = -\alpha w_c$.

Thus, (2.5) leads to

$$\lambda(v_b w_c - v_c w_b) + \nu(u_b v_c - u_c v_b) = 0$$

on M. By transvecting $v^b w^c$ and using (1.23), it follows that

$$\lambda(1 - \lambda^2 - \mu^2 - \nu^2) = 0$$

on the set. But, in a consequence of Remark 1 in section 1, $\lambda^2 + \mu^2 + \nu^2 \neq 1$ on M_α. Consequently we have the function λ vanishes on M_α. So we should have $v_c = 0$ on M_α. Hence (1.3) yields $\nu^2 = 1$ and $\mu = 0$ on the set. Thus, (1.10) gives $\alpha = 0$. It contradicts the definition of the set M_α. This completes the proof.

LEMMA 2.2. *Let M be a k-antiholomorphic hypersurfaces with normal $(f, g, u, v, w, \lambda, \mu, \nu)$-structure of $S^n(1/\sqrt{2}) \times S^n(1/\sqrt{2})$. Then we have the function μ vanishes identically.*

Proof. Since the hypersurface M is k-antiholomorphic, that is, the function α vanishes on M, (2.8)–(2.11) reduces respectively to

(2.12)
$$\mu\nu\theta(1 + \lambda^2) = \lambda\mu\nu(\mu^2 - \nu^2),$$

(2.13)
$$\theta(\lambda^2\nu^2 - \mu^2) = \lambda(-1 + \lambda^2 + 2\mu^2 + 2\nu^2 - \nu^4 + \mu^2\nu^2),$$

(2.14)
$$\mu\theta(\nu^2 + \lambda^2 - \mu^2 + \lambda\theta) = \lambda\mu(\mu^2 - \nu^2),$$

(2.15.)
$$\mu\nu(-1 + 2\mu^2 + 2\nu^2 + \lambda^4 + 2\mu^2\lambda^2 + 2\lambda^2\nu^2) = 0$$

If $(\mu\nu)(p) \neq 0$ for some point p of M, then the expresion above can be written as

$$\theta = (\mu^2 - \nu^2)/(1 + \lambda^2), \tag{2.16}$$

$$\theta(\nu^2 + \lambda^2 - \mu^2 + \lambda\theta) = \lambda(\mu^2 - \nu^2), \tag{2.17}$$

$$2\mu^2 + 2\nu^2 + \lambda^4 + 2\mu^2\lambda^2 + 2\lambda^2\nu^2 = 1 \tag{2.18}$$

for such p of M. Substituting (2.16) into (2.17), we find

$$(\mu^2 - \nu^2)(1 + \lambda^2 + \mu^2 - \nu^2) = 0$$

at the point p. Comparing (2.18) with the last expression, we have

$$(\mu^2 - \nu^2)(3\mu^2 + \nu^2 + \lambda^2 + \lambda^4 + 2\mu^2\lambda^2 + 2\lambda^2\nu^2) = 0$$

at $p \in M$. Since $(\mu\nu)(p) \neq 0$, it follows that $\mu^2 - \nu^2 = 0$ at the point. so (2.16) leads to $\theta(p) = 0$ and hence (2.13) means $\lambda(-1+\lambda^2+4\mu^2) = 0$ at the point.

Differentiating $\lambda^2(-1+\lambda^2+4\mu^2) = 0$ covariantly and taking account of (1.16) and the orginal expression, we find

$$2\mu\lambda(w_c - \lambda k_c - l_{ce}u^e) = \lambda^2 v_c$$

at $p \in M$. If we transvect this with k^c and make use of (1.10) and the fact that $\theta(p) = \alpha(p) = 0$, we get $\lambda\mu = 0$ at the point, which is contradictory because of Remark 1. Thus $\mu\nu = 0$ on the whole space M. So (2.13) and (2.14) becomes respectively

$$\theta(\lambda^2\nu^2 - \mu^2) = \lambda(-1 + \lambda^2 + 2\mu^2 + 2\nu^2 - \nu^4), \tag{2.19}$$

$$\mu\theta(\lambda^2 - \mu^2 + \lambda\theta) = \lambda\mu^3. \tag{2.20}$$

We know consider a set given by

$$M_0 = \{p \in M \mid \mu(p) \neq 0\},$$

M_0 is an open set in M. Then, the function ν should be vanish on M_0 because of the fact $\mu\nu = 0$ on M. Hence (2.4), (2.19) and (2.20) can be respectively written as on M_0

