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PARTIAL INTERNATIONAL COORDINATION OF MONETARY POLICIES

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〈ABSTRACT〉

This paper studies a partial coordination situation where a set of countries coordinate their monetary policies among themselves;while the rest of the world choose their policies independently.

Using a three-country orewlapping generations model, it is shown that nash partial-coordination equilibrium does not exist. This paper also studies the partial coordination under unanticipated productivity shocks.

This paper studies a partial coordination situation where a set of countries coordinate their monetary policies among themselves;while the rest of the world choose their policies independently. The motivation for this study stems from what we observe today. We observe attempts to coordinate the economic policies of the G-7 countries or the European community rater than global coordination agreements.

While most of the existing literature on international coorination compares the Nash equilibrium with the "full" coordination equilibrium, there are three exceptions. Turnovsky(1988), Canzoneri and Henderson(1991), and Espinosa-Vega and Yip(1994) study the issues of partial international coordination. However, to our knowledge, our paper is the first attempt to study partial coordination of monetary policies in a dgnamic general eguilibrium framework.

Section 1 describes the model:A three-country overlapping generations model with fiat monies. We present the competative michanisms in which representative agents solve their optimization problems, given the policy choices of governments.

Section 2 shows that when two countries coordinate their policies while playing a Nash game with respect to the third country;steady-state Nash partial-coordination equilibrium does not exist.

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Coordinating countries want to choose less inflationary policy;while non-coordinating country wants to choose more inflationary policy. No solution exists. However, we propose, with caution, that:if coordinating countries adhere to their optimal money growth rates;then non-coordinating country will follow. As more countries join policy coordination;inflation rates go down;world outputs increase;and welfare improves. Steady-state utility levels of coordinating countries and non-coordinating countries are the same.

Section 3 studies the monetary policy choices under unanticipated productivity shocks. When the differences in the productivities-of coordinating;and of non-coordinating countries-are <u>not</u> large, Stackelberg warfare arises. If the coordinating countries paly the role of "leader";world output shrink more than the case when non-coordinating country plays the "leader". When the differences in the productivities=of coordinating;and of non-coordinating countries-are large:the country(s) with higher productivity prefers to be the "follower";while the country(s) with lower productivity wants to be the "leader".

1. The Model

In this section, we solve for the competitive equilibrium. Given monetary policies of governments;representative agents make labor supply and portfolio decisions, maximizing utility of leisure and consumption. Equilibrium labor supply, consumption, and prices are derived;as function of future money growth rates.

[Assumption 1] There are three countries. They are indexed by the variable k, k=1, 2 and 3. Representative agents of each country live for two periods. During each time period, two generations are alive-one young and one old. Young generations work but do not consume;old generations consume but do not work.

Utility function of country k's representative agent has the form

$$U(L_k, C_k) = -L_k + \ln C_k \tag{1}$$

where L_k = labor supply when young

C_k = consumption when old

for k=1, 2, 3.

[Assumption 2] There is only one type of good. It is not storable. Labor is the only input for production of goods. Production functions are

$$Y_k = \theta_k \cdot L_k \tag{2}$$

where Y_k = output of country k

θ_k = productivity of labor of country k

L_k = labor supply of country k

for k = 1, 2, 3.

[Assumption 3] There are three kinds of fiat monies-of country 1, 2, and 3, issued by each government. There are no transactions costs of whatsoever. Goods and labor in both countries can be paid for with any kind of monies. There are no restrictions on goods and portfolio transactions.

[Assumption 4] Economy extends over discrete time, t, t = 1, 2, · · · ∞. The tool at the disposal of government is monetary policy. In period t, government chooses (possibly negative) lump-sum payments, S_{kt}, to be distributed in the following period to what is the young generation of the current period. Country k's money transfer, S_{kt}, is given to period-t young generation of country k only; when period t+1 comes.

[Assumption 5] Every period, there is extremely small (close to zero) productivity shocks will arrive in the next period. Productivity shocks of both countries are equal, but less-than-perfectly correlated.

The optimization problem of period-t young generation of country k, k = 1, 2, 3, is to choose labor supplies, L_{kt}, and portfolio of currencies, M_{1t}^{dk}, M_{2t}^{dk} and M_{3t}^{dk};which is carried over to the next period, to buy the consumption goods when old. The decision is made; given wages, W_{kt}, prices, P_{nt} and P_{nt+1}, for n = 1, 2 and 3; and money transfers s_{kt}.

$$\text{Max}_{L_{kt},\, M_{1t}^{dk},\, M_{st}^{dk},\, M_{3t}^{dk}} \; -L_{kt} + \ln C_{kt+1} \tag{3}$$

subject to

$$\frac{M_{1t}^{dk}}{P_{1t}} + \frac{M_{2t}^{dk}}{P_{2t}} + \frac{M_{3t}^{dk}}{P_{3t}} = \frac{W_{kt} \cdot L_{kt}}{P_{kt}} \tag{4}$$

$$\frac{M_{1t}^{dk}}{P_{1t}} + \frac{M_{2t}^{dk}}{P_{2t}} + \frac{M_{3t}^{dk}}{P_{3t}} + \frac{S_{kt}}{P_{kt+1}} = C_{kt+1} \tag{5}$$

$$0 \leq L_{kt}, \quad 0 < C_{kt+1} \tag{6}$$

where

C_{kt+1} = (country k) period-t young generation's consumption in t+1 when they become the old

w_{kt} = nominal wage of country-k labor in t, expressed in currency of country k

P_{nj} = price of consumption goos in period j, j = t, t+1, expressed in currency of country n, n = 1,2,3.

M_{nt}^{dk} = young generation of country k's demand for currency of country n, in period t; n = 1,2,3.

