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Nonparametric Stock Price Prediction

Sung-Sup Choi* and Joo Hean Park**

Abstract

When we apply parametric models to the movement of stock prices, we don't know whether they are really correct specifications. In the paper, any prior conditional mean structure is not assumed. By applying the nonparametric model, we see if it better performs (than the random walk model) in terms of out-of-sample prediction. An interesting finding is that the random walk model is still the best. There doesn't seem to exist any form of nonlinearity (not to mention linearity) in stock prices that can be exploitable in terms of point prediction.

I. Introduction

The issue of correctly specifying the asset return distribution has received substantial attention in the literature. In the context of stock returns, the history goes as far back to the turn of the century. Bachelier (1900) developed a model in which the lower frequency asset price changes, being the sum of a very large number of high-frequency transaction price changes, are approxi-mately normal.

Empirically, however, high frequency data such as the daily or weekly data do not confirm this prediction, being approximately symmetric but nevertheless highly leptokurtic. These char-acteristics led researchers like Mandelbrot (1963) to the use of symmetric stable distributions for return-generating processes. The symmetric stable distributions, however, contradict the empiri-cal fact that the aggregated asset return data follow normal distribution, even though capturing major characteristics of high

* Dept. of Business Administration, KyungWon University, SungNam, Korea.
**Dept. of Economics, Dongduck Women's University, Seoul, Korea.

frequency data.[1]

In dealing with high frequency data, a more sophisticated parametric class of model, GARCH model, has been developed and popularized. In this model, the variance of the error term at time t, conditional on past information, is a nonlinear function of past error terms(AR part) and past error variances(MA part). GARCH model creates simulated data series that exhibit persistence and busting, which in turn accounts very well for the stylized facts of high frequency data.[2] In addition to these favorable properties, its unconditional density converges to normality under temporal aggregation.

Now, a possible question is whether the GARCH model is the correct specification of asset return dynamics. If this GARCH characterization is correct, any sort of nonlinearity in the condi-tional mean (not to mention linearity) cannot be exploited to improve point predictions. Hsieh(1989) showed that, using exchange rate, GARCH filtering does not remove all the hidden struc-tures from the data. In a theoretical development, Sims (1984) shows that general equilibrium asset pricing models imply martingale asset-price behavior only at arbitrarily short horizons. The message is that neither theoretical nor empirical results can rule out the potential presence of more complicated dynamics in asset returns.

In fact, strong evidence of nonlinearity has been found in the prediction errors of linear models.[3] The nonlinearity may be either in conditional mean park or in the error term, otherwise in both. However, the source of nonlinearity has not yet been identified. If the conditional mean part is in itself nonlinear, the source of nonlinearity in the prediction error of the linear model cannot be identified because of misspecification of the conditional mean part. The empirical fact that no nonlinear model outperforms the random walk systematically in the prediction, may be explained by two possible reasons. One is that the conditional mean is in itself linear while the other is that the true nonlinear conditional mean has not yet found since the class of plausible alternative non-linear models are huge.

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1) Note that the stable Paretian family remains stable Paretian under temporal aggregation, and this point was forcefully argued by Fama(1976).

2) A variety of high frequency asset returns is described ad linearly unpredictable, but conditionally heteroskedastic, and unconditionally leptokurdic.

3) for example, Granger and Anderson(1978), Hinich and Patterson(1985), Engle, Lillien and Robins(1987), Hsieh(1989), Scheinkman and LeBaron(1989) detect statistically significant nonlinearity in asset prices.

In terms of theoretical works, there currently exist few economic theories that yield a unique a priori specification of a nonlinear model to test. Grandmont(1985) proposed an alternative dynamic structure which follows nonlinear deterministic dynamic paths. Unfortunately, however, he found that from very simple models of intertemporal optimization behavior there is an infinite quantity of dynamic paths the economy can take, including chaotic.

On the other hand, Meese and Rogoff (1983) showed empirically that a random walk model outperforms a variety of other economic models in terms of out-of-sample forecast.[4] All the models considered by them, however, are linear models; they didn't look at a validity of nonlinear models. The overwhelming variety of plausible candidate nonlinear models makes model selection a difficult task. In fact, a large variety of nonlinear models that have received attention lately[5] may be a small subset of the class of plausible yet unfound nonlinear models.

In this paper, we investigate whether the random walk model for the(high frequency) stock price data will be outperformed in terms of out-of-sample forecast. The design of the research becomes as follows: to determine initially if stock returns have complicated structure, we fit a random walk model, get the residuals, and do the Brock-Dechert-Scheinkman (1987) (BDS) test. Recognizing that the GARCH model has been well known to capture some form of nonlinearities that might exist in real data, we also fit the GARCH model, and apply the same steps. If there remain hidden structures not captured by a random walk and/or GARCH form of nonlinear models, the BDS test will be rejected. When rejected, we need to choose an alternative model.

In this study, we do not specify the conditional mean function parametrically and make use of kernel nonparametric estimation techniques in estimating conditional mean. The validity of more complicated return structure will then be considered in terms of out-of-sample forecast.[6] More precisely, the out-of-sample forecast based on nonparametric form will be compared to that of random walk. If the nonparametric model outperforms, this implies that the nonlinearity would be embedded in the condi-

4) Their candidate models included a flexible price monetary model, a sticky price monetary model, a sticky price monetary model with current account effects, six univariate time series models, a vector autoregressive model, and the forward rate.

5) For example, nonlinear moving average, bilinear, threshold, exponential autoregressive, and so forth.

tional part so the search for more complicated models will be useful and the random walk model is not going to be as much valid. Otherwise, the ture prediction model may be a linear model with errors having a nonlinear structure.

In the next section, the random walk hypothesis, the simple form of nonlinear model (which is the GARCH model in this case), the BDS test, and the nonparametric kernel estimation technique will be very briefly discussed. And then the empirical results and further research suggestions follow in the last section.

Ⅱ. RANDOM WALK MODEL, GARCH MODEL, BDS TEST, KERNEL ESTIMATION

1. RANDOM WALK MODEL

The hypothesis of market efficiency implies the following price equation form;

$$P(t)=E\left[\sum_{s=t}^{\infty}\frac{D(s)}{(1+r)^{s-t}}\mid I(t)\right] \tag{1}$$

where $P(t)$, $D(t)$ and $I(t)$ represent stock price, dividend and the set of information available to market participants at time t respectively and r is a discount rate. With a transversality condi-tion, the above equation is equivalent to the statement that, for all t:

$$P(t)=E\left[\frac{P(t+1)}{1+r}\right]E[D(t)] \tag{2}$$

or to the statement that

6) Meese and Rogoff(1983) have shown that a random walk model outperforms any other form of linear models for exchange rate process (i.e., a random walk model is the best linear model in terms of out-of-sample forecast). Although the similar empirical studies need to be done among linear models with stock return process, a random walk model is likely to be the best linear model in terms of out-of-sample forecast with stock price data as well. Assuming that a random walk model is the best, we effectively investigate the validity of nonlinear model by comparing a random walk model with that of non-parametric form in terms of out-of-sample forecast.

$$E[R(t)]=E\left[\frac{P(t+1)}{P(t)}+\frac{D(t)}{P(t)}-1\right]=r \tag{3}$$

At the same time, the above equation implies the following :

$$R(t)=r+\varepsilon(t) \tag{4}$$

where $R(t)$ denotes return at time t and $\varepsilon(t)$ is serially uncorrelated and orthogonal to any element to $I(t)$. Market efficiency is normally tested by adding regressors drawn from $I(t)$ to the above equation and testing the hypothesis that their coefficients equal zero, or by testing the hypothesis that $\varepsilon(t)$ follows a white noise process.

2. GARCH MODEL

Let $y(t)$ be a discrete time stochastic process. In general, the GARCH (p, q) regression model is determined as follows :

$$y(t) \mid I(t) \sim N(x(t)'\theta, h(t)) \tag{5}$$

where

$$\varepsilon(t) \equiv y(t)-x(t)'\theta, \tag{6}$$

$$h(t) \equiv \alpha(0)+\sum_{i=1}^{p} \alpha(i)\varepsilon^2(t-i)+\sum_{j=1}^{p} \beta(j)\mathrm{h}(t-j) \tag{7}$$

where $p, q>0$; $\alpha(i)\geq 0$ for $i=0, 1, 2, \cdots, \mathrm{q}$; $\beta(j)\geq 0$ for $j=0, 1, 2, \cdots, \mathrm{p}$

The stability condition for this regression model is given by

$$\sum_{i=1}^{\max(p,q)} \alpha(i)+\beta(i)\leq 1 \tag{8}$$

which corresponds to having the characteristic roots of the expansion of unconditional residual variance lie outside the unit circle.

The estimation of a GARCH regression model can be done using log likelihood maximization. Identification of p and q can be achieved using standard Box-Jenkins on the squared OLS residuals. In this paper, the GARCH (1,1) model is specified as an alternative to the random walk model:

$$R(t)=r+\varepsilon(t), \text{ where}$$
$$\varepsilon(t) \mid I(t-1) \sim N(0, h(t)),\ h(t)=\alpha_0+\alpha_1\varepsilon^2(t-1)+\beta_1 h(t-1) \tag{9}$$

3. BDS TEST

The BDS statistic, developed by Brock, Dechert, and Scheinkman (1987), tests the null that the data is independently and identically distributed(*iid*) against the alternative of either nonlinear stochastic or deterministic structure. The test looks at the N-space filling properties of N-streams (streams of length N) of data. If the test discovers enough areas in N-space with low observation density, it will reject the null of random iid data. If the data were truly random, N-space would not contain any holes, but would be filled given enough observations. It needs to be noted that the BDS statistic cannot be used globally in N-space as it is highly dependent on the underlying distribution of the data. In other words, the global implications of the intuitive description above are used only for expository purposes. The BDS test is evaluated locally in the neighborhood of an N-stream.

Given a time series $x(t)$ where $t=1, 2,\cdots, T$, let an N-history of $x(t)$ be defined by $x(t, N)=x(t),\cdots, x(t+N-1)$. The correlation dimension($C_N(\varepsilon, \infty)$), developed by Grassberger and Procaccia (1983), is defined as follows:

$$C_N(\varepsilon, \infty)=prob[(x(t, N), x(s, N) \mid \| x(t, N)-x(s, N) \| < \varepsilon] \tag{10}$$

where the double vertical line($\|\cdot\|$) indicates the max norm and the epsilon(ε) is a number from the non-negative real line. The correlation dimension thus measures the fraction of the N-histories that are epsilon away from one another as the sample size goes to infinity. If the process $x(t)$ is truly iid, then it can be shown that :

$$prob[(x(t,N), x(s,N) \mid \| x(t,N)-x(s,N) \| < \varepsilon] = \prod_{j=0}^{N-1} prob[(x(t+j), x(s+j) \mid \| x(t+j)-x(s+j) \| < \varepsilon] \tag{11}$$

If we assume that the data generating process is stationary, then it follows that:

$$C_N(\varepsilon, \infty) = [C_1(\varepsilon, \infty)]^N \tag{12}$$

In finite samples, we are able to derive a central limit theorem for *iid* and weakly dependent processes such that the sample correlation dimensions converge to the population dimensions de-fined above. We can therefore show the following:

$$T^{1/2}[C_N(\varepsilon, \infty) - [C_1(\varepsilon, \infty)]^N] \rightarrow N(0, V_N(\varepsilon)) \tag{13}$$

where $V_N(\varepsilon)$ is the covariance matrix, and Hsieh and LeBaron(1988) showed that this can be estimated in sample.

Conceptually, the BDS test looks at the dispersion of the points in a number of spaces with dimension going to from 2 to n. This dispersion will be either in line or at odds with the assumption of white noise. The BDS test then amounts to a test of the difference between the dispersion of the observed data in these consecutive spaces with the dispersion that a white noise process would generate in these same spaces. For details, readers are advised to consult Brock (1986) or the extensive survey of Eckman and Ruelle (1985). In this paper, the residuals from the random walk model and the GARCH(1,1) model (after normalizing the variances) are run through the BDS program, written by W. D. Dechert. As we are investigating low order structure such as dimension 2, rejections of the iid null at low embedding dimensions will be of special interest.

4. NONPARAMETRIC KERNEL ESTIMATION

The conditional expectation function represented by equation (14) is usally used for out-of-sample forecast.

$$g(x) \equiv E[y|x] = \int y f(y|x)dy = \int y \frac{f(y, x)}{f(x)} dy \tag{14}$$

A sample analog estimator of the conditional expectation function can be obtained by substituting nonparametric estimates of the underlying conditional densities into the above equation(14). Different nonparametric methods make use of different nonparametric estimates of the underlying densities.

Nevertheless, all of nonparametric estimators have the following form:

$$\hat{g}(x^*) = \sum_{t=1}^{T} W_T(x(t); x^*)y(t) \tag{15}$$

where W_T is a weighting function, varying with sample size T. A nonparametric estimate of $\hat{g}(x^*)$ is simply a weighted average of the dependent variables $y(t)$. The rule of weighting is that the nearer the distance between x and x^* is (in some measure), the more weight x will get.

The nonparametric estimation method employed in this paper is the kernel method, the weighting function of which will be as follows :

$$W_T(x(t); x^*) = \frac{K\left[\frac{x^*-x(t)}{h(T)}\right]}{\sum_{t=1}^{T} K\left[\frac{x^*-x(t)}{h(T)}\right]} \tag{16}$$

where $K(.)$ is a kernel function, $h(T)$ is a bandwidth depending on sample size. The kernel regression estimates at x^* is defined accordingly :

$$g(x^*) = \frac{\sum_{t=1}^{T} K\left[\frac{x^*-x(t)}{h(T)}\right]y(t)}{\sum_{t=1}^{T} K\left[\frac{x^*-x(t)}{h(T)}\right]} \tag{17}$$

Under regularity conditions, the kernel estimator is proven to be consistent and asymptotically normally distributed.[7]

7) Robinson(1983) extended the consistency and asymptotic normality results to time-series context; it requires a little bit more strict conditions.

The kernel methods reflect a few judgmental decisions, such as size of bandwidth and choice of kernel function. In fact, theory gives only a set of conditions which the bandwidth and the kernel function must satisfy. A symmetric density function is often taken for a kernel function.

Many Monte-Carlo studies, however, show that the estimation result is relatively robust to the choice of kernel functions. Of far greater importance is the choice of bandwidth size, h(T), which determines how sharply the weights reduce as the distance between x* and x(t) becomes widened. The bandwidth must go to zero with sample size, but at a slower rate. The condition the band-width should satisfy is the following:

$$h(T) \rightarrow 0,\ Th(t)^{p} \rightarrow \infty, \text{ as } T \rightarrow \infty \tag{18}$$

where p is the number of explanatory variables. In doing so, the shrinking bandwidth size contains progressively nearer points to x^*. This allows the reduction of bias along with variance, enabling consistency to be achieved. The practical problem is, however, that there exists no way of determining the size of bandwidth in a finite sample. For this reason, several bandwidths will be considered in this paper.

In our dynamic model, the stochastic conditioning variables x(t) is composed of lagged dependent variables; $x(t)=\{y(t-1), \cdots, y(t-p)\}$. Thus, we shall work with the following structure:

$$\begin{aligned} &y(t)=g(y(t-1), \cdots, y(t-p))+\varepsilon(t), \\ &E[\varepsilon(t) \mid y(t-1), \cdots, y(t-p)]=0 \end{aligned} \tag{19}$$

where $g(.)$ is a unknown function. The future value $y(t)$ is predicted by estimating $g(y(t-1), \cdots, y(t-p))$ nonparametrically. This prediction scheme has an intuitive interpretation. Suppose we try to predict $y(t)$ based on $\{y(t-1), \cdots, y(t-p)\}$. Out of a whole series of $y(t-j)$, the subseries $\{y(s-1), \cdots, y(s-p), s<t\}$ which has the most similar pattern to the conditioning vector $\{y(t-1), \cdots, y(t-p)\}$ will be given the highest weights to $y(t)$. Likewise, the subseries having less similar pattern will have less weights. The idea is to predict the future in terms of the most exact experiences that

we' ve gone through.

Ⅲ. EMPIRICAL IMPLEMENTATION

1. DATA DESCRIPTIONS

The initial data set consists of more than 6,000 daily stock returns (from Jan. 1, 1962 to Dec. 31, 1986) on the value-weighted portfolio of the Center for Research in Security Prices at the University of Chicago (CRSP). With a daily observation interval, the biases associated with nontrading, the bid-ask spread, asynchronous prices, and others (including weekend effect and/or Monday effect) may become statistically significant. A formal model of the market microstructure may be needed to deal with the biases, which will probably make the analysis complexed. Here, we use 1279 weekly stock return data constructing from the initial data in order to avoid complexities.

2. EMPIRICAL RESULTS

We do first in-sample analysis of the correlation dimension test designed by Brock, Dechert, and Scheinkman (1987). This test is done for the residuals of both random walk model and GARCH model.

Under the null hypothesis of an iid series, the BDS statistic is distributed asymptotically as a standard normal random variable. BDS procedure may be useful for testing *iid* behavior when likely alternatives are not known a priori. Nevertheless, there are some caveats that need to be kept in mind. Rejections of *iid* behavior are sometimes taken to imply that the series is linearly or nonlinearly forecastable. But, this may not always be the case since the rejection could be due to outliers, structural shifts, conditional dependence in even-ordered moments, etc. Even if a time series is linearly or nonlinearly predictable, the BDS test is unable to distinguish between linear stochastic, linear deterministic, nonlinear deterministic, or nonlinear stochastic data generating process.

In any application of a test that relies on its asymptotic distribution, we must make sure that the asymptotic distribution is actually well approximated by the finite sample distribution. For the BDS statistics, there are more complications since the BDS test is computed for a given embedding dimension N and a given disturbance e(in number of standard deviations of the data, σ). The choice of N and σ must therefor be determined priori. Many Monte Carlo simulations suggest that the embedding dimension N should be 5 or lower, e should be between 0.5σ to 2σ. And the asymptotic distribution can be approximated well by the finite sample distribution of the BDS statistic for 500 or more observations.

We pick $N=1$, $\varepsilon=0.5\sigma$, 0.75σ, σ, 1.25σ, 1.5σ and our data points are 6,000 which may be sufficient to approximate the asymptotic distribution. BDS test results appear in table I. For all cases considered, the BDS test finds strong evidence of non-*iid* behaviors.[8] As previously mentioned, however, the test is silent on the form of likely deviations from the null. In particular, we would expect the BDS test to reject *iid* behavior due to the well known conditional heteroskedasticity in error terms even if there were no presence of nonlinearity in conditional mean part. For this reason, We proceed to fit the GARCH parametric model in order to filter out the possible conditional heteroskedasticity, and normalize the residuals,[9] and then reapply the BDS tests which also appear in table I. Again, the BDS test still finds strong evidence of non-*iid* behaviors.

Now, let' s begin with the nonparametric kernel estimation. To start with, we choose the normal density as a kernel function. As we all know, the normal density is represented by two moments. The first moment, mean, is chosen by zero. The second moment, variance-covariance matrix, is taken by estimates of the autocovariances from the sample.[10] With respect to the bandwidth $h(T)$, we perform a sensitivity analysis. The bandwidth size is taken by

$$h(T)=cT^{\frac{1}{p+d}}, \text{ where } c,\ d>0 \tag{20}$$

8) Numbers in table I are z-values, all of which indicate significance at the 1 percent significance level.

