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An EOQ Model for Deteriorating Items with Linearly Increasing Demand

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Abstract

In this paper an inventory model is presented for determining the ordering schedule in which the demand rate is changing linearly with time and the decay is assumed to be a constant rate of the on-hand inventory. An easy to use heuristic is developed to find the times and sizes of replenishments so as to keep the total of ordering, inventory carrying and deteriorating costs as low as possible. Solutions of the model to test problems show that our heuristic model outperforms other existing models in the literature without sacrificing the computational complexity. When there is no deterioration, the model developed is related to the corresponding model of nondeteriorating items.

1. Introduction

A convenient way of determining optimum order quantity is the popular 'basic EOQ model'. To be able to use the basic EOQ model, several conditions should be satisfied. Among them are the requirement of constant demand rate and lack of deterioration of inventoried items.

The assumption of constant demand has been relaxed by Donaldson [5], Ritchie [8], Triantaphyllou [11] and Dave [2]. Their papers are concerned with an inventory policy for the case of linearly increasing demand up to a known time horizon H. Their analysis of demand up to a time horizon H is just a device to aid the mathematical solution and not essential feature of real life. Silver [10] developed a heuristic

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solution procedure for the linearly increasing demand to reduce the computational effort needed in Donaldson's work.

Dave and Patel [3] have developed a (T, S_i) policy inventory model incorporating the possibility of deterioration with linearly increasing demand. The model is extended by Sachan [9] and Goswami et al [6] to cover the backorder case. However, all the works referred above incorporating deterioration assumed equal replenishment periods. This is really an unnecessary restriction. Since demand rate increases over time, both the order quantity and frequency of the orders should increase by the passage of time in order to achieve the total relevant cost. Keeping the replenishment cycle constant and only increasing the order quantity will result in higher relevant cost than adjusting both ordering cycles and order quantity.

Bahari-Kashani [1] relaxed constraint on equal replenishment periods and developed a heuristic solution procedure to find the reorder times and sizes of replenishments. The heuristic procedure uses two demand rates, one for the beginning of the horizon and one for the end of the planning horizon. As demand increases over time, the replenishment cycle length is reduced from one extreme to the other. This model produced a better result than the restricted optimal model which is suggested by Dave and Patel.

In the present paper, we have developed a heuristic inventory model for an item having a linear trend in demand. Shortages in inventory are not allowed and it is assumed that the inventory deteriorates over time at a constant rate, θ of the on-hand inventory. The outline of the solution procedure is as follows: Instead of determining the first replenishment and all others so as to minimize the total relevant cost up to the horizon, we determine the size of the first replenishment to minimize the total relevant cost per unit time over the duration of the first replenishment only. The procedure is repeated for each replenishment. The solutions of the heuristic are suboptimal, but outperforms other existing models in the literature without sacrificing the computational complexity.

2. The Mathematical Model

A heuristic inventory model has been developed with the following notation and assumptions :

(i) $f(t) = (a + bt)$ is the demand rate at any time t.

(ii) A is the ordering cost per order.

(iii) r is the inventory carrying cost per unit per year.

(iv) p is the purchase cost per unit.

(v) A constant rate, θ, of the on-hand inventory deteriorates per unit of time.

(vi) T is the duration of the first replenishment, in periods.

(vii) TRC(T) is the total relevant cost per unit time.

Here we consider the case of the first replenishment only and determine its size so as to minimize total relevant costs per unit time over the duration of the first replenishment only.

Since $\int_0^T (a+bt)\ dt$ is the total demand occurring during the first replenishment, the number of units that deteriorates during the first teplenishment is [7]

$$\int_0^T (a+bt)e^{\theta t}\ dt - \int_0^T (a+bt)\ dt \qquad (1)$$

The number of units in inventory during the first replenishment is

$$\int_0^T t(a+bt)e^{\theta t}\ dt \qquad (2)$$

Hence, the total relevant cost per unit time during the first replenishment is [7]

$$TRC(T) = \frac{A + p\left[\int_0^T (a+bt)e^{\theta t}dt - \int_0^T (a+bt)dt\right] + r\int_0^T t(a+bt)e^{\theta t}dt}{T} \qquad (3)$$

We wish to determine the value of T which minimizes TRC(T). A necessary condition for TRC(T) to be a minimum is that

$$\frac{dTRC(T)}{dT} = 0 \qquad (4)$$

Condition (4) leads to

$$\frac{dTRC(T)}{dT} = \frac{1}{T^2}\{pT[(a+bT)e^{\theta T} - (a+bT)] + rT^2(a+bT)e^{\theta T} - A$$

$$- p\left[\int_0^T (a+bt)e^{\theta t}dt - \int_0^T (a+bt)dt\right] - r\int_0^T t(a+bt)e^{\theta t}dt\} = 0 \qquad (5)$$

Since θ is usually very small, manipulating the condition (5) and neglecting the second (θ^2) or higher order terms in the expansion of $e^{\theta T}$ gives

$$\frac{dTRC(T)}{dT} = \frac{\frac{3rb\theta}{4}T^4 + \frac{2}{3}(ra\theta + rb + pb\theta)T^3 + \frac{1}{2}(ra + pa\theta)T^2 - A}{T^2} = 0 \qquad (6)$$

which leads to biquadratic equation,

$$\frac{3rb\theta}{4}T^4 + \frac{2}{3}(ra\theta + rb + pb\theta)T^3 + \frac{1}{2}(ra + pa\theta)T^2 = A \qquad (7)$$

