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PLASMA WAVE PROPAGATION IN THE BLACK HOLE IONOSPHERE

PARK, SEOK JAE
Korea Astronomy Observatory
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ABSTRACT

An axisymmetric, stationary electrodynamic model of the central engine of an active galactic nucleus has been well formulated by Macdonald and Thorne. In this model the relativistic region around the central black hole must be filled by highly conducting plasma. We analyze plasma wave propagation in this region and discuss the results. We find that the ionosphere cannot exist right outside of the event horizon of the black hole. Another interesting aspect is that certain resonance phenomena can occur in this case.

Key Words : black holes, plasma waves, active galactic nuclei

I. INTRODUCTION

An axisymmetric, stationary electrodynamic model was well formulated in the language of the "3+1"-spacetime formalism by Macdonald and Thorne(1982, hereafter MT), which consists of the supermassive black hole surrounded by a magnetized accretion disk. For the full analysis readers are also directed to Thorne, Price and Macdonald(1986, hereafter TPM).

In Park(1992) we defined the "ionosphere" of a supermassive black hole which lies at the center of an active galactic nucleus (hereafter, AGN). In this paper we analyze plasma wave propagation in the ionosphere. Our first goal is to analyze the redshift effect. Another goal is to find whether there are forbidden frequencies or not. If any resonance can occur in the ionosphere, it also will be theoretically interesting.

In §2 we will derive fundamental equations and in §3 we will solve them. Finally, we will discuss the conclusions in §4. Throughout this paper $(- + ++)$ signs will be used with units such that $c = G \equiv 1$. Greek indices will run 0 to 3 while Latin indices 1 to 3.

II. BASIC EQUATIONS

(a) Maxwell Equations

A fiducial observer (hereafter FIDO, see TPM) measures physical quantities in his neighborhood. Each FIDO is at rest with respect to the black hole and he never moves from his fixed location. Throughout this paper we define all the electrodynamic quantities at a point in the ionosphere as those measured by the FIDO at that point, using his own proper time τ. Since τ is not a global coordinate time, we also use the universal time t to define a slicing of spacetime. They are related to each other by the lapse function α,

$$\alpha \equiv \frac{d\tau}{dt}. \tag{2.1}$$

¿From the point of view of the 3+1-formalism, spacetime is a foliation of spacelike hyperspaces connected by timelike curves. If n^μ is the orthonormal timelike vector of the time slice, then $\gamma_{\mu\nu}$, the intrinsic curvature of the hyperspace, is defined by

$$\gamma_{\mu\nu} = g_{\mu\nu} + n_\mu n_\nu, \tag{2.2}$$

where $g_{\mu\nu}$ means the usual 4-dimensional spacetime metric tensor. Now $g_{\mu\nu}$ is set by (Arnowitt, Deser, and Misner

1962)

$$g_{\mu\nu} = \begin{pmatrix} -\alpha^2 + \beta_k\beta^k & \beta_j \\ \beta_i & \gamma_{ij} \end{pmatrix}, \tag{2.3}$$

where β^i the shift vector with $\beta_i = \gamma_{ij}\beta^j$.

In this paper we assume that the hole is so slowly-rotating that we may use the Schwarzschild metric as a good approximation. Since the rotational energy of the black hole in the M-T model is being extracted by the Blandford-Znajek process (Blandford and Znajek 1977), this assumption may fit well at the end of the evolution of galactic nuclei.

According to this assumption, nonvanishing coefficients of the metric in the spherical coordinates (r, θ, φ) are close to those of the Schwarzschild metric,

$$\alpha \simeq \left(1 - \frac{2M}{r}\right)^{1/2}, \tag{2.4a}$$

$$\gamma_{rr} \simeq \left(1 - \frac{2M}{r}\right)^{-1/2}, \tag{2.4b}$$

$$\gamma_{\theta\theta} \simeq r^2, \tag{2.4c}$$

and

$$\gamma_{\varphi\varphi} \simeq r^2 \sin^2\theta. \tag{2.4d}$$

In terms of these the FIDO frame in absolute space will be (TPM, eq. [2.3*a*])

$$\mathbf{e}_{\hat{r}} \simeq \left(1 - \frac{2M}{r}\right)^{1/2} \frac{\partial}{\partial r}, \tag{2.5a}$$

$$\mathbf{e}_{\hat{\theta}} \simeq \frac{1}{r}\frac{\partial}{\partial \theta}, \tag{2.5b}$$

and

$$\mathbf{e}_{\hat{\varphi}} \simeq \frac{1}{r\sin\theta}\frac{\partial}{\partial \varphi}. \tag{2.5c}$$

Hereafter we skip the coordinated basis symbol as $\mathbf{e}_{\hat{\varphi}} \rightarrow \mathbf{e}_{\varphi}$.