Market clearing conditions are

$$\text{Labor: } \frac{W_{nt}}{P_{nt}} = \boldsymbol{\theta}_{nt} \tag{7}$$

$$\text{Money:} M_{nt}^{d1} + M_{nt}^{d2} + M_{nt}^{d3} = M_{nt} \tag{8}$$

$$\text{Goods: } \boldsymbol{\theta}_{1t} \cdot L_{1t} + \boldsymbol{\theta}_{2t} \cdot L_{2t} + \boldsymbol{\theta}_{3t} \cdot L_{3t} = C_{1t} + C_{2t} + C_{3t} \tag{9}$$

where M_{nt} = money supply of country n in period t for n = 1,2,3; and t = 1,2, $\cdots$, ∞.

The necessary and sufficient conditions for the optimality, of the above decision problem (3); which is for young generation of country k; are

$$\begin{aligned}
\frac{1}{(W_{kt}/P_{kt})} &= \left[\frac{M_{1t}^{dk}}{P_{1t}} + \frac{M_{2t}^{dk}}{P_{2t}} + \frac{M_{3t}^{dk}}{P_{3t}} + \frac{S_{kt}}{P_{kt+1}}\right]^{-1} \cdot \frac{P_{1t}}{P_{1t+1}} \\
&= \left[\frac{M_{1t}^{dk}}{P_{1t}} + \frac{M_{2t}^{dk}}{P_{2t}} + \frac{M_{3t}^{dk}}{P_{3t}} + \frac{S_{kt}}{P_{kt+1}}\right]^{-1} \cdot \frac{P_{2t}}{P_{2t+1}} \\
&= \left[\frac{M_{1t}^{dk}}{P_{1t}} + \frac{M_{2t}^{dk}}{P_{2t}} + \frac{M_{3t}^{dk}}{P_{3t}} + \frac{S_{kt}}{P_{kt+1}}\right]^{-1} \cdot \frac{P_{3t}}{P_{3t+1}}
\end{aligned} \tag{10}$$

which is the marginal condition between leisure and consumption; when wage incomes can be transformed into consumption, via either currency of country 1, 2, and 3.

Period-t exchange-rate between currency of country k and country q, e_{kqt}, is defined as p_{kt}/p_{qt}. From the optimality condition (10),

$$\frac{P_{1t}}{P_{1t+1}} = \frac{P_{1t}}{P_{2t+1}} = \frac{P_{1t}}{P_{3t+1}} \tag{11}$$

which can be rewritten as

$$\frac{P_{kt}}{P_{qt}} = \frac{P_{kt+1}}{P_{qt+1}} \tag{12}$$

for $k=1, 2, 3;\ q=1, 2, 3;$ and $k \neq q$

If agents are subject to no portfolio restrictions, then in equilibrium $e_{kqt}=e_{kqt+1}$ $(=e_{kqt+j})$. Equilibrium exchange rates are constant over time. There cannot be an anticipated change in the equilibrium exchange rate. Currencies are perfect substitutes. In equilibrium, returns from holding money balances are equal for every currency.

Portfolio compositions are such that; the post-transfer portfolio weights are identical for agents of each country.

That is,

$$(M_{1t}^{d1}+s_{1t}) : M_{2t}^{d1}:M_{3t}^{d1}$$
$$=M_{1t}^{d2} : (M_{2t}^{d2}+S_{2t}) : M_{3t}^{d2}$$
$$=M_{1t}^{d3} : M_{2t}^{d3} : (M_{3t}^{d3}+S_{3t})$$
$$=M_{1t} : M_{2t} : M_{3t}$$

where : denotes ratio. $(x : y=x/y)$

Suppose that either productivity shocks, or new information on the future productivity shocks, arrive in period $t+1$. Then, governments will adjust their monetary policies to those shocks; and the equilibrium exchange rates will change. There is extremely small (close to zero) probability that shocks could arrive in period $(t+1)$. Shocks of each country are equal; but less-than-perfectly correlated (Assumption 5). In order to diversify the exchange-rates risks - unanticipated changes in the equilibrium exchange-rates, agents hold diversified portfolio. By holding portfolio as expressed in (13); agent k's consumption share, $(C_{kt+1}/Y^*_{t+1};Y^*_{t+1}=$ the world output in $_{t+1})$, is invariant, to the unanticipated changes in the equilibrium exchange-rate of $(t+1)$.

The optimality condition (10) can be rewritten as; using the budget constraint (4) and (5); money clearing condition (8); and the exchange-rate equality (11),

$$\frac{1}{(W_{kt}/P_{kt})} = [C_{kt}+1]^{-1} \cdot \frac{P_{nt}}{P_{nt+1}} \tag{14}$$

where

$$C_{kt+1} = \left[\frac{M_{1t+1}}{P_{1t+1}} + \frac{M_{2t+1}}{P_{2t+1}} + \frac{M_{3t+1}}{P_{3t+1}}\right] \cdot \frac{\left[\frac{W_{kt} \cdot L_{kt}+S_{kt}}{P_{kt}}\right]}{\left[\frac{M_{1t+1}}{P_{1t}} \quad \frac{M_{2t+1}}{P_{2t}} \quad \frac{M_{3+1}}{P_{3t}}\right]}$$

for k=1, 2, 3;n=1, 2, 3.

$(M_{1t+1}/P_{1t+1}+M_{2t+1}/P_{2t+1}+Mct+1/P_{3t+1})$ is the world output, Y^{*}_{t+1}, in period (t+1). From (14), labor supply is derived as function of wage, prices, and money transfers.

$$L_{kt} = \frac{1}{W_{kt}/P_{kt}} \cdot \left[A_{kt} - \frac{S_{kt}}{P_{kt}}\right] \tag{15}$$

where $A_{kt}=$

$$\left[\frac{M_{1t+1}}{P_{1t+1}} + \frac{M_{2t+1}}{P_{2t+1}} + \frac{M_{3t+1}}{P_{3t+1}}\right] \cdot \frac{^{-1}W_{kt}}{P_{kt+1}} \cdot \left[\frac{M_{1t+1}}{P_{1t+1}} + \frac{M_{2t+1}}{P_{2t+1}} + \frac{M_{3t+1}}{P_{3t+1}}\right]$$

for k=1, 2, 3.

Using the exchange-rate equality (11); labor supply (15) becomes, simply,

$$L_{kt} = 1 - \frac{S_{kt}}{W_k} \tag{16}$$

for k=1,2,3.

P_{nt+1}'s do not appear in (16), since the utility function for consumption is ln C.

In the optimality condition (14), country-k agent's share of consumption, which we define as δ_{kt+1}, is

$$\delta_{kt+1}=\frac{\left[\dfrac{W_{kt}\cdot L_{kt}+S_{kt}}{P_{kt}}\right]}{\left[\dfrac{M_{1t+1}}{P_{1t}}+\dfrac{M_{2t+1}}{P_{2t}}+\dfrac{M_{3+1}}{P_{3t}}\right]} \tag{17}$$

for k=1,2,3.

By the budget constraints (4) and (5); and the money market clearing condition (8),

$$\delta_{1t+1}+\delta_{2t+1}+\delta_{3t+1}=1 \tag{18}$$

Substituting (16) for labor supply in (17); using $w_{kt}=P_{kt}\cdot\theta_{kt}$, and (18);$\delta_{kt+1}$ is simply,

$$\delta_{kt+1}=\frac{\theta_{kt}}{\theta^*_t} \tag{19}$$

where $\theta^*_t=\theta_{1t}+\theta_{2t}+\theta_{3t}$;k=1,2,3.

(19) shows that country-k agent's consumption share, δ_{kt+1} ($\equiv C_{kt+1}/Y^*_{t+1}$; Y^*_{t+1}=world output in t+1), is <u>independent</u> of money transfer s_{kt}. The reasons for this simple result are : in our model, production function is linear in labor; and utility is linear in leisure ($-L_{kt}$), too. Therefore in the optimality condition (14), the left-hand side of equality is a given 'constant' for agent of country k. Marginal utility of leisure is 1; and its price is θ_{kt} (=w_{kt}/p_{kt}). if the government of country k changes its money transfers S_{kt}; agent will adjust labor supply L_{kt}, to the level where the optimality condition (14) is satisfied. In the optimality condition (14), consumption share δ_{kt+1} is linear in θ_{kt}. Combining with (18), we obtain (19) : Consumption share is determined by the productivities. In sum, when governments change monetary policies, agents <u>do</u> adjust their labor supplies. However, consumption shares, per se, remain unchanged; due to the linearity in production and utility (of leisure) functions.

Money demands are derived as function of productivities : from the portfolio equation (13), and the expression for consumption share (19).

$M_{kt}^{dk}=M_{kt+1}\cdot(\theta_{kt}/\theta^*_t)-S_{kt}$
$M_{qt}^{dk}=M_{qt+1}\cdot(\theta_{qt}/\theta^*_t)$
for k=1, 2, 3; q=1, 2, 3; q≠k

As shown in Kareken and Wallace (1981), in an OLG model with fiat monies, equilibrium exchange rate is indeterminate. For our research, the indeterminacy problem is solved as follows. Define μ_{kt} as $\mu_{kt} \equiv (M_{kt}+S_{kt})/M_{kt}$; which is the (gross) growth rate of country-k money supply, between period t and t+1. Without loss of generality, the first-period money supplies are normalized such that $M_{11}=M_{21}=M_{31}$. Suppose both governments maintain constant money growth rates μ for all t, $t=1,2,\cdots,\infty$. That is, $\mu_{1t}=\mu_{2t}=\mu_{3t}=\mu$ for all t. Then we fix the equilibrium exchange rate as 1. That is, $e_{kq}=1$ for $k=1, 2, 3$; $q=1, 2, 3$; $q\neq k$; where e_{kq} is the price of currency of country q, in terms of currency k. (Equilibrium exchange rates are constant over time by (12). $e_{kq}=p_{k1}/p_{q1}=p_{kt}/p_{qt}=p_{kt+i}/p_{qt+i}$; $i=1, 2, \cdots, \infty$. Suppose governments change their monetary policies, and choose the suquence of money growth rates $\{\mu_{kt}\}$, for $k=1, 2, 3$: $t=1, 2, 3$: $t=1, 2, \cdots, \infty$. Then, by the neutrality of money, the equilibrium exchange-rate e_{kq}, will be

$$e_{kq}=\frac{\bigcap_{t=1}\mu_{kt}}{\bigcap_{t=1}\mu_{qt}} \tag{20}$$

where $\bigcap_{t=1}\mu kt \equiv \mu k1\cdot\mu k2\cdot\cdot\cdot\mu kt\cdot\cdot\cdot\mu k\infty$ for $k=1,2,3$.

From the budget constraints (4) and (5); and the goods and money market clearing conditions (8) and (9); we obtain,

$$\frac{M_{1t}}{P_{1t}}+\frac{M_{2t}}{P_{2t}}+\frac{M_{3t}}{P_{3t}}=\theta_{1t}\cdot L_{1t}+\theta_{2t}\cdot L_{2t}+\theta_{3t}\cdot L_{3t} \tag{21}$$

Goods trading is achieved via money balances; with money velocity of one; for every period. We can derive the exchange-rate equation (20) by mathematical induction.

Finally, equilibrium labor supply, output, prices and wages are derived as function of money growth rates. Using the equalities;$s_{kt}=M_{kt}\cdot(\mu_{kt}-1)$ and $w_{kt}=p_{kt}\cdot\theta_{kt}$; labor supply (16), can be rewritten as

$$L_{kt}=1-\frac{M_{kt}}{\theta_{kt}\cdot P_{kt}}\cdot(\mu kt-1) \tag{22}$$

for $k=1, 2, 3$.

Substituting (22) for L_{kt} in (21), we get

$$\frac{M_{1t}}{P_{1t}}\cdot\mu_{1t}+\frac{M_{2t}}{P_{2t}}\cdot\mu_{2t}+\frac{M_{3t}}{P_{3t}}\cdot\mu_{3t}=\theta^{*}_{t} \tag{23}$$

where $\theta^*_t = \theta_{1t} + \theta_{2t} + \theta_{3t}$.

From the exchange rate equation (20)

$$p_{kt} = e_{kq} \cdot p_{qt} = \frac{\bigcap_{j=1} \mu_{kj}}{\bigcap_{j=1} \mu_{qj}} \cdot p_{qt} \tag{24}$$

for k=1,2,3; q=1,2,3; q≠k.

From (23) and (24), equilibrium prices $\{p_{kt}\}$, k=1,2,3; t=1,2, · · · , ∞, are derived as function of funture moneey growth rates.

$$\begin{aligned} p_{1t} &= \frac{\Omega}{\theta^*_t \cdot (\bigcap_{j=t} \mu_{2j}) \cdot (\bigcap_{j=t} \mu_{3j})} \cdot M_{1t} \\ p_{2t} &= \frac{\Omega}{\theta^*_t \cdot (\bigcap_{j=t} \mu_{3j}) \cdot (\bigcap_{j=t} \mu_{1j})} \cdot M_{2t} \\ p_{3t} &= \frac{\Omega}{\theta^*_t \cdot (\bigcap_{j=t} \mu_{1j}) \cdot (\bigcap_{j=t} \mu_{2j})} \cdot M_{3t} \end{aligned} \tag{25}$$

where $\Omega \equiv$

$(\bigcap_{j=t}\mu_{2j})(\bigcap_{j=t}\mu_{3j})\mu_{1t} + (\bigcap_{j=t}\mu_{3j})(\bigcap_{j=t}\mu_{1j})\mu_{2t} + (\bigcap_{j=t}\mu_{1j})(\bigcap_{j=t}\mu_{2j})\mu_{3t}$

for t=1,2, · · ·, ∞; $\theta^*_t \equiv \theta_{1t} + \theta_{2t} + \theta_{3t}$.

Labor supplies $\{L_{kt}\}$, k=1,2,3; t=1,2, · · ·, ∞, are derived as function of future money growth rates; by substituting (25) for M_{kt}/p_{kt} in labor supply functions (22).

$$\begin{aligned} L_{1t} &= 1 - \frac{\theta^*_t}{\theta_{1t}} \cdot \frac{(\bigcap_{j=t} \mu_{2j}) \cdot (\bigcap_{j=t} \mu_{3j}) \cdot (\mu_{1t} - 1)}{\Omega} \\ L_{1t} &= 1 - \frac{\theta^*_t}{\theta_{2t}} \cdot \frac{(\bigcap_{j=t} \mu_{3j}) \cdot (\bigcap_{j=t} \mu_{1j}) \cdot (\mu_{2t} - 1)}{\Omega} \\ L_{1t} &= 1 - \frac{\theta^*_t}{\theta_{3t}} \cdot \frac{(\bigcap_{j=t} \mu_{1j}) \cdot (\bigcap_{j=t} \mu_{2j}) \cdot (\mu_{3t} - 1)}{\Omega} \end{aligned} \tag{26}$$

where Ω is defined in (25),

for t=1, 2, · · ·, ∞; $\theta^*_t = \theta_{1t} + \theta_{2t} + \theta_{3t}$.

World outputs, $\{Y^*_t\}$, are derived as function of funture money growth rates; by substituting

(26) for labor supplies in $Y^*_t = \theta_{1t} \cdot L_{1t} + \theta_{2t} \cdot L_{2t} + \theta_{3t} \cdot L_{3t}$.

$$Y^*_t = \frac{\theta^*_t[(\bigcap_{j=t} \mu_{1j})(\bigcap_{j=t} \mu_{2j})(\bigcap_{j=t} \mu_{2j})(\bigcap_{j=t} \mu_{3j})(\bigcap_{j=t} \mu_{3j})(\bigcap_{j=t} \mu]_{1j})}{\Omega} \tag{27}$$

where Ω is defined in (25),

for $t=1,2,\cdots,\infty$; $\theta^*_t = \theta_{1t} + \theta_{2t} + \theta_{3t}$.

2. Steady-State Equilibrium

The present section studies the policy choices in a steady-state. We show that Nash partial-coordination equilibrium does not exist. However, we propose that in a steady-state : if coordinating countries adhere to their optimal money growth rates : then, non-coordinating country will choose the same money growth rates. As more countries join policy coordination : countries choose less-inflationary policies; output increases; welfare improves.

2.1. Government objective Functions

Country 1 chooses its monetary policy independently, maximizing the utility of its representative agents. Country 2 and 3 choose monetary policies cooperatively to maximize the sum of utilities; of their representative agents. Productivities of all countries are the same and constant for every period; i.e., $\theta_{1t} = \theta_{2t} = \theta_{3t} = \theta$ for all t, $t=1,2,\cdots,\infty$.