For the case of increasing demand(b>0) (7) can be effectively solved by itèration. The value of T obtained on the kth iteration on (7) can be written as

$$T^{(k)} = \sqrt{\frac{A}{\frac{3rb\theta}{4}[T^{(k-1)}]^2 + \frac{2}{3}(ra\theta + rb + pb\theta)T^{(k-1)} + \frac{1}{2}(ra + pa\theta)}} \qquad (8)$$

The initial estimate for T is $T^{(0)} = \sqrt[3]{3A/(2br)}$. Convergence occurs when at iteration i, $T^{(i)} = T^{(i-1)}$. A few iterations are normally required for convergence. Since

$$\frac{d^2TRC(T)}{dT^2} = \frac{\frac{3}{2}rb\theta T^5 + \frac{2}{3}(ra\theta + rb + pb\theta)T^4 + 2AT}{T^4} > 0 \qquad (9)$$

for the case of increasing demand (b>0), the solution of (8) is unique. When there is no deterioration ($\theta = 0$), the equation (7) can be written as

$$\frac{2}{3}rbT^3 + \frac{1}{2}raT^2 = A \qquad (10)$$

which is the equation (4) in Silver [10].

3. Numerical Comparisons

To show the usefulness of the heuristic model proposed in this paper comparisons were made with the results in the literature. These were

(i) The result of Dave and Patel [3] where they imposed the restriction of equal replenishment interval.

(ii) The result of Bahari-Kashani [1] where they proposed a heuristic procedure that does not impose the restriction of equal replenishment interval. But this procedure does not guarantee optimal solution.

We shall consider an example with the following parameter values :

$a = 0$, $b = 1600$, $A = 256$ per order, $p = 1.67$ per unit , $r = 0.56$ per unit per year, $H = 10$ years, $\theta = 0.003$.

The first replenishment is made at time $t = 0$ (at time 0, $a = 0$). Now starting with $T^{(0)} = \sqrt[3]{3A/(2br)} = 0.754$, the successive iterations using equation (8) produce the data given in Table 1.

Table 1.

k	1	2	3	4
$T^{(k)}$	0.748	0.752	0.751	0.751

Convergence of the iteration process in Table 1 shows that $t_1 = T_1 = 0.751$. Now we know that the second replenishment occurs at time t_1. At that time the demand rate is given by

$$f(t_1) = 0 + bt_1 = 1600(0.751) \doteqdot 1201.6$$

Taking $T^{(0)}$ equal to the T value found for the previous replenishment, here 0.751 , the successive iterations using equation (8) produce the data given in Table 2.

Table 2.

k	1	2	3	4	5	6
$T^{(k)}$	0.568	0.612	0.601	0.604	0.603	0.603

Convergence of the iteration process in Table 2 shows that $T_2 = 0.603$. Hence the third replenishment occurs at $t_2 = t_1 + T_2 = 1.354$. Continuing in this fashion we get a pattern of thirty replenishments in Table 3. The computation can be carried out on personal computer using spreadsheet software.

According to the heuristic, we are actually to choose $T = 0.096$ on the last replenishment. The boundary at $H = 10$ forces a shorter duration of the last replenishment interval. If the linear pattern in demand continued beyond time 10, the T value of the last replenishment would be 0.235, not just 0.096.

The total cost from this heuristic model is $ 14639.32, which is 4.32% less than Dave and Patel's model and 1.70% less than Bahari-Kashani's model. To evaluate the consistency of the heuristic model, the parameters have

Table 3. The timing of replenishments

i	T_i	t_i
1	0.751	0.751
2	0.603	1.354
3	0.525	1.879
4	0.474	2.353
5	0.439	2.792
6	0.411	3.203
7	0.390	3.593
8	0.372	3.965
9	0.357	4.322
10	0.344	4.666
11	0.333	4.999
12	0.323	5.322
13	0.314	5.636
14	0.306	5.942
15	0.299	6.241
16	0.292	6.533
17	0.286	6.819
18	0.280	7.099
19	0.275	7.374
20	0.270	7.644
21	0.266	7.910
22	0.262	8.172
23	0.258	8.430
24	0.254	8.684
25	0.250	8.934
26	0.247	9.181
27	0.244	9.425
28	0.241	9.666
29	0.238	9.904
30	0.096	10.000

varied from one extreme to the other. The result is given in Table 4-7.

For all test problems, the heuristic model proposed here produces best solution. The average savings by our heuristic compared to Bahari-Kashani's model is 1.87%. Our heuristic model incur a small cost penalty compared to Bahari-Kashani's model because of the detrimental effect of the demand termination point at the horizon of $H=10$. In practice demand will not usually cease at the horizon, our heuristic model actually will give better result than the result given in Table 4-7.

4. Conclusions

We have developed the inventory replenishment policy for deteriorating item with linearly increasing demand. Instead of determining the first replenishment and all others so as to minimize the total relevant cost up to the horizon, we determine the size of the first replenishment to minimize the total relevant cost per unit time over the duration of the first replenishment only. In practice, when the time horizon is unlikely to be known in advance, our model can be useful for deteriorating items with linearly increasing demand. Solutions of the model to test problems show that our heuristic model outperforms other existing models in the literature without sacrificing the computational complexity.

Table 4. Deