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THE WAVELENGTH OF GRAVITATIONAL WAVES PRODEUCED BY EXTENDED INFLATION

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ABSTRACT

In contrast to conventional belief that extended inflation ends when the Universe percolates, we find inflation may continue at least many Hubble times even after the Universe percolates. What is observed is that inflation will not stop unless the global equation of state changes from inflationary one into radiation one. Thus the energy density of shorter wavelength gravitational waves induced by bubble collision at near the end of inflation should be at least *Order* $(10^2) \sim O(10^3)$ times greater than previous estimation of Turner and Wilcek(TW).

Key Words: Inflation, Percolation, Equation of state, Gravitational waves

I. INTRODUCTION

For most inflation models, it is not easy to pinpoint when inflation ends. For second-order phase transition inflation models such as new inflation models, we tacitly assume that inflation ends when the inflaton field reaches to the local minima of the inflaton potential. For first-order inflation, we assume that inflation ends when the Universe percolates (Guth 1981). As noticed by Guth and Weinberg, the percolation time refers to the moment when the bubble nucleation parameter $\epsilon(t)$, a dimensionless number defined as the number of true vacuum bubbles devided by Hubble four volume H^4, $\epsilon \equiv (\lambda/H^4)$, exceeds a certain value, $\epsilon \sim 0.24$ (Guth and Weinberg 1982). Physically this is the moment when true vacuum bubbles nucleated collide each other to form an ***infinite*** network. This moment has also been understood as the moment when the global equation of state changes from inflation one to the radiation one. In this work, we check whether the percolation time indeed guarantees the end of inflation. What will be shown is that a successful percolation does not necessarily guarantee the end of inflationary epoch. The Universe may percolate but it is possible that inflation can continue many Hubble times thereafter. Therefore, it becomes necessary to check rigorously whether at percolation, the global equation of state indeed varies from the inflationary one to the radiation one.

This work is inspired in part by recent report on extended inflation models in connection with the production of gravitational waves. TW (Turner and Wilczek 1990) pointed out that bubble collisions occurring at near the end of extended inflation can supply a potentially detectable source of gravitational waves. The importance of their work cannot be emphasized in that other than well-known cosmic background radition, they newly provide the second method to test the inflation models. In their work, in order to compute the characteristic wavelength of shorter gravitational waves, TW assumed that a typical wavelength may be determined by a characteristic bubble size $\bar{\lambda}$, where

$$\bar{\lambda} \approx H^{-1}(t_e).$$

Here t_e refers the moment when inflation ends. Thus the charateristic bubble size is determined by the Hubble size at the same moment.

II. BUBBLE PERCOLATION

It is natural to assume that the moment when the the global equation of state (of the Friedmann-Robertson-Walker (FRW) universe) is transformed from the inflationary $\dot{\rho}_\sigma = 0$ (or $+\rho_\sigma = -p_\sigma$), to that of radiation $p_\sigma(t) = (1/3)\rho_\sigma$, inflation terminates. (Here, ρ_σ and p_σ denotes the energy density, and pressure of cosmic matter, respectively.) For first-order inflation scenarios, false vacuum decays via nucleation of true vacuum bubbles. Unstable false phase decays into the stable true phase via quantum tunneling process. For each tunneling, true vacuum bubbles are nucleated randomly in false vacuum. As a true vacuum bubble nucleates, its wall quickly expands (almost) at the speed of light. In the process, the false vacuum energy swept by expanding bubble wall is deposited as a kinetic energy of the expanding bubble wall (La and Steinhardt 1989). For bubbles nucleated within event horizon of others eventually collide, and eventually, the kinetic energy of the bubble wall is transformed into radiation. It is this radiation which reheats interiors of empty bubbles (Hawking *et al* 1982). One may conjecture that during the reheating process, the reheated bubble interiors will exert some kind of 'back reaction' to the global equation of state. Therefore, inflationary equation of state given above becomes no more exact as inflation nears its end. This argument is incomplete in a sense that most first-order inflation ends rather abruptly. Therefore, there is a sudden change of the global equation of state.

During inflation we note that the 'average' energy density in a (physical) volume is not a well-defined quantity. This comes from an observation that an observer cannot measure the total energy density in a volume. The problem is that an observer, in the false vacuum will not aware of the presence true vacuum bubbles, since neighboring space is expanding exponentially fast (i.e., 'superluminally'). Only signals emitted within the observer's event horizon will be received. Likewise, an observer in the true vacuum will not aware the presence of the false vacuum, since the bubble wall, separating the false to true vacua, is expanding (almost) at the speed of light. Thus even if the true vacuum bubbles form an infinite network, i.e., even if the Universe percolates, the global equation of state will remain as an inflationary one. But let us question again what happens if all bubbles nucleated at each spatial points of false vacuum and grows to a similar size to collide with others. If this occurs in a typical Hubble time $\sim H^{-1}$, then a global transformation of the equation of state is guaranteed. This indeed is a feature characteristic at near the end of extended inflation.

The percolation refers to the event when true vacuum bubbles collide with each other, merge, and form an 'infinite' network. It has shown in the simplest extended inflation scenario that (La and Steinhardt 1989)

$$t_{pc} \;=\; H_B^{-1}\,\omega\,\left(\frac{\epsilon_{cr}}{\epsilon_o}\right)^{\frac{1}{4}},$$

where $\epsilon_{cr} \approx 0.03 \sim 0.24$ is the Guth-Weinberg bound for percolation and ϵ_o is the bubble nucleation parameter at the beginning of inflation (Guth and Weinberg 1982). Now let us check whether at percolation, the total energy density of radiation in a physical volume containing the percolated network exceeds that of the false vacuum. Guth and Weinberg showed that the probability of a point remaining in the false vacuum in a universe p_f, where true vacuum bubbles are randomely nucleating, is

$$p_f(t,t_B) \;=\; exp\left[-\int_{t_B}^{t} dt'\lambda'(t')R^3(t')\frac{4\pi}{3}\,\left(\int_{t'}^{t}\frac{dt''}{R(t'')}\right)^3\right],$$

where t_B denotes the beginning of inflation, $R(t)$ is the scale factor of the FRW universe, and $\lambda(t')$ is the bubble nucleation rate per unit time per unit volume (Guth and Weinberg 1982). The exact value of λ can be computed whenever an effective potential for the Higgs-like matter-fields responsible for inflation is given. The physical volume of the false vacuum is $p_f R^3(t)$. For conventional (second-order) exponential inflation models $R \sim e^{H_B t}$, where $H_B = const.$, is the Hubble parameter at the beginning of inflation. It is obvious that a decrease in the physical volume of the false vacuum (due to nucleating true vacuum bubbles) is always exceeded by an exponentially fast growing scale factor $R^3 \propto e^{3H_B t}$. Thus, during inflation, the physical volume of the false phase is ever-increasing: $p_f(t,t_B)R^3(t) = exp\left[(3 - \frac{4\pi}{3}\epsilon_o)H_B(t-t_B)\right] \to \infty$ as $t \to \infty$. Hence, there is no end of inflation. This of course is the 'graceful exit' problem; the fatal flaw of the old inflation theory.

III. END OF EXTENDED INFLATION

For first-order inflation models, however, or for models in a background of Brans-Dicke (-like) non-minimal gravity[Note1], the scale factor of the universe grows as $R(t) \propto t^{\omega}$. (*We refer to theories of time-varying gravitational coupling* $G = G(\phi(t))$, *where a non-minimaly coupled dilatonic-field* ϕ *is directly coupled to the Ricci scalar* $\mathcal{R}$: $\mathcal{L} \sim \xi\phi^2\mathcal{R}$+*(Kinetic+potential terms of the* ϕ*-field). The non-minimal coupling parameter* $\xi \equiv (1/8\omega)$, *where* ω *is the conventional Brans-Dicke parameter.*) Here ω is the Brans-Dicke parameter. At present, solar system experiments strongly constrains $\omega \geq 500$. For the power-law expansion, the fraction of the false vacuum during inflation is

$$p_f(t, t_B) \approx exp\left\{-\frac{\pi\epsilon\omega}{3}\left[\left(\frac{H_B t}{\omega}\right)^4 - \left(\frac{H_B t_c}{\omega}\right)^4\right]\right\}.$$

Here $t_c \equiv H_B^{-1}\omega$. The physical volume of the false vacuum becomes

$$p_f(t, t_B)R^3(t) \approx$$

$$exp\left\{-\frac{\pi\epsilon_o\omega}{3}\left[\left(\frac{H_B t}{\omega}\right)^4 - \left(\frac{H_B t_c}{\omega}\right)^4\right]\right\}\left(\frac{H_B t}{\omega}\right)^{\omega+\frac{1}{2}} \rightarrow 0$$

as $t \rightarrow \infty$. As inflation proceeds, the fraction of the false vacuum eventually decreases to a negligible. There is a moment

$$\frac{d}{dt}\left[p_f(t, t_B)R^3(t)\right]|_{t=t_*} = 0,$$

where

$$t_* = H_B^{-1}\omega\left[\frac{9}{4\pi\epsilon_o}\right]^{\frac{1}{4}},$$

after which the physical volume becomes exponentially decreasing. (Here, we set $\epsilon_o \equiv \lambda/H_B^4$) Thus inflation will end sometimes after the bubble nucleation parameter exceeds its percolation limit $t = t_{pc}$. Thus we find