In 3+1-electrodynamics the field tensor $F^{\mu\nu}$ splits into the electric field $\mathbf{E}$ and the magnetic field $\mathbf{B}$ which reside in hyperspace and evolve with the time. If we fix the observer as the FIDO at the given point around a Schwarzschild black hole, the Maxwell equations for the ionosphere are given by (TPM, eq. [2.10])

$$\nabla \cdot \mathbf{E} = 4\pi\rho, \tag{2.6a}$$

$$\nabla \cdot \mathbf{B} = 0, \tag{2.6b}$$

$$\nabla \times (\alpha \mathbf{E}) = -\dot{\mathbf{B}}, \tag{2.6c}$$

and

$$\nabla \times (\alpha \mathbf{B}) = \dot{\mathbf{E}} + 4\pi\alpha \mathbf{j}, \tag{2.6d}$$

where $\dot{\mathbf{B}} \equiv \partial \mathbf{B}/\partial t$ and so on.

(b) Plasma Wave Equations

The continuity equation and motion equation are given by

$$\frac{1}{\alpha}\dot{\rho} + \nabla \cdot (\rho \mathbf{v}) = 0 \tag{2.7a}$$

and

$$\frac{1}{\alpha}\dot{\mathbf{v}} + \mathbf{v} \cdot \nabla \mathbf{v} = \frac{q}{m}(\mathbf{E} + \mathbf{v} \times \mathbf{B}), \tag{2.7b}$$

respectively, where ρ is the density and $\mathbf{v}$ is the velocity. Here we neglect gravitational acceleration which is equal to $-\nabla\alpha/\alpha$.

As usual, we consider small perturbations such as,

$$\rho = \rho_0 + \rho_1, \tag{2.8a}$$

$$\mathbf{v} = \mathbf{v}_0 + \mathbf{v}_1, \tag{2.8b}$$

$$\mathbf{E} = \mathbf{E}_0 + \mathbf{E}_1, \tag{2.8c}$$

and

$$\mathbf{B} = \mathbf{B}_0 + \mathbf{B}_1, \tag{2.8d}$$

where the 0-suffixed quantities are the unperturbed and the 1-suffixed are small perturbations.

We assume that all the perturbations are proportional to $e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)}$. Then from equations (2.6) and (2.7) we get the useful first-order equations as

$$-\frac{i\omega\rho}{\alpha} + \rho_0\nabla\cdot\mathbf{v} = 0, \tag{2.9a}$$

$$-\frac{i\omega}{\alpha}\mathbf{v} = \frac{q}{m}(\mathbf{E} + \mathbf{v}\times\mathbf{B}_0), \tag{2.9b}$$

$$\nabla\times\mathbf{E} = \frac{i\omega}{\alpha}\mathbf{B}, \tag{2.9c}$$

and

$$\nabla\times\mathbf{B} = -\frac{i\omega}{\alpha}\mathbf{E} + 4\pi\mathbf{j}, \tag{2.9d}$$

where, and from now on, the 1-suffices are skipped for simplicity. Since we are interested in perturbations of electrons m and q in equation (2.9b) are mass and charge of electron respectively.

Equation (2.9b) can be rewritten as

$$(-\frac{i\omega}{\alpha} + \omega_c\times)\mathbf{v} = \frac{q}{m}\mathbf{E}, \tag{2.10a}$$

where

$$\omega_c \equiv \frac{q}{m}\mathbf{B}_0, \tag{2.10b}$$

is the cyclotron frequency. The solutions of equation (2.10a) are

$$\mathbf{v}_{\|} = \frac{q}{m}\frac{i\alpha}{\omega}\mathbf{E}_{\|} \tag{2.11a}$$

and

$$\mathbf{v}_{\perp} = \frac{\alpha q}{m}\frac{(i\omega + \alpha\omega_c\times)}{\omega^2 - \alpha^2\omega_c^2}\mathbf{E}_{\perp}, \tag{2.11b}$$

where $\|$ and $\perp$ mean the components parallel and perpendicular to the magnetic field, respectively.

If we employ a rectangular coordinate system whose z-axis is parallel to the magnetic field, then we have

$$\begin{aligned}\mathbf{j} &= \rho_0\mathbf{v}\\ &= \frac{\alpha\omega_p^2}{4\pi}\left(\frac{1}{\omega^2 - \alpha^2\omega_c^2}[(i\omega\mathbf{e}_x + \alpha\omega_c\mathbf{e}_y)E^x + (-\alpha\omega_c\mathbf{e}_x + i\omega\mathbf{e}_y)E^y] + \frac{i}{\omega}\mathbf{e}_z E^z\right)\\ &\equiv \vec{\vec{\sigma}}\cdot\mathbf{E},\end{aligned} \tag{2.12}$$