Throughout this chapter, we find out <u>time-consistent</u> policies. First, we present the objective function of country-1 government. Government of country 1 in period t, chooses money growth rates $\{\mu_{1t+i}\}$, $i=0,1,\cdots,\infty$, maximizing a discounted sum of utilities; of present and future generations. Ex-ante optimization of period-t government is :

$$\begin{aligned} &\text{Max}_{\{\mu_{1t+i}\}} \quad \Sigma_{i=0,1,\cdots,\infty}\, \sigma^i \cdot (-L_{1t+i} + \ln C_{1t+i}) \\ &\text{subject to } 0 < \mu_{1t+i} \\ &\qquad\qquad 0 \le L_{1t+i} \\ &\qquad\qquad 0 < C_{1t+i} \\ &\text{where } \sigma = \text{time preference, } 0 < \sigma < 1 \end{aligned} \tag{28}$$

Labor supplies $\{L_{1t+i}\}$, for $i=1,2,\cdots,\infty$, are given in (26). (θ^*_t/θ_{1t}) in (26) is replaced with 3; since we assumed that $\theta_{1t} = \theta_{2t} = \theta_{3t} = \theta$ for all t, $t=1,2,\cdots,\infty$. $(\theta^*_t = \theta_{1t} + \theta_{2t} + \theta$

$_{3t}=3\theta$) We have shown in Section 1 above, that consumption share δ_{1t+i}, ($\delta_{1t+i}=C_{1t+i}/Y^*_{t+i}$; Y^*_{t+i} =world output in t+i) does <u>not</u> depend on the monetary policy. We obtained in (28), that $\delta_{1t+i}=C_{1t+i}/Y^*_{t+i}=\theta_{1t+i-1}/\theta^*_{t+i-1}$. Consumption share is determined by the productivities of the previous period. Since $(\theta_{1t+i-1}/\theta^*_{t+i-1})=1/3$ in a steady-state,

$$C_{1t+i}=\frac{1}{3}\cdot Y^*_{t+i} \qquad (29)$$

for i =0,1, ···, ∞

Detailed explanations for this result are provided in Section 1 (below (18)). We obtain this result since (i) utility functions and production functions are homogeneous across countries; and (•¢) utility function is linear in leisure (−-L); and production function is linear in labor. When young generation of period (t+i−1) makes labor supply decision; both marginal utility of leisure and real wage, do not depend on labor supply. Marginal utility of leisure is 1; and real wage is θ_{1t+i-1}. For a given monetary policy, labor supply is chosen at the level which will make their consumption in period (t+i) to be $(\theta_{1t+i-1}/\theta^*_{t+i-1})Y^*_{t+i}$; which is a third of Y^*_{t+i} in a steady-state. The objective function in (28)−ex-ante optimization of period-t government, can be rewritten as :

$$\text{Max}_{\{\mu_{1t+i}\}}\ \Sigma_{i=0,1,\cdots,\infty}\ \sigma^i\cdot[-L_{1t+i}+\ln(\frac{1}{3}\cdot Y^*_{t+i})]$$

$$\text{subject to } 0<\mu_{1t+i} \qquad (30)$$

$$0\leq L_{1t+i}$$

$$0<\frac{1}{3}\cdot Y^*_{t+i}$$

Ex-ante optimization problem in period t, for coordinating countries, countries 2 and 3, are : to choose $\{\mu_{2t+i}\}$, $\{\mu_{3t+i}\}$, maximizing the sum of utilities of present and future generations. $C_{2t+i}=C_{3t+i}=(1/3)\cdot Y^*_{t+i}$ by the same reasons explained above.

$$\text{Max}_{\{\mu_{2t+i}\},\{\mu_{3t+i}\}}\ \Sigma_{i=0,1,\cdots,\infty}\ \sigma^i\cdot\begin{bmatrix}-L_{2t+i}+\ln(\frac{1}{3}\cdot Y^*_{t+i})\\ -L_{3t+i}+\ln(\frac{1}{3}\cdot Y^*_{t+i})\end{bmatrix} \qquad (31)$$

subject to $0<\mu_{2t+i}$, μ_{3t+i}

$0 \leq \mu_{2t+i},\ \mu_{3t+i}$

$0 < C_{1t+i}$

$$0 < \frac{1}{3} \cdot Y^{*}_{t+i}$$

for $i=0,1, \cdots, \infty$; σ=time preference, $0<\sigma<1$.

Labor supply L_{ct+i}, $c=1,2$, and world outputs Y^{*}_{t+i} are as expressed in (26) and (27); replacing $(\theta^{*}_{t+i}/\theta_{ct+i})$ with 3.

2.2. Partial-Coordination Equilibrium : Steady-State

In a time-consistent policy, steady-state money growth rates of a given country is constant over time. For currencies of each country to co-exist in a steady-state; steady-state money growth rates of every country should be equal. A country can not maintain a higher money growth rates forever; than the other. Suppose that steady-state money growth rate of country 1, μ_1, is higher than that of coordinating countries, μ_c, $c=2,3$. If $\mu_1 > \mu_c$, then, by the exchange-rate equation (20);

$$e_{1c} = \frac{(\mu_1)^{\infty}}{(\mu_1)^{\infty}} = \infty$$

The value (in consumption good) of currency of country 1 is zero. ($p_1=\infty$) Money demand for currency of country 1 is zero. In a Partial-Coordination situation, losing the authority of issung its currency is never optimal; for any country. Therefore, Maintaining an equal money growth rates in a steady-state, is an optimality condition for every country. That is,

$$\mu_1 = \mu_c \tag{32}$$

for $c=2,3$.

