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Fast Approximate Dose Model Used in Arc Therapy

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아크 치료를 위한 고속 근사선량모델 개발

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Abstract—Using beam data and accurate 3D dose model, a study of the spatial dose distribution for various arcs was carried out. The dose dirstibution generated by the accurage dose model could be represented by a simple approximate analytic form which is convenient and very efficient for calculating dose distribution iteratively in the optimization procedure. We developed an empirical cylindrical dose model to compute dose for one full rotational arc or partial rotational arc. After a tedious search for fits to a collection of 200 points of accurate dose data, we found simple formular with 7 parameters search. As a consequence, the programs required approximately less than 1 second to compute dose for one single arc on a 20 by 20 matrix (400 points) using fast approximate dose model. In conclusion the fast approximate dose model give dose distributions similar to the accurate dose model, which makes this fast dose model an attractive alternative to the accurate 3D dose model.
Key words : 3D *dose model, cylindrical dose model, arc*

요약—선량데이타와 정확한 3차원 선량모델을 이용하여 여러 아크에 대한 선량분포를 조사하였다. 정확한 선량모델에 의해 계산된 선량값은 간단한 실험식으로 표현이 가능했으며 이는 선량 최적화 과정에서 반복적으로 선량값을 계산하는 데 매우 유용하였다. 360도 아크와 부분 아크에 대한 선량값을 빠른속도로 계산하기 위하여 실험적으로 구해진 실린더형 선량모델을 개발하였다. 200개 위치의 정확한 선량값을 비선형식으로 피팅하여 7개의 변수를 포함하는 실험식을 개발하였다. 결과적으로 이 모델을 이용하는 경우 한 아크에 대한 선량 계산시 400개의 위치를 계산하는 데 PC-486으로 1초이내에서 계산이 가능하였다. 결론적으로 개발된 고속선량모델은 정확한 선량모델에 의한 선량값과 유사한 값을 제공함으로써 계산속도가 늦은 일반 3차원 선량모델을 대치할 수 있을 것으로 사료된다.
중심단어 : 삼차원 선량모델, 실린더선량모델, 아크

INTRODUCTION

Stereotactic radiosurgery is a technique for obliterating intracranial tumors which are inaccessible or unsuitable for open surgical techniques, using tightly collimated beams of ionizing radiation. The aim of stereotactic radiosurgery is to deliver, with a high degree

of spatial accuracy, a large radiation dose to the target volume within the brain, while maintaining the smallest possible dose to the rest of the brain tissue. At present, there are three types of radiation beams that can be used for radiosurgery : focused beams obtained from specially designed units incorporating several hundred cobalt sources (the Gamma Knife) [1-3] ; heavy charged particle beams (protons, deuterons, helium ions, etc.) obtained from cyclotrons or synchrocyclotrons [4, 5] : and x-rays obtained from medium energy (4 to 10 MV) isocentrically mounted linear accelerators (LINAC) [6-11]. LINAC-based stereotactic radiosurgery is a less expensive alternative to the two proven radiosurgical techniques. The LINAC method is based on the combination of multiple isocentric arc irradiation with small fields centered in the stereotactic target. This optimizes the dose fall off outside the target volume.

Several dose algorithms have been described, which calculate three dimensional dose distributions for LINAC arc-based raiosurgery. These 3-D algorithms are based on an isocentric model [12], which is one of the empirical dose models that describe the dose for a single field, and is used as a basis for superimposed fields. For radiosurgery planning, an algorithm is needed that permits the calculation of dose distribution for any single fixed circular field in any orientation, as well as of a number of superimposed fields.

The design of an optimal radiosurgery pla nning system which uses 3-D patient data and multiple arcs represents a significant challenge. This is in part due to the lack of 3-D information about target volumes and anatomic structures, and also by the many arc parameters involved in treatment planning. Furthermore, since 3-D arc calculations take a very long time, interactive modification of treatment using exact dose model may not be practical.

The 3-D dose algorithm presently used is required to calculate doses at any point in the patient for any specified gantry angle, turntable angle, collimator angle and collimator size. Since the dose algorithm operates on a 3-D patient representation stored as set of multiple transverse sectional cntours, calculating dose in 3-D space using the exact 3-D dose model is time consuming. The use of the exact dose model in optimization procedure is not efficient. A faster and more efficient dose calculation algorithm for the arc is required to simulate the exact dose model.