where

$$\omega_p^2 \equiv \frac{4\pi\rho_0 q}{m}, \tag{2.13a}$$

is the plasma frequency and

$$\vec{\vec{\sigma}} = \frac{\alpha\omega_p^2}{4\pi}\begin{pmatrix} \frac{i\omega}{\omega^2-\alpha^2\omega_c^2} & \frac{-\alpha\omega_c}{\omega^2-\alpha^2\omega_c^2} & 0 \\ \frac{-\alpha\omega_c}{\omega^2-\alpha^2\omega_c^2} & \frac{i\omega}{\omega^2-\alpha^2\omega_c^2} & 0 \\ 0 & 0 & \frac{i}{\omega} \end{pmatrix}, \tag{2.13b}$$

is the conductivity tensor.

If we reset the right hand side of equation (2.9*d*) with the dielectricity tensor $\vec{\vec{\epsilon}}$

$$\nabla \times \mathbf{B} = -\frac{i\omega}{\alpha}\vec{\vec{\epsilon}}\cdot\mathbf{E}, \tag{2.14a}$$

we have

$$\vec{\vec{\epsilon}} = \vec{\vec{1}} + \frac{4\pi i\alpha}{\omega}\vec{\vec{\sigma}} = \begin{pmatrix} \epsilon_1 & i\epsilon_2 & 0 \\ -i\epsilon_2 & \epsilon_1 & 0 \\ 0 & 0 & \epsilon_3 \end{pmatrix}, \tag{2.14b}$$

where

$$\epsilon_1 \equiv 1 - \frac{\alpha^2\omega_p^2}{\omega^2 - \alpha^2\omega_c^2}, \tag{2.14c}$$

$$\epsilon_2 \equiv -\frac{\omega_c}{\omega}\frac{\alpha^3\omega_p^2}{\omega^2 - \alpha^2\omega_c^2}, \tag{2.14d}$$

and

$$\epsilon_3 \equiv 1 - \frac{\alpha^2\omega_p^2}{\omega^2}. \tag{2.14e}$$

In equation (2.14*b*) $\vec{\vec{1}}$ is the unit tensor.

If we approach the black hole more and more, we have $\alpha \to 0$ and $\vec{\vec{\epsilon}} \to \vec{\vec{1}}$ because $\vec{\vec{\sigma}} \to \vec{\vec{0}}$. This means that the strong gravity of the black hole tends to erase the role of the dielectric tensor. It seems, therefore, that the ionosphere cannot exist right outside of the event horizon of the black hole.

Since we assumed that all the perturbations are proportional to $e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)}$, substituting equation (2.9*c*) into equation (2.14*a*), we get

$$\mathbf{E} - \mathbf{e}_k(\mathbf{e}_k\cdot\mathbf{E}) = \frac{1}{\alpha^2 n^2}\vec{\vec{\epsilon}}\cdot\mathbf{E}, \tag{2.15a}$$

where

$$n \equiv \frac{k}{\omega} \tag{2.15b}$$

is the refraction rate and $\mathbf{e}_k$ is the unit vector in the $\mathbf{k}$ direction.

Let θ be the angle between $\mathbf{r}$ and $\mathbf{B}_0$(i.e., z-axis) and set

$$\mathbf{e}_k = \mathbf{e}_y \sin\theta + \mathbf{e}_z \cos\theta. \tag{2.16}$$

After some calculation equation (2.15) becomes

$$\begin{pmatrix} 1 - \frac{\epsilon_1}{\alpha^2 n^2} & -i\frac{\epsilon_2}{\alpha^2 n^2} & 0 \\ i\frac{\epsilon_2}{\alpha^2 n^2} & \cos^2\theta - \frac{\epsilon_1}{\alpha^2 n^2} & -\sin\theta\cos\theta \\ 0 & -\sin\theta\cos\theta & \sin^2\theta - \frac{\epsilon_3}{\alpha^2 n^2} \end{pmatrix}\begin{pmatrix} E^x \\ E^y \\ E^z \end{pmatrix} = 0. \tag{2.17}$$

To have a meaningful solution the determinant of the matrix should be zero. After some calculation that condition gives us the Appleton-Hartree equation in this case as

$$\tan^2\theta = -\frac{\epsilon_3(\alpha^2 n^2 - \epsilon_L)(\alpha^2 n^2 - \epsilon_R)}{(\alpha^2 n^2 - \epsilon_3)(\alpha^2 n^2 \epsilon_1 - \epsilon_L\epsilon_R)}, \tag{2.18}$$

where

$$\epsilon_L \equiv \epsilon_1 + \epsilon_2 = 1 - \frac{\alpha^2\omega_p^2}{\omega(\omega + \alpha\omega_c)} \tag{2.19a}$$

and

$$\epsilon_R \equiv \epsilon_1 - \epsilon_2 = 1 - \frac{\alpha^2\omega_p^2}{\omega(\omega - \alpha\omega_c)}. \tag{2.19b}$$