[Non-existence of Nash Partial-Coordination Equilibrium : Steady-State (NPCES)]

We prove the non-existence of NPCES as follows : For NPCES to exist, countries should maintain equal steady-state money gorwth rates with each other. Suppose that both country 1 and coordinating countries c, $c=2,3$, maintain equal money growth rates μ, for period $(t+1)$, $(t+2)$, $\cdots, \infty$. Then, in a time-consistent policy; governments of period t, choose $\mu 1t$ and μct; maximizing their objective functions given above; with the conditions that $\mu_{kt+i}=\mu$ for all i, $i=1,2, \cdots, \infty$, and $k=1,2,3$. We show that reaction function of country 1, and that of coordinating countries, do not cross with each other : which proves that NPCES does not exist.

Periodt government of country 1 chooses μ_{1t}; given (μ_{2t}, μ_{3t}), and $\mu_{kt+i}=\mu$ for i=1,2, · · ·, ∞; k=1,2,3. In a time-consistent policy, objective function of ex-ante optimization problem (30) becomes,

$$\underset{\mu_{1t}}{\text{Max}} \quad -L_{1t}+\ln\left(\frac{1}{3}\cdot Y^{*}_{t}\right) \tag{33}$$

subject to $0<\mu_{1t}$

$0\leq L_{1t}$

$0<\frac{1}{3}\cdot Y^{*}_{t}$

$$\text{where } L_{1t}=\frac{1}{\mu_{1t}} \tag{34}$$

$$Y^{*}_{t}=\theta^{*}t\cdot\left[\frac{1}{3}\cdot\left(\frac{1}{\mu_{1t}}+\frac{1}{\mu_{2t}}+\frac{1}{\mu_{3t}}\right)\right] \tag{35}$$

(34) is obtained from labor supply function (26);(35) is obtained from Y^{*}_{t} of (27) : using the conditions that $\theta^{*}_{t}/\theta_{1t}=3$ and $\mu_{kt+i}=\mu$ for c=1, 2, · · ·, ∞; k=1, 2, 3.

The necessary and sufficient condition for the optimality is

$$\frac{\partial U_{1t}}{\partial U_{1t}} : 0=\frac{1}{\underset{\mu_{1t}}{2}}+\left[\frac{\mu_{2t}+\mu_{3t}}{\mu_{1t}\cdot\mu_{2t}+\mu_{2t}\cdot\mu_{3t}+\mu_{3t}\cdot\mu_{1t}}-\frac{1}{\mu_{1t}}\right] \tag{36}$$

Period-t governments of coordinating countries choose (μ_{2t}, μ_{3t}); given μ_{1t} and $\mu_{kt+i}=\mu$ for i= 1, 2, · · ·, ∞; k=1, 2, 3. In a time-consistent policy, objective function of ex-ante optimization problem (31) becomes

$$\underset{\substack{\mu_{2t}\\ \mu_{3t}}}{\text{Max}} \begin{bmatrix} -L_{2t}+\ln\left(\frac{1}{3}\cdot Y^{*}_{t}\right) \\ -L_{3t}+\ln\left(\frac{1}{3}\cdot Y^{*}_{t}\right) \end{bmatrix} \tag{37}$$

subject to $0<\mu_{2t}, \mu_{3t}$

$0\leq L_{2t}, L_{3t}$

$0<\frac{1}{3}\cdot Y^{*}_{t}$

$$\text{where } L_{ct} = \frac{1}{\mu_{ct}} \text{ for } c=2, 3 \tag{38}$$

Y^{*}_{t} is the same as above in (35). (38) is obtained from labor supply function (26); using the conditions that $\theta^{*}_{t}/\theta_{2t} = \theta^{*}_{t}/\theta_{3t} = 3$ and $\mu_{kt+i} = \mu$ for $i=1,2, \cdots, \infty$; $k=1,2,3$.

The necessary and sufficient condition for the optimality are :

$$\frac{\partial U_{ct}}{\partial \mu_{2t}} : 0 = \frac{1}{2\mu_{2t}^2} + 2\left[\frac{\mu_{1t} + \mu_{2t}}{\mu_{1t} \cdot \mu_{2t} + \mu_{2t} \cdot \mu_{3t} + \mu_{3t} \cdot \mu_{1t}} - \frac{1}{\mu_{2t}}\right] \tag{39}$$

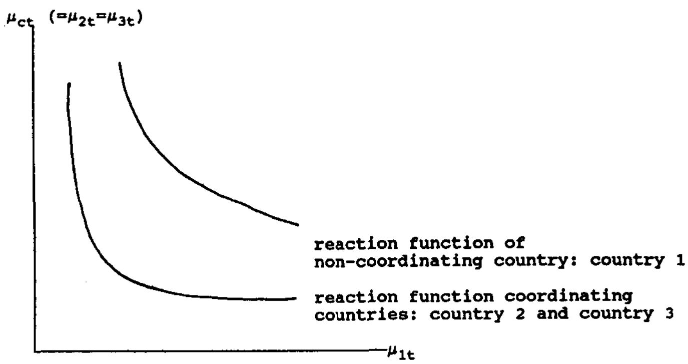
$$\frac{\partial U_{ct}}{\partial \mu_{3t}} : 0 = \frac{1}{2\mu_{3t}^2} + 2\left[\frac{\mu_{1t} + \mu_{2t}}{\mu_{1t} \cdot \mu_{2t} + \mu_{2t} \cdot \mu_{3t} + \mu_{3t} \cdot \mu_{1t}} - \frac{1}{\mu_{3t}}\right]$$