(2.21)
$$(1-\mu^2)T_{ac} + \lambda\mu(k_{ae}f^e_c + k_{ce}f^e_a) - k_{be}w^b(f^e_a v_c + f^e_c v_b)$$
$$= \theta(v_a w_c + v_c w_a) + 2\lambda v_a w_c + \lambda\mu(k_a w_c - k_c w_a) + \mu^2(k_a v_c - k_c v_a),$$

$$\theta\mu^2 = \lambda(1-\lambda^2-2\mu^2), \tag{2.22}$$

$$\lambda\theta(\lambda+\theta) - \theta\mu^2 = \lambda\mu^2. \tag{2.23}$$

Transvecting (2.21) with $k^a k^c$ and taking account of (1.5),(1.8),(1.10),(1.19 and (2.2), we find

$$\mu(k_{be}k^e_a)w^b w^a = -\mu\theta^2 - \mu\lambda\theta$$

on the set because of $\alpha = 0$, or using (1.3) with $\nu = 0$ and (1.4),

$$\mu(1-\mu^2-\theta^2) = -\mu\theta^2 - \mu\lambda\theta.$$

Thus, it follows that

$$1 + \lambda\theta - \mu^2 = 0 \tag{2.24}$$

on M_0.

Comparing (2.22), (2.23) and (2.24), we get on M_0

$$\lambda(\theta+\lambda)^2 = 0$$

or, using (2.22)

$$\lambda^2(1-\lambda^2-\mu^2) = 0.$$

Taking account of Remark 1 in section 1, the function μ must be vanish and consequently the set M_0 is void. Therefore our assertion is true.

LEMMA 2.3. *Under the same assumptions as those stated in Lemma 2.2, we have* $\lambda = \theta = 0$ *on* M.

Proof. Since the function μ vanishes identically, we see from the second equation of (1.18) that

$$w_c - \lambda k_c - l_{ce} u^e = 0. \tag{2.25}$$

If we transvect k^c to this and make use of (1.6) and (1.19) with $\alpha = 0$, it means

$$\theta = \lambda. \tag{2.26}$$

Thus, (2.19) with $\mu = 0$ gives

$$\lambda(\nu^4 - 2\nu^2 + \lambda^2\nu^2 - \lambda^2 + 1) = 0,$$

that is,

$$\lambda(\nu^2 - 1)(\lambda^2 + \nu^2 - 1) = 0.$$

Owing to Remark 1, it implies

$$\lambda^2 + \nu^2 = 1 \tag{2.27}$$

on M and hence $v_c = 0$ because of (1.3). Hence λ and ν are both constant because of Remark 1.

Therefore, the third equation of (1.16) yields

$$k_{ce} w^e = 0. \tag{2.28}$$

So (1.8) with $\alpha = 0$ leads to

$$f_{ce} k^e = 0.$$

Transvecting f_b^c means

(2.29) $$k_b = \lambda w_b - \nu u_b$$

with the help of (1.1), (1.10) with $\alpha = 0$ and (2.26). Hence, (2.25) reduces to

(2.30) $$l_{ce}u^e = (1-\lambda^2)w_c + \lambda\nu u_c.$$

If we take account of (2.28),(2.29), Lemma 2.2 and the fact that $v_c = 0$, (2.4) turned out to be

$$(1-\nu^2)T_{cb} = 0,$$

or,equivalently

(2.31) $$\lambda(l_{ce}f_b^e + l_{be}f_c^e) = 0,$$

where we have used (2.2) and (2.27). We now suppose that the function λ does not vanish at some point p of M, then (2.31) means