III. SOLUTIONS

In the theoretical models of AGNs there is a critical luminosity associated with accretion called Eddington luminosity

$$L_E = \frac{4\pi M m_p}{\sigma_T} \simeq 1.3 \times 10^{46} M_8 \quad \text{ergs s}^{-1}, \tag{3.1}$$

where M is the mass of the accreting object, i.e., the central supermassive black hole in this paper, m_p is the mass of proton, σ_T is the cross section of Thompson scattering, and M_8 is M in the unit of $10^8 M_\odot$. The related accretion rate

$$\dot{M}_E = L_E \simeq 10^{25} M_8 \quad \text{g s}^{-1} \tag{3.2}$$

has an important role in AGN models. For example, the central engines of QSOs are believed to accrete matter with this rate (e.g., see Begelman *et al.* 1984).

The e-folding time for the mass of a black hole accreting at $\dot{M}_E$

$$t_E = \frac{M}{\dot{M}_E} \simeq 4 \times 10^8 \quad \text{yr}, \tag{3.3a}$$

the particle density

$$n_E = \frac{1}{\sigma_T M} \simeq 10^{11} M_8^{-1} \quad \text{cm}^{-3}, \tag{3.3b}$$

and

$$B_E = \left(\frac{8\pi m_p}{\sigma_T M}\right)^{1/2} \simeq 4 \times 10^4 M_8^{-1/2} \quad \text{G} \tag{3.3c}$$

are related critical quantities.

If we adopt these Eddington quantities for the ionosphere of a black hole with mass $M_8 \simeq 1$, we have

$$\omega_c = \frac{qB_E}{m} \simeq 5 \times 10^{11} \quad \text{s}^{-1}, \tag{3.4a}$$

and

$$\omega_p \simeq 10^{10} \quad \text{s}^{-1}. \tag{3.4b}$$

We, therefore, have

$$\omega_p < \omega_c. \tag{3.5}$$

The solution of equation (2.18) in this case is well known. In this paper we investigate only the waves propagating along the magnetic field lines. If we set $\theta = 0$, equation (2.18) splits as

$$\alpha^2 n^2 = \epsilon_L, \tag{3.6a}$$

$$\alpha^2 n^2 = \epsilon_R, \tag{3.6b}$$

and

$$\epsilon_3 = 0. \tag{3.6c}$$

The last equation is meaningless but the other equations have their own cut-off frequencies

$$\omega_L \simeq \alpha\omega_c \frac{\omega_p^2}{\omega_c^2} \tag{3.7a}$$

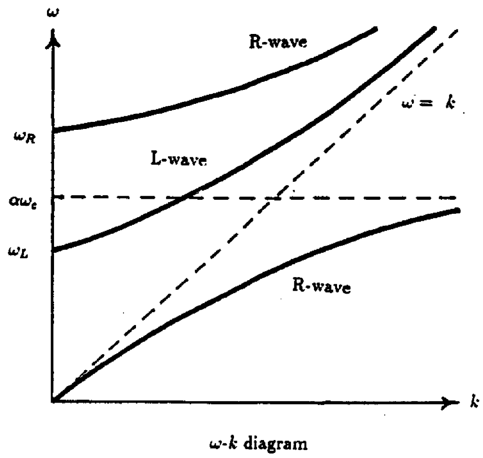

Fig. 1. •-k diagram.

and

$$\omega_R \simeq \alpha\omega_c \left(1 + \frac{\omega_p^2}{\omega_c^2}\right). \qquad (3.7b)$$

due to relation (3.5). Each value of equation (3.7) makes k to be equal to zero.

Therefore, we have

$$\omega_L < \alpha\omega_c < \omega_R \qquad (3.8)$$

and we get R-wave for $\omega_R < \omega$, L-wave for $\omega_L < \omega$, and another R-wave for $\omega < \alpha\omega_c$. Here R-wave and L-wave mean the right-hand and left-hand polarized waves. For details readers are directed to basic plasma textbooks.

The ω-k diagram in this case is shown in Figure 1.

IV. DISCUSSIONS

We find that the ionosphere cannot exist right outside of the event horizon of the black hole. We may conclude, therefore, that we do not have to consider the redshift seriously in the black hole ionosphere.

In Figure 1 we find that there are no forbidden regions. This means that plasma wave with any frequency can propagate along the magnetic field lines at the center of an AGN.

One interesting point is that $k \to \infty$ as $\omega \to \alpha\omega_c$. This means that there are resonance phenomena around $\omega \simeq \alpha\omega_c$. The role of the resonance is beyond the scope of this paper. It may possess an important role at the center of an AGN.

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