METHODS AND MATERIALS

In order to investigate the possible solution associated with an efficient fast dose model used in arc-based stereotactic radiosurgery, two major steps are discussed : (1)Development of exact 3D dose data using reference head model for a study of the spatial dose distribution for single arc ; (2) Development of a fast approximate dose model from the fit of exact dose data ;

Exact Dose Data for Reference Head Model

Dose distribution was examined using the exact geometry of a reference head model and an exact 3-D dose computation algorithm. A spherical head model with a centrered target position was used for the standard reference head model since it is geometrically simple, and represents a comparable result about dose distribution with a real patient head. The dose shape is not changed much except in an extreme case (near the surface) as the target position varies in arc irradiation technique. Therefore, useful information such as the isodose shape, obtained from this reference model can be used as a general guideline for an individual patient condition. From the modification of single isocentric dose model [13], the formula to express the dose at defined point m for a single beam with gantry and table

orientation i, j can be written by

$$D_{ijm}(C_j, STD_{ijm}, d_{ijm}, r_{ijm}) = D_{Ref} \times ROF(C_j) \times TMR(w_{ijm}, d_{ijm}) \times (SAD/STD_{ijm})^2 \times OAR(C_j, STD_{ijm}, r_{ijm}) \quad (1)$$

where

- i = gantry angle orientation
- j = turntable angle orientation
- C = Collimator size defined at SAD
- STD= source to target distance
- SAD= source to sxis distance = 100cm
- w = field size at point of interest m expressed by w = C (STD/SAD)
- d = depth of point of interest m
- r = off-axis distance
- D_m = the dose at point of interest m
- D_{ref} =the dose for the reference set-up
- ROF= relative output factor defined by D (C, STD=100, d_{max}, r=o) /D (C_{ref}, SSD=100, d_{max}, 0)
- TMR= tissue maximum ratio defined by D (w, d)/D (w, d_{max})
- OAR= off-axis ratio defined by D(C, STD, d, r)/D(C, STD, d, r=0)

If the dose matrix and beam setting para meters are determined, the calculation of the dose on the dose matrix requires d_{ijm}, r_{ijm}, STD_{ijm}, and w_{ijm} from our dose model in Eq.(1). The unknown beam parameters can be determined from the geometrical relationship between a cartesian frame coordinates in spherical head (radius, R_0) and beam position (Fig. 1) [14].

The final algorithm to express dose at the point of interest m in the defined dose grid for multiple arcs is expressed in the form given by

$$D_m = \sum_j \sum_i D_{ijm}$$

where D_{ijm} is given by Eq.(1). The unknown beam parameters for one gantry (θ_i) and turntable orientation (ϕ_j) of each arc j are determined in the cartesian reference frame coordinate system. TMR and OAR values are then obtained from the measured data.

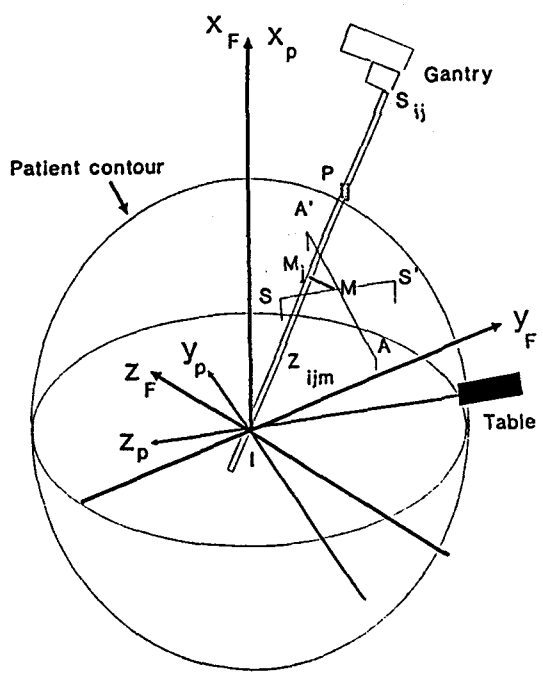

Fig. 1 Geometrical relationship between three coordinate systems.

Fast Dose model

The dose distribution generated by the exact dose model could be represented by a simple approximate analytic form which is convenient and very efficient for calculating dose distribution iteratively in the optimization procedure. Two fast dose models were developed to represent the dose in 3-D space : (1) for full rotational ; (2) partial rotational arcs.

Brahme et al.[15] developed an analytical approach for a solution of an integral equation encountered in rotation therapy. In this study we develop an empirical cylindrical dose model which may be used to represent the dose distribution for one full rotation arc and one partial rotational arc.

1) Full Rotational Arc

One simple analytical form was found, which well represents both radial and axial dose distribution for one full rotational arc with small

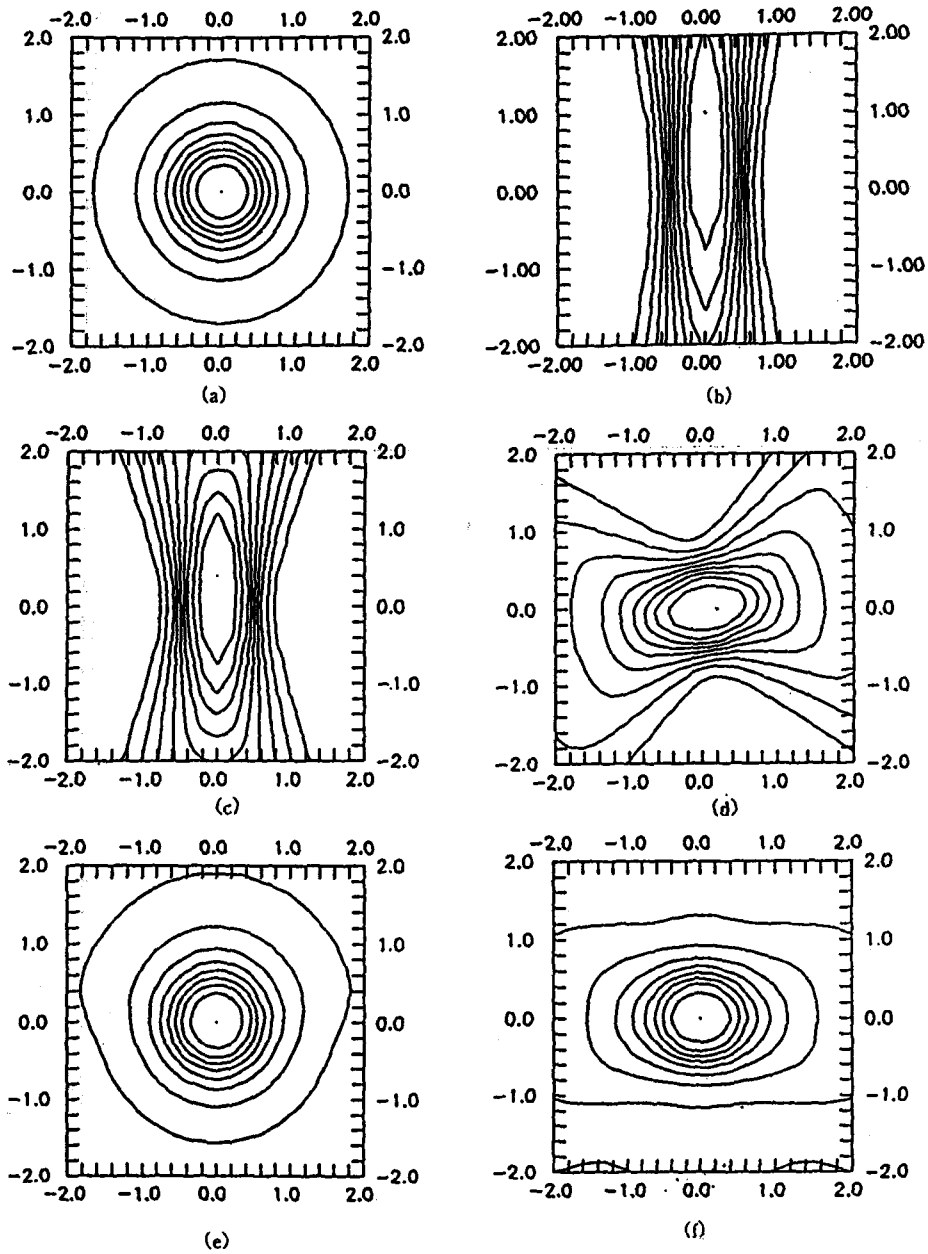

Fig. 2 Isodose distributions on the arc plane, calculated by 3D-dose computation model with various partial rotational arcs.
(a)360 °, (b)25 °, (c)50 °, (d)100 °, (e)180 °, (f)260 °. The isodose lines displayed are from 90 to 10 in 10 decrements and normalized by maximum dose.