(2.32) $$l_{ce}f_b^e + l_{ce}f_b^e = 0,$$

at the point. Transvecting (2.32) with f_a^b and making use of (2.30), we find

$$-l_{cb} + (1-\lambda^2)w_c u_b + \lambda\nu u_c v_b + (l_{ce}w^e)w_b + l_{de}f_c^e f_b^d = 0,$$

from which, taking the skew-symmetric part and using (2.27), we have

(2.33)
$$(l_{ce}w^e)w_b - (l_{be}w^e)w_c + \nu^2(w_c u_b - w_b - w_b u_c) = 0$$

at the point.

If we trnsvect w^b to the last relationship and use (1.3) and (2.27), then

$$(2.34) \qquad \lambda^2 l_{ce} w^e = \{l_{de} w^d w^e + \lambda\nu^3\} w_c + \lambda^2\nu^2 u_c$$

at $p \in M$ because of $\mu = 0$. On the other side, by transvecting (2.29) with l_c^b and considering (1.19) with $\alpha = 0$, we obtain

$$\lambda l_{ce} w^e - \nu l_{ce} u^e = 0,$$

which together with (2.30) and (2.34) gives

$$l_{ce} w^c w^e = 0$$

at the point. Thus (2.34) becomes

$$(2.35) \qquad \lambda l_{ce} w^e = \nu^3 w_c + \lambda^2 u_c$$

because the function λ does not vanish at p. Differentiating (2.29) covariantly and taking account of (1.13),(1.15) and (1.18), we get

$$(2.36) \qquad k_{be} l_c^e = \lambda l_{ce} f_b^e + \nu f_{cb}$$

at $p \in M$, where we have used the fact that $\alpha = 0$, λ and ν are constant. If we differentiate (2.35) covariantly, we find

$$\lambda(\nabla_c l_{be}) w^e + \lambda l_b^e \nabla_c w_e = \nu^3 \nabla_c w_b + \lambda\nu^2 \nabla_c u_b$$

at the point. Since λ and ν are constant, which togrther with (1.13) and (1.15) gives

$$\lambda(\nabla_c l_{be}) w^e + \lambda l_b^e(-\nu k_{ce} - l_{ca} f_e^a)$$
$$= \nu^3\{-\nu k_{cb} - l_{ce} f_b^e\} + \lambda\nu^2\{-\lambda k_{cb} + f_{cb}\}$$

because $\mu = 0$, or taking the skew-symmetric part and using (1.11), (2.28) and (2.32),

$$\lambda\nu l_{be}k_c^e + \lambda l_{be}l_{ca}f^{ea} = \nu^3 l_{ce}f_b^e - \lambda\nu^2 f_{cb}$$

at $p \in M$. Subsituting (2.36) into this, we find at the point

$$\lambda l_e^a l_{ca} f_b^e = \nu l_{ce} f_b^e$$

with the aid of (2.32). If we transvect the last expression with f^{cb} and make use of (1.1) with $v_c = 0$, we get at $p \in M$

$$\lambda l_e^a l_{ca}(g^{ec} - u^e u^c - w^e w^c) = \nu l_{ce}(g^{ec} - u^e u^c - w^e w^c),$$

which together with (2.30) and (2.35) yield

$$\lambda l_{cb} l^{cb} = \nu l_e^e.$$

Since $v_c = 0$, (1.14) gives

$$\nu l_e^e = -2(n-1)\lambda.$$

Thus, the last two relationships mean

$$\lambda\{l_{cb}l^{cb} + 2(n-1)\} = 0$$

at $p \in M$. So the set M_0 must be void. Consequently the function λ vanishes identically and hence $\theta = 0$ on M. This completes the proof of the Lemma.