9) Having fitted GARCH(1,1) model, we get the series of adjusted residuals and variances. Then the normalization is done by dividing the adjusted residuals by the variance; if GARCH model is correct specification, this normalization must generate iid process.

10) Many Monte-Carlo studies show that a good nonparametric estimate can be obtained by taking a kernel function which reflects the structure of the underlying population as much as possible.

Table 1. BDS test results for the residuals of random walk model

	0.5σ*	0.75σ	σ	1.25σ	1.5σ
residuals of random walk model	7.34	8.33	8.34	8.28	8.60
residuals of the GARCH(1.1) model	5.84	5.23	10.47	14.25	18.67

* σ is the sample standard deviation. The embedding dimension is 2. so the test proceeds by comparing 2-histories using the sup norm.

Fixing $d=1$, we explore a wide range of c from 0.2 to 1 in order to see the sensitivity of the analysis to bandwidth size.

We now turn to the one-step-ahead out-of-sample prediction analysis. We estimate non-parametric autoregressions of order 1 and 3. Each model is estimated on the basis of the most up-to-date information available when forecasting. This is done by using rolling regression to reestimate each model for each forecasting period. In out-of-sample forecast, the choice of the first period is arbitrary. We reserve the last 279 samples for the out-of-sample forecast. It is likely that the result for the one-step-ahead would be reinforced for the one that performs better as we increase the step size. Since the focus here is on the high frequency data, we don' t deal with the issue related to the low frequency data, such as mean reversion. Mean reversion is found to follow the business cycle frequency, 3-5 years. If this is the case, the 24 years of data (which is sampling period of this paper) is not big enough in detecting such a phenomenon. Remember that a big sample in financial data is not something that we can determine by looking at number of observations.

The results of the out-of-sample forecast for the nonparametric models of autoregression of order 1 and 3 (in comparison with those for the random walk model) are reported in table II. A wide range of bandwidth, c, from 0.2 to 1.0, is also reported, but as we mentioned earlier the results are not sensitive to the changes in bandwidth. RMSE stands for Root Mean Square Error while LQ and UQ indicate Lower Quantile, Upper Quantile, respectively. LQ, UQ, and MEDIAN are calculated based on the order statistics of the absolute value of residuals. Thus, the lower value of LQ, UQ, MEDIAN, the better out-of-sample forecast. The striking feature of the empirical

Table 2. One Step shead out-of-sample forecast comparison

Model	band-width	RMSE	LQ	MEDIAN	UQ
Nonparametric Model : $y(t)=g(y(t-1))+\varepsilon(t)$	0.2	13.955	3.9204	8.0758	13.5061
	0.4	13.6060	3.9481	8.0758	13.0441
	0.6	13.5327	3.9113	7.9850	13.0441
	0.8	13.4880	4.0055	7.9479	12.9781
	1.0	13.4509	4.1326	7.9064	13.2481
Nonparametric Model : $y(t)=g(y(t-1),y(t-2), y(t-3))+\varepsilon(t)$	0.2	17.3082	5.0769	10.7254	17.1009
	0.4	16.1925	4.3857	10.0977	16.1530
	0.6	15.2719	4.2931	9.0701	15.6103
	0.8	14.6167	4.2003	8.7164	14.5693
	1.0	14.2007	4.0373	8.6061	13.8199
Random Walk	N/A	12.8832	4.0252	7.6204	13.1276

*All entries are converted into stock prices from the stock returns data, where the basis price is set to be 100.

result is that the random walk model almost invariably has the lowest RMSE across all the different bandwidths, which means the random walk model outperforms the non-parametric model in terms of out-of-sample forecast. The empirical results summarized in table II together with table I, imply that a true underlying asset return model would have a linear conditional mean and a nonlinear structure of error terms. Therefore, it may not be useful to search for more complicated nonlinear conditional mean model to improve the out-of-sample forecast.