collimators. After a tedious search for fits to a collection of 200 points of exact dose data we found good results with the equation :

$$D_j = q_1 \exp[(-q_2 Z^2 + q_3 Z - q_4 r)/C] + q_5 \exp[(-q_6 r^2 - q_7 Z^2)/C^2] \cdots\cdots\cdots (3)$$

where r and Z and radial and axial distances from isocenter in cylindrical geometry and C

is a collimator diameter. D_j is the ose contributed from arc j. The q parameters are fitting parameters. The parameters obtained by non-linear least square (NLLS) are given in Table 1.

A dose distribution for multiple arcs is ob tained simply by adding each contribution from each arc. A dose model for multiple arcs is expressed in the form given by :

$$D = \sum_j D_j \quad \cdots\cdots (4)$$

where D_j is given by Eqs. (3).

2) Partial Rotational Arc

When a full 360 ° rotation arc is used, the isodose curves tend to be circles on the arc plane (Fig. 2a). If partial rotation is used, this symmetry is lost and the region of highest dose is shifted from the axis of rotation away from the missing sector(Fig. 2b to 2f). If the missing sector is a small fraction of the 360 ° rotation, the effect is negligible. As the sector becomes larger the distortion also becomes larger. This is true no matter what the shape of the contour is.

We adapted the cylindrical dose model for full rotational arcs to the development of a more useful dose model for a partial rotational arcs. We accomplished this by transforming the cylindrical dose model for full rotation arcs to the modified cylindrical dose model for partial rotation arcs with an arc correction factor, which is dependent on varying the length and position of the arc.

When we consider partial rotation arcs, we obtain an asymmetry factor by comparing the dose istribution obtained from the cylindrical dose model for full rotation with that obtained from the exact dose model for partial rotation arcs. The arc correction factor, g, is now defined by :

$$g = \frac{D_P}{D_F} \quad \cdots\cdots (5)$$

where D_F is the dose for a full rotation arc and is given by Eqs. (3). Here, we consider a 1cm collimator for the demonstration. D_p is the dose from the exact dose model for partial rotation at the same position. Since it is not practical to find a correction factor at every position, it is desirable to find the approximate correction factor over the major arc plane for the possible partial rotational arcs (100 °, 180 °, etc.). The arc correction facotrs for 50 °, 100 ° and 180 ° partial rotation arcs were found for the different angle positions about five isodose lines (20, 30, 50, 70 and 80) of full rotational arcs (dashed circles in Fig. 3a to c).

The arc correction factors for 100 ° and 180 ° partial rotation arcs are well represented by a simple quadratic form

$$g = g_0 + g_1 \mu + g_2 \mu^2 \quad \cdots\cdots (6)$$

where

$$\mu = \cos \theta \quad \cdots\cdots (7)$$

where θ is defined as the angle from the

Table 1. Fitting parameters for fast dose model

q_1	q_2	q_3	q_4	q_5	q_6	q_7
0.387	6.178	1.182	0.395	0.695	2.077	3.551
g_{01}	g_{02}	g_{11}	g_{12}	g_{21}	g_{22}	
−0.327	1.663	0.114	−0.116	2.725	−3.260	

$$g = g_0 + g_1 \mu + g_2 \mu^2$$
$$g_0 = g_{01} + g_{02} D_F$$
$$g_1 = g_{11} + g_{12} D_F$$
$$g_2 = g_{21} + g_{22} D_F$$

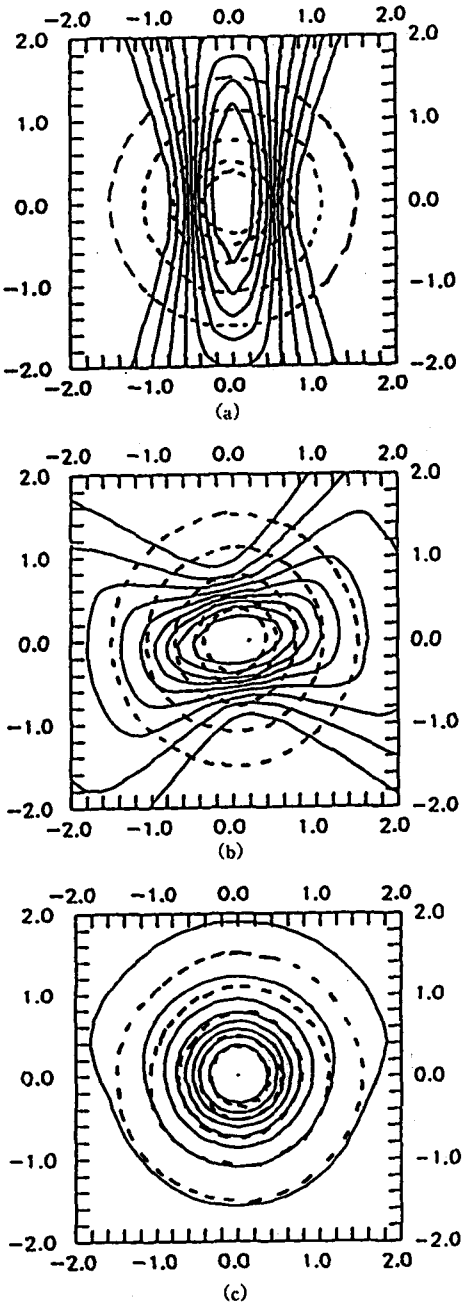

Fig. 3. Comparision of isodose distributions between partial rotational arcs (solid isodose lines) and full rotational arc (dashed isodose lines). (a) 50 °, (b)100 °, (c)180 ° arc.

position of the angle of the bisector of the arc, θ_b. θ_b can be expressed in terms of starting and ending gantry angle θ_{Gs}, θ_{Ge} for the arc by :

$$\theta_b = \frac{\theta_{Gs} + \theta_{Ge}}{2} \quad \cdots\cdots(8)$$

The values for g_0, g_1 and g_2 are scattered in a linear fashion for a 100 ° rotation arc.

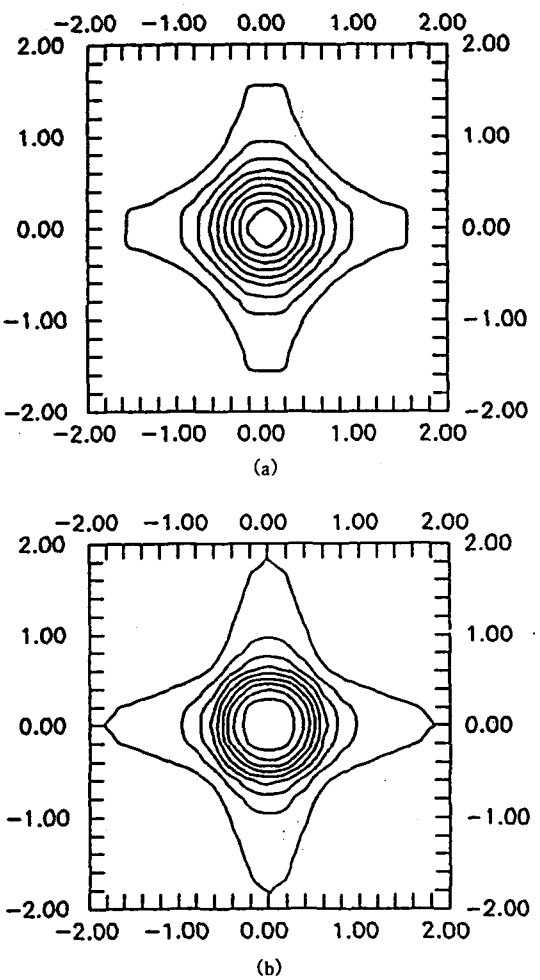

Fig. 4. Isodose distributions on the coronal planes vertical to two orthogonal full rotation arcs, calculated by a fast dose model (a), and accurate dose model (b). The isodose lines displayed are from 90 to 10 in 10 decrements.

The coefficients of linear regression are shown in table 1.

The dose for a partial rotation arc can be now expressed in terms of a full rotation dose model with an arc correction factor g in the cylindrical coordinate system. (r, Z and Φ)

$$D_p = D_F\ (C,\ r,\ Z)\ g(C, \Phi) \cdots\cdots\cdots\cdots (9)$$

A dose distribution for multiple arcs is obtained simply by adding each contribution from each arc, and can be represented on dose grids defined. A dose model for multiple arc is expressed in the form given by :

$$D = \sum_j D_j \cdots\cdots\cdots\cdots (10)$$

where j is an arc number and D_j is given by Eq. (8).

RESULTS

Figure 4 gives a montage of the spatial contours of the dose distribution (on the coronal plane) from two orthoganal full rotational arcs with isocenter position (0, 0, 0) and collimator size 1 cm. The isodose shape in Figure 4a shows similar trends with that obtained from

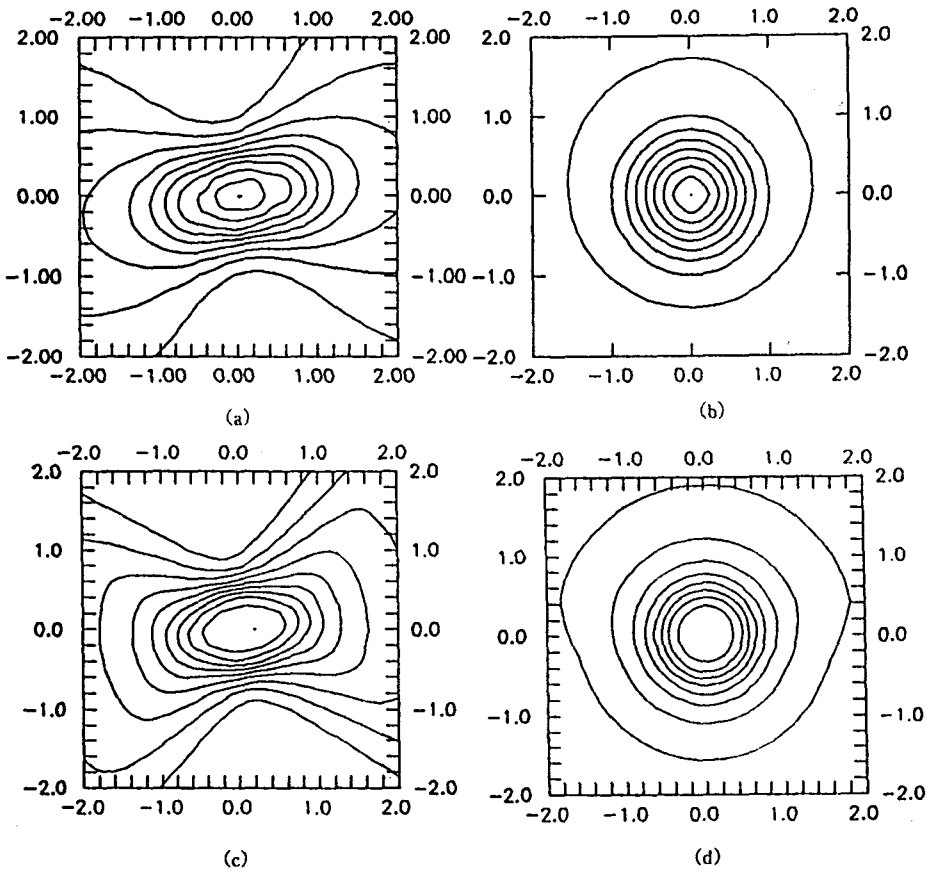

Fig. 5. Isodose distributions calculated by a fast dose model (a, b) and exact dose model (c, d) partial rotational arcs. (a) and (c) represent isodose distributions for 100 ° arc (gantry 30 ° to 130 °). (b) and (d) represent isodose distributions for 180 arc (gantry −90 ° to 90 °). Isodose lines displayed are from 90 to 10 in decrements 10.