Combining Theorem 1.2, 1.3 with Lemma 2.3, we conclude:

THEOREM 2.4. *Let M be a k-anyiholomorphic hypersurfaces with nomal $(f, g, u, v, w, \lambda, \mu, \nu)$-structure of $S^n(1/\sqrt{2}) \times S^n(1/\sqrt{2})(n > 1)$. Then M is a minimal Sasakian C-Einstein manifold.*

Moreover, M as a submanifold of codimension 3 of a Euclidean $(2n+2)$-space is an intersection of a complex cone with generator C and a $(2n+1)$-sphere $S^{2n+1}(1)$.

LEMMA 2.5. *Let M be a hypersurface with normal $(f, g, u, v, w, \lambda, \mu, \nu)$-structure of $S^n(1/\sqrt{2}) \times S^n(1/\sqrt{2})$. If the function θ vanishes identically. then M is k-antiholomorphic.*

Proof. Since $\theta = 0$ on M, (2.8)–(2.10) reduce respectively to be

(2.37)
$$\lambda(\alpha + \mu\nu)(\mu^2 - \nu^2) = 0,$$
(2.38)
$$\lambda(-1 + \lambda^2 + 2\mu^2 + 2\nu^2 + 2\alpha\mu\nu - \nu^4 + +\mu\nu^2) = 0,$$
(2.39)
$$\lambda(\mu^2 - \alpha\nu - \mu\nu^2 - \alpha^2\mu) = 0.$$

From (2.37) and (2.39), we have

(2.40)
$$\lambda(\nu + \alpha\mu)\alpha(\alpha + \mu\nu) = 0.$$

If we suppose that the function $\alpha(\alpha + \mu\nu) \neq 0$ for some point p of M, then $\lambda(\nu + \alpha\mu) = 0$ at the point. By the definition of the function λ and Remark 1, it can not vanish at $p \in M$, So we have $\nu + \alpha\mu = 0$ at $p \in M$. Thus (2.37) reduces to

$$\lambda\mu^2(1 - \alpha^2)(\alpha + \mu\nu) = 0$$

and hence $\lambda(1 - \alpha^2) = 0$ at the point.

Therefore, it follows that $\mu(p) = 0$ and consequently $\nu(p) = 0$ because of Lemma 2.1. So (2.38) gives $\lambda(1 - \lambda^2) = 0$ at the point, which is contrdictory by virtue of Remark 1. Thus, we have $\alpha(\alpha + \mu\nu) = 0$ on the whole space M. In the next place, we consider a set given by

$$N_0 = \{p \in M \mid \alpha(p) \neq 0\}.$$

Then N_0 is an open set in M. We have

(2.41)
$$\alpha + \mu\nu = 0$$

on N_0. Therefore (2.49) implies

$$\lambda\mu(\mu^2 - \alpha^2) = 0$$

on ths set. Since the function λ cannot be zero, it follows from (2.41) that $\mu^2(1 - \nu^2) = 0$ and hence $1 - \nu^2 = 0$ on N_0. Therefore, the last relationship of (1.3) gives $\mu(p) = 0$ for $p \in N_0$. Consequently (2.41) yields $\alpha = 0$ on N_0, which is contradictory. Hence the hypersurface is k-antiholomorphic. Thus, Lemma 2.5 is proved.

According to Lemma 2.3 and Lemma 2.5, we can state :

THEOREM 2.6. *Let M be a hypersurface with normal $(f, g, u, v, w, \lambda, \mu, \nu)$-structure of $S^n(1/\sqrt{2}) \times S^n(1/\sqrt{2})$. The hypersurface M is k-antiholomorphic if and only if the vector w^a and k^a are mutually orthogonal.*

From the Theorem 2.4 and Theorem 2.6, we have immediately :

COROLLARY 2.7. *Let M be a hypersurface with normal $(f, g, u, v, w, \lambda, \mu, \nu$ structure of $S^n(1/\sqrt{2}) \times S^n(1/\sqrt{2})(n > 1)$. If the vector w^a and k^a are mutually orthogonal, then M is the same type as that of Theorem 2.4.*

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Department of Mathematics
University of Ulsan
Ulsan 680–749, Korea