Given μ_{1t}, optimal money growth rates of coordinating countries, μ_{2t} and μ_{3t}, are the same. Define them as μ_{ct} : $\mu_{ct} \equiv \mu_{2t} = \mu_{3t}$. Using the feasibility constraints that $\mu_{kt} > 0$ for all k, k=1, 2, 3 : reaction functions (36) and (39) are simplified. The reaction function of country 1, (36), becomes :

$$\mu_{ct} = \frac{2 \cdot \mu_{1t}}{\mu_{1t-1}} \tag{40}$$

The reaction function of coordinating countries, (39) becomes :

$$\mu_{ct} = \frac{2 \cdot \mu_{1t}}{2 \cdot \mu_{1t-1}} \tag{41}$$



[Reaction Functions]

[PROPOSITION]

Steady-state Nash partial-coordination equilibrium does not exist. Coordinating countries want to choose less inflationary policy; while non-coordinating countries want to choose more inflationary policy. However, if coordinating countries adhere to their own optimal money growth rates; then, non-coordinating countries will follow. Steady-state utility levels of coordinating and non-coordinating countries are the same. As more countries join monetary policy coordination : inflation rates go down; world output increases; and welfare improves.

Let us compare the optimality condition of non-coordinating country, (36); and that of coordinating countries, (39). For both equations, the first term appearing on the right-hand side of equality is, marginal effects of increasing money growth rates on leisure; the second term is marginal effects on consumption.

At any given μ, $\mu=\mu_{1t}=\mu_{2t}=\mu_{3t}$ (>0);marginal benefits (on utility of leisure) of increasing money growth rates are the same for both coordinating and non-coordinating countries. But, marginal costs (on utility of consumption) are larger (twice) for coordinating countries. Coordinating countries want to choose less inflationary policies; while non-coordinating countries want to choose more inflationary policies. Reaction functions do not cross with each other. Marginal negative effects of enjoying one more unit of leisure, on the world output, are the same for every country. Agents of coordinating countries consume larger portion of world output : they prefer less-inflationary, more-output world.

In a steady-state, every country should maintain the same money growth rates. No country can maintain higher growth rates forever; than the rest of the world. Value of its currency goes to zero. If coordinating countries adhere to their optimal growth rates (1.5 in the diagram); non-coordinating countries should also maintain the same growth rates. However, non-coordinating countries has always incentives to raise money growth rates (above from 1.5). Once in a while, non-coordinating countries can choose higher money growth rates. In that sense, we should accept the proposition with caution. For the same reasons explained above, as more countries join coordination; countries will choose less inflationary policy; world output increases; and welfare improves.

3. Adjustment Policies under Unanticipated Shocks : Partial Coordination

We assume that productivity shocks for coordinating countries are equal : $\theta_{2t}=\theta_{3t}=\theta_{ct}$. Unanticipated productivity shocks (θ_{1t}, θ_{ct}) arrive in the economy, in period t. Shocks are transitionary. Economy has been in a steady-state up to period (t−1); will return to steady-state from period (t+1).

Nash partial-coordination equilibrium does not exist. We show that when the differences in the productivities are large, "leader"-"follower" relations of Stackelberg solution is determined. When $\theta_{1t} \gg \theta_{ct}$, country 1 is better off by becoming the follower; while coordinating countries prefer to play the game as leader. When $\theta_{1t} \ll \theta_{ct}$, the reverse is the solution. Country 1 prefers to be the leader; while coordinating countries are better off by becoming the follower. When the differences in the productivities are not large, stackelberg warfare is the result. Both coordinating countries and non-coordinating countries want to be the leader.

[A] Stackelberg Solution : Non-Coordinating Country - "Follower"; Coordinating Countries - "Leader"

Government of country-1 - "follower" chooses money growth rates, μ_{1t}, maximizing the utility of representative agents; given the coordinating countries are choosing μ_{ct}.

$$\max_{\mu_{1t}} \quad -L_{1t} + \ln\left(\frac{1}{3} \cdot Y^*_t\right) \tag{42}$$

subject to $0 < \mu_{1t}$

$0 \leq L_{1t}$

$0 < \frac{1}{3} \cdot Y^*_t$

$$\text{where } L_{1t} = 1 - \frac{\theta^*_t}{3 \cdot \theta_{1t}} \cdot \left[1 - \frac{1}{\mu_{1t}}\right] \tag{43}$$

$$Y^*_t = \theta^*_t \cdot \left[\frac{2 \cdot \mu_{1t} + \mu_{ct}}{3\mu_{1t} \cdot \mu_{ct}}\right] \tag{44}$$

(43) is obtained from the labor supply function (26); using the conditions that $\mu_{kt+i} = \mu$ for k= 1, 2, 3; i=1,2, $\cdots$, ∞. (44) is obtained from the world output function (27); using the same conditions. (The reasons for choosing (42) as an objective function of government are explained in Section 2.1) The solution of the above optimization problem; of the follower, is :

$$\mu 1t = \frac{(\theta^*_t / \theta_{1t})\mu_{1t}}{\mu_{ct} - 2 \cdot (\theta^*_t / \theta_{1t})} \tag{45}$$

Government of coordinating countries — "leader" choose money growth rates, μ_{ct}, maximizing the utility of representative agents; knowing the choice of government of country 1 — "follower", which is (45).

$$\max_{\mu_{ct}} \quad -L_{2t}+\ln\left(\frac{1}{3}\cdot Y^*_t\right) -L_{3t}+\ln\left(\frac{1}{3}\cdot Y^*_t\right) \tag{46}$$

subject to $0<\mu_{ct}$

$0\leq L_{2t}, L_{3t}$

$0<\frac{1}{3}\cdot Y^*_t$

$$\text{where } L_{ct}=1=-\frac{\theta^*_t}{3\cdot\theta_{ct}}\cdot\left[1-\frac{1}{\mu_{ct}}\right] \tag{47}$$

$$Y^*_t=\theta_{1t} \tag{48}$$

(47) is obtained from the labor supply function (26); using the conditions that $\mu_{kt+i}=\mu$ for $k=1, 2, 3; i=1, 2, \cdots, \infty$. (48) is obtained from the world output function (27); using the same conditions; and substituting the choice of the follower, (45), for μ_{1t}.