3. SUGGESTIONS FOR FUTURE WORKS

The thrust of this paper is this: when we fit some parametric model, we don't know whether it is really a correct specification. Thus, we don't impose any a priori conditional mean structure; we neither force it to be linear, nor it to be nonlinear. An interesting finding of this paper is that the random walk model is still the best in terms of out-of-sample forecast. The well-known work by Meese and Rogoff (1983) provides graphic illustration of the failure of a variety of other linear models to outperform the random walk model in out-of-sample prediction in terms of exchange rate data. This result is reinforced by Diebold and Nason (1989) again with exchange rate data. Their finding seems to be that there doesn' t exist any form of nonlinearity (not to mention linearity) exploitable in terms of point prediction with exchange rate data. Our study shows that their finding still prevails in terms of stock price data. From the findings that no nonparametric model seems to outperform the random walk model in terms of out-of-sample forecast, the true stock price model would be sketched like a linear conditional mean with a error having nonlinear structure.

Closing the concluding remarks, we need to indicate some problems in this study for possible future works.

First, we have to indicate two stock market anomalies relevant to this study among others.

One is the week of the month effect, and the other is the January effect. Both effects would show up in weekly data which are used here. The week of the month effect was found by Ariel (1987) who showed that for the period 1963 to 1981 all of the market' s cumulative advance occurred around the first half of the month, the second half contributing nothing to the cumulative increase. The January effect can be removed by using a dummy variable. But, there are several recent studies which show that this January effect is primarily a size effect and appears to be insignificant in the value weighted index. Since we are using the value weighted index, it seems acceptable to ignore the January effect.

Second, in terms of data-related problem, we might have 'time deformation problem' ; economic time and calendar time might differ. For example, the appropriate time scale for stock markets might 'speed up' in calendar time in periods when an usually large amount of news must be processed by the market.[11] Stock (1987) explored the possibility that the relationship between economic and calendar time depends on the economic history of certain variables which indicate acceleration or deceleration of economic time, and develops a test statistic for time deformation which amounts to a set of linear restrictions in a vector autoregression (VAR). More careful empirical study needs to be filtered through this kind of test. But, given the fact that time defor-

11) Clark's(1973) model of this phenomenon subordinates asset prices to an information arrival process; Clark shows how this framework can potentially explain the observed leptokurtosis in asset returns.

mation generates spurious nonlinearity, we doubt that it affects the result of this study that the random walk model can' t be outperformed by the nonparametric model.

Lastly, including our study, most nonparametric evidence for incremental predictability of financial asset returns above and beyond simple random walks is negative. It appears that parametric methods of prediction that have high power at detecting and forecasting structures "when we know what we are looking for" might have to be used to have more success at predicting financial asset returns. But, as we indicated earlier, the overwhelming variety of plausible candidate models makes parametric model selection a difficult task. Maybe, what is really needed is an economic theory which might give a specific guidance as to appropriate functional form of parametric model. Even after we settle on this issue of choosing a parametric form, it might be necessary to measure predictive gains in units like trading profits which are more mean-ingful than usual measures such as reduction in root mean squared error. Furthermore, we might also need to use more delicate statistical methods such as those used in the Brock, Lakonishok, LeBaron (1990) since the gains are likely to be small.