$$t_* < t_{pc} < t_{end}.$$

We now seek for the desired moment when the total energy of radiation of a given physical volume is greater than that of false vacuum:

$$p_f(t, t_B)R^3(t)\rho_F < \mathcal{V}_{bubble}(L_{ph})\rho_\gamma.$$

Here $\mathcal{V}_{bubble}$ is the total physical volume of the bubbles in L_{ph}^3, where the quantity L_{ph}^3 is an arbitrary chosen (physical) volume. After percolation, the total energy density of radiation in interiors of the percolated true vacua will not be much differ from that of the (unpercolated) false vacuum: $\rho_F \approx \rho_{radiation}$. During this period, bubbles may overlap, but it leaves negligible correction (Guth and Weinberg 1982; La and Steinhardt 1989). Thus there holds an inequality

$$p_f(t_e, t_B)\left[H(t_e)L_{ph}(t_e, t_B)\right]^3\ \rho_F < \int_0^{H(t_e)x} d[H(t_e)x(t_e, t_o)]\rho_\gamma$$

$$\frac{4\pi}{3}\{1 + H(t_e x(t_e, t_o)\}^3\ \frac{dN_B}{d[H(t_e)x(t_e, t_o)]}.$$

Here, the quantity $[Hx]$ refers to the size of bubble nucleated at $t_o > t_B$, grown until when inflation ends $t = t_{end}$. For convenience, we adopted the dimensionless unit $[Hx]$, where for $H(t_{end}) \sim 10^{11}GeV$, and x for the present size of the universe $x \sim O(10^3)Mpc$, $[H(t_{end})x(t_{end})] \sim 10^{25}$. The bubble spectrum

$$\frac{dN_B(t, t_o)}{d[H(t)x(t, t_o)]} \approx \frac{C_B(t, t_o, t_B)}{[1 + H(t)x(t, t_o)]^{4+\frac{4}{\omega}}}$$

is the same as the one found in previous literature (La and Steinhardt 1989). This quantity offers the total number of bubbles nucleated at $t = t_o$ grown until t. We note that the constant C_B is evaluated to take a form

$$C_B(t_{end}, t_o, t_B) = p_f(t_{end}, t_B)[1 + H(t_{end})x(t_{end}, t_o)]^{\beta}\epsilon(t_{end})[H(t_e)L_{ph}(t_{end}, t_B)]^3,$$

where

$$\beta \approx \frac{4\pi\epsilon(t)}{3[1 + H(t_{end})x]^{\frac{4}{\omega}}}.$$

Now let us substitute this into the previous inequality. Then

$$1 + \frac{3[\beta - (4/\omega)]}{4\pi\epsilon(t_{end})} < [1 + H(t_{end})x(t_{end}, t_B)]^{\beta - 4/\omega}.$$

This is the desired formula. After percolation, where $\epsilon(t_{end}) \geq 0.03 \sim 0.24$, $\beta \sim 3$. Therefore above inequality constrains the maximum of

$$H(t_{end})x(t_{end}, t_B) \leq Order(1).$$

Therefore, inflation end rather shortly after $t \sim t_{pc}$, but at a non-trivial elapse: $t_{end} \sim Order(1) \times t_{pc}$.

Consequently, the vast majority of bubbles nucleated at the final stage of the first-order inflation will have sizes $\sim Order(1) \times H^{-1}(t_{end})$. Therefore,

$$\bar{\lambda} \approx Order(1) \times H^{-1}(t_e).$$

Thus we find that the wavelength of the shorter wavelength gravitational waves produced at the end of inflation has been increased by a factor $\sim O(1)$ times: hence, its energy density $\sim O(10^2) \sim O(10^3)$ times, at least. What would be other non-trivial consequences? We argue that a typical size of primordial blackholes (PBHs), supposedly formed in regions of isolated false vacua surrounded by percolated true vacuum, should also vary as the wavelength of the gravitational waves. Thus Hawking, Moss and Stewart's PBH, its mass will be changed as $M_{BH} \sim M_p^2/O(1) \times H(t_e)$(Hawking *et al* 1982). For $H \sim 10^{11} GeV$, $M_{BH} \sim 10^2 g$. This size PBHs will evaporate before $10^{-18} sec$, so that they are not likely to leave any observable effects to this day; except, possibly altering the baryon-photon ratio, η. They may, perhapse, influence a model baryogenesis scenario which occurs at $t \sim 10^{-20} sec$ after the big-bang.

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