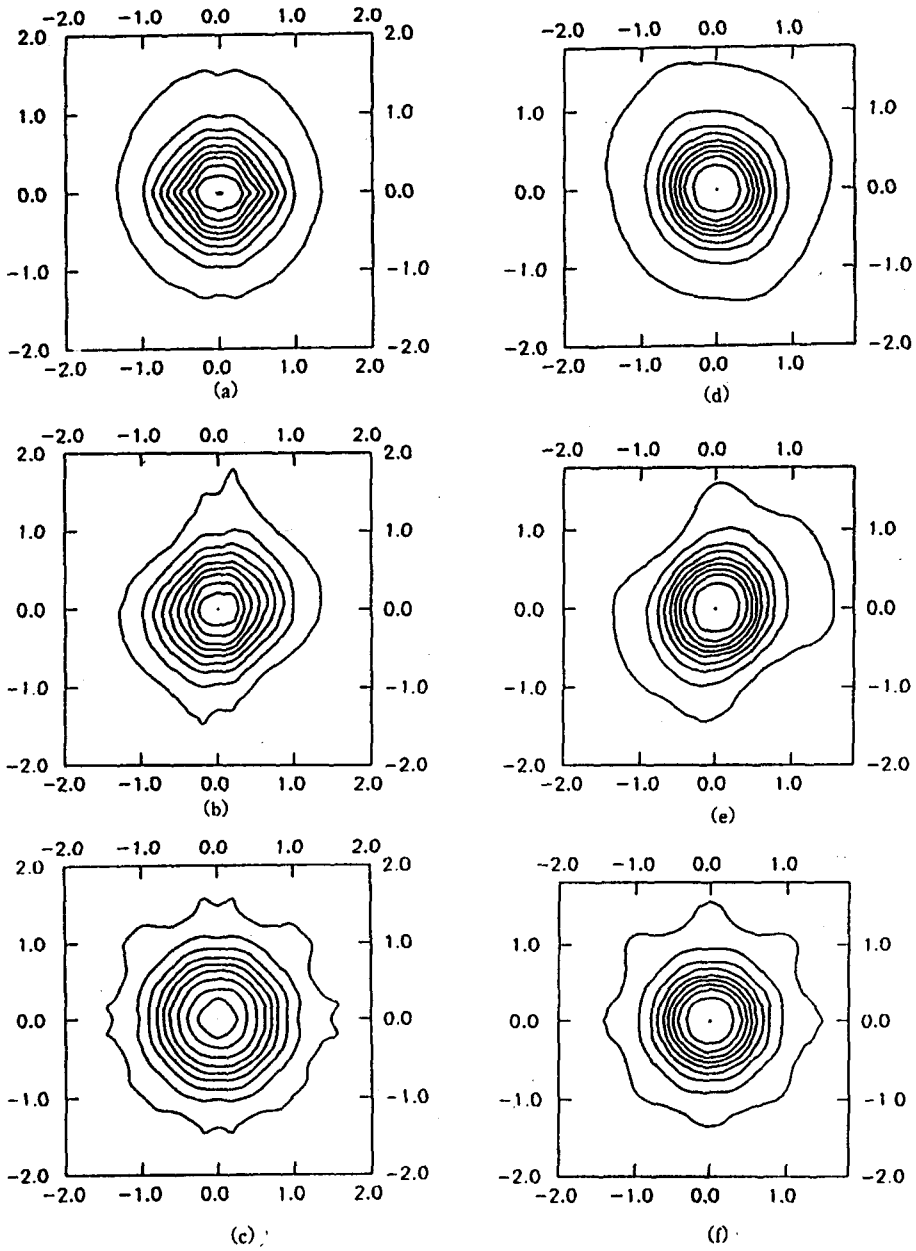

Fig. 6. Isodose distributions calculated by a fast dose model (a, b, c) and exact dose model (d, e, f) for four standard arcs (100 ° arc with turn-table orientation 0 °, 45 °, −45 °, and −90 °) with 1 cm collimator size. The distributions are displayed onto the transverse (a,d), sagittal (b,e) and coronal planes (c, f). Isodose lines displayed are from 10 to 90 in increments 10.

the exact dose model (Fig. 4b)

Figures 5a and b give montages of spatial contours of dose distribution from the dose model developed on major arc planes for 100 ° and 180 ° partial rotation arcs with isocenter position (0, 0, 0) and collimator size 1 cm.