References

1. Ariel, Robert A., "A Monthly Effect in Stock Returns", *Journal of Financial Economics* 18 (1987). 1-14
2. Bachelier, L., "Theory of Speculation", reprinted in P. Cootner eds., *The Random Character of Stock Market Prices* : MIT Press, Cambridge, Massachusetts; 1964
3. Black, Fisher, "Studies of Stock Price Volatility Change" Proceeding of the 1976 Meetings of the Business and Econ. Stat. Section : *American Statistical Association*; 1976
4. Bollerslev, Tim, "Generalized Autoregressive Conditional Heteroskedasticity", *Journal of Econometrics* 31(1986), 307-27
5. Brock, Lakonishok, LeBaron, "Technical Analysis as a Specification Test", *Manuscript*, Univ. of Wisconsin-Madison and Univ. of Illinois-Urbana Champaign;1990
6. Brock, W., Dechert, W., and Scheinkman, J., "A Test for Independence Based on the Correlation Dimension", *SSRI working Paper* #8702, Dept. of Economics. University of Wisconsin, Madison; 1987
7. Brock, William A. "Distinguishing Random and Deterministic Systems", *Journal of Economic Theory* 40(1986), 168-195
8. Clark, P.K, "A Subordinated Stochastic Process Model with Finite Variance for Speculative Prices", *Econometrica* 41 (1973), 135-155
9. Diebold, F., and Nason, J., "Nonparametric Exchange Rate Prediction" Manuscript : *Federal Reser e Board*, Washington, D.C.; 1989
10. Eckman & Ruelle, "Ergodic Theory of Chaos and Strange Attraction", *Review of Modern Physics* 57 (1985), 617-656
11. Engle, R., Lillien, D., and Robbins, R., "Estimating Time-varying Risk Premia in the Term Structure : ARCH-M Model", *Econometrica* 55 (1987), 391-408
12. Engle, Robert, "Autoregressive Conditional Heteroskedasticity with Estimates of the Variance of UK Inflation", *Econometrica* 50 (1982), 987-1008
13. Fama, D. A., *Foundations of Finance*, New York : Basic Books; 1989
14. Fama, Eugene, "Efficient Capital Markets : Review of Theory and Empirical Work", *Journal of Finance* 25 (1970), 383-417
15. Fama, French, "Permanent and Temporary Components of Stock Prices", *Journal of*

Political Economy 96 (1988), 246-273

16. French , Roll, "Stock Return Variances : The arrival of Information and the Reaction of Traders", *Journal of Financial Economics* (1986)
17. Grandmont, "Nonlinear Economic Dynamics : Introduction", *Journal of Economic Theory* 40 (1985), 3-12
18. Granger, Anderson, *An Introduction to Bilinear Time Series Models*, Vandenhoeck & Ruprecht, Gottingen, 1978
19. Grassberger & Procaccia, "Estimation of the Kolmogorov Entrophy from a Chaotic Signal", *Physical Review* 28 (1983), 2591-2593
20. Hinich, M. & D. Patterson, "Evidence of Nonlinearity in Daily Stock Retrurns", *Journal of Business and Economic Statistics* 3 (1985), 69-77
21. Hsieh, D.A., "Testing for Nonlinear Dependence in Foreign Exchange Rates :1974-1983", *Journal of Business* 62 (1989), 339-369
22. Hsieh, P.A. & B. LeBaron, "Finite-Sample Properties of the BDS Statistics", *Manuscript*, Graduate School of Business, University of Chicago; 1988
23. Lo, Mackinlay, "Stock Market Prices Do Not Follow Random Walks", *Review of Financial Studies* 1 (1987), 41-66
24. Mandelbrot, B., "The Variation of Certain Speculative Prices", *Journal of Business* 36 (1963), 394-419
25. Meese, R.A. & K. Rogoff, "Empirical Exchange Rate Models of the Seventies : Do they fit out of sample?", *Journal of International Economics* 14 (1983), 3-24
26. Porterba, Summers, "Mean Reversion in Stock Prices : Evidence and Implications", *NBER working paper* #2343; 1988
27. Robinson, P.M., "Nonparametric Estimators for Time Series", *Journal of Time Series Analysis* 4 (1983), 185-207
28. Scheinkman, J.A. & B. LeBaron, "Nonlinear Dynamics and Stock Returns", *Journal of Business* 61 (1989), 311-337
29. Sims, C.A., "Martingate-like Behavior of Prices and Interest Rates", *Discussion Paper* #205, Dept. of Economics, University of Minnesota; 1984
30. Stock, J.H., "Nonparametric Policy Analysis", *Journal of Americal Statistical Associaltion* 84 (1989), 567-575