[Stackelberg Partial-Coordination : Country 1 – "Follower"; Coordinating Countries – "Leader"]

(i) When $\theta_{1t}<\theta_{ct}$

	Country 1	corrdinating countries
μ_t	$\frac{\theta^*_t}{3\cdot\theta_{1t}}$ (>1)	∞
L_t	$2-\frac{\theta^*_t}{3\cdot\theta_{1t}}$ (<1)	$1-\frac{\theta^*_t}{3\cdot\theta_{ct}}$
U_t	$-2+\frac{\theta^*_t}{3\cdot\theta_{1t}}+\ln\frac{\theta_{1t}}{3}$	$-1+\frac{\theta^*_t}{3\cdot\theta_{ct}}+\ln\frac{\theta_{1t}}{3}$

(ii) When $\theta_{1t}>\theta_{ct}$

μ_t	1	$\frac{\frac{\theta^*_t}{3\cdot\theta_{1t}}}{\quad}$

		$[\frac{\theta^*_t}{3\cdot\theta_{1t}} -1]$
L_t	1	0
U_t	$-1+\ln\frac{\theta_{1t}}{3}$	$\ln\frac{\theta_{1t}}{3}$

$Y^*_t=\theta_{1t}$ for both cases of (i) and (ii).

World output is constant at θ_{1t}—(48). Whatever level of money growth rates do the coordinating countries (leader) choose, country 1 (follwoer) chooses its money growth rates at the level which will make the world output constant. Knowing that; coordinating countries choose hyperinflationary policy. Coordinating countries supply very little labor compared to the non-coordinating country. Utility level of the leader is higher than that of the follower.

[B] Stackleberg Solution : Non-Coordinating Country— "Leader"; Coordinating Countries— "Follower"

Since all the details are similar to Section [A], we describe the steps and the results only. Coordinating countries solve (46) above as a follower, choosing μ_{ct}.

Y^*_t of (48) is replaced with Y^*_t in (44). We obtain

$$\mu_{ct}=\frac{2\cdot(\theta^*_t/\theta_{1t})\mu_{1t}}{2\cdot\mu_{1t}-(\theta^*_t/\theta_{1t})} \tag{49}$$

Substituting (49) for μ_{ct} in the world output function (44),

$$Y^*_t=2\cdot\theta_{ct} \tag{50}$$

Country 1 (leader) solves (42) with Y^*_t given in (50).

[Stackelberg Partial Coordination : Country 1—Leader; Coordinating Countries—Follower]

(i) When $\theta_{1t}<\theta_{ct}$

	Country 1	corrdinating countries
	$\frac{\theta^*_t}{}$	

μ_t	$\frac{3\cdot\theta_{1t}}{[\frac{\theta^*_t}{3\cdot\theta_{1t}}-1]}$	1
L_t	0	1
U_t	$\ln\frac{2\cdot\theta_{ct}}{3}$	$-1+\ln\frac{2\cdot\theta_{ct}}{3}$

(ii) When $\theta_{1t}>\theta_{ct}$

μ_t	∞	$\frac{\theta^*_t}{3\cdot\theta_{ct}}$ (>1)
L_t	$1-\frac{\theta^*_t}{3\cdot\theta_{1t}}$	$2-\frac{\theta^*_t}{3\cdot\theta_{ct}}$
	1	0
U_t	$-1+\frac{\theta_{1t}}{3\cdot\theta_{ct}}+\ln\frac{2\cdot\theta_{ct}}{3}$	$-1+\frac{\theta_{1t}}{2\cdot\theta_{ct}}+\ln\frac{2\cdot\theta_{1t}}{3}$

The results are reverse of Section [A]. Country 1 Chooses hyper-inflationary policy; supplying very little labor, compared to the coordinating countries. World output is $2\cdot\theta_{ct}$: which suggests that when the differences between the productivities are not large; world output shrinks more when the coordinating countries play the Stackelberg game as the "leader".

If $\theta^*_t/(3\cdot\theta_{ct})+\ln(\theta_{1t}/3)<\ln(2\cdot\theta_{ct}/3)$, $(\theta_{ct}\gg\theta_{1t})$,

then coordinating countries want to be the follower. Opportunity costs of leisure are high. If they take the leader's role; they can enjoy more leisure; buy their consumption is low. By becoming the follower, they work more. But their utility gains from larger consumption dominate the utility losses from less leisure. If $\ln(\theta_{1t}/3)>-2+\theta^*_t/(3\cdot\theta_{ct})+\ln(2\cdot\theta_{ct}/3)$, $(\theta_{ct}\ll\theta_{1t})$; then by the same reasons; country 1 wants to be the follower.

Suppose there are n countries in the world (n : large).

(n-1) countries are coordinating their policies; while the last one country is choosing its policy independently. The results above suggest that (n-1) countries would, more often, want to be the follower : providing incentives for the last one country not joining the coordination.

FOOTNOTE

1. The pioneering papers on international policy coordinations are of Hamada(1976, 1979). Recent contributions in monetary policy coordination investigate the nature of inefficiencies associated with the non-cooperative policy-game and the benefits of intenational policy coordination [Turnvosky, Basar and d'Orey(1988); Oudiz and Sachs(1985);Canzoneri and Gray(1985);Currie and Levine(1985);Miller and Salmon(1985); Carlozzi and Taylor(1985)]. Rogoff(1985) and van der Ploeg(1988) demonstrate that international moneary coordination may be counter productive.

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