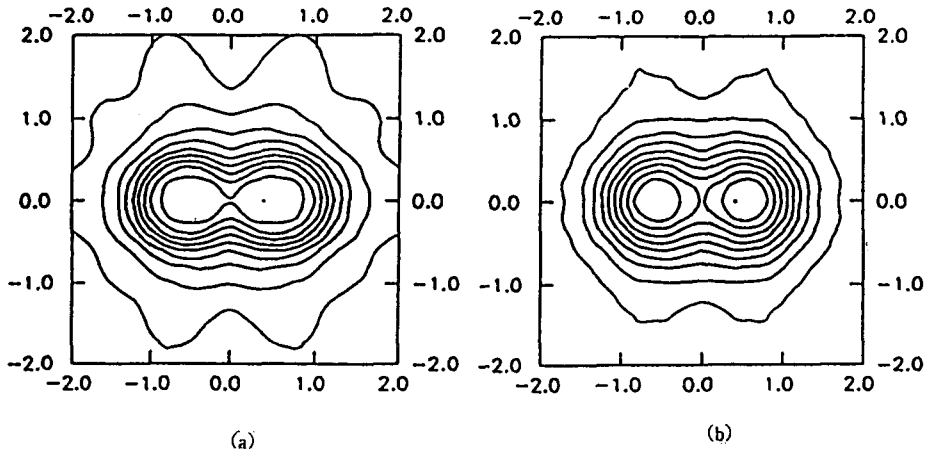

Fig. 7. Isodose distributions on the coronal plane for eight standard arcs with two isocenters, calculated by (a) exact dose model, (b) modified cylindrical dose model (partial rotational).

The isodose shape in figures 5a and b is in similar pattern with that obtained from the exact dose model (Figures 5c and d).

figures 6a to c give montages of spatial contours of dose distribution from a fast dose model on three orthogonal planes (axial, sagittal and coronal) for four standard 100-arcs (gantry 30 °~130 ° used in routine stereotactic radiosurgery) with isocenter position (0, 0, 0) and collimator size 1 cm. The isodose shape in figures 6a to c is in good agreement with that obtained from the exact dose model (Figures 6d and f).

Figure 7b give montages of spatial contours of dose distribution from a modified cylindrical dose model on the coronal plane for eight standard arcs with two isocenters, (0, −0.6, 0) and (0, 0.6, 0) (four arcs about each isocenter) and collimator size 1 cm. The isodose shape in figures 7b is in good agreement with that obtained from the exact dose model (Fig. 7a).

As a consequence, the programs required approximately 30 seconds to calculate dose for four standard arcs on a 20×20 matrix (400 points) using the modified cylindrical dose model for partial rotational arcs and an additional 7 minutes to perform grid generation to represent isodose curves with a Zenith 386/16 computer without floating point processor. This is much faster (approximately 20 times) than the exact dose model to calculate dose for four standard arcs. Thus, with the use of a fast dose model optimum irradiation parameters can be found 20 times faster than with the exact dose model, which makes it possible to perform real-time optimization during computer dosimetry treatment planning.

DISCUSSION

We conclude that in optimal dose distribution for arc-based radiosurgery, it is important to use approximate forms for dose calculation in 3-D space. In the current work we use a cylindrical dose model to simulate the exact dose distribution for an arc. Since the dose distribution from the fast dose model is similar to that from the exact dose model, we conclude that dose optimization with a fast dose model is an efficient and paractcal alternative to the trial and error method using the exact dose model. Several improvements are expected in the future. One is finding an arc correction factor for any length of arc in addition to the standard arcs (100 ° and 180 °). This modification makes it possible to use the start and stop positions of gantry angle as variables in

dose optimization. Since the dose distributions are not changed much as target position or head contour vary, it may not be necessary to correct for different target positions and head contours.

In the present work, the cylindrical dose model was used with an arc correction factor. Although these methods showed some good results in simple examples, this method may not be the best choice. After testing some other model with analytic approach or experimental, the most reliable method may be applied to practical dose calculation for an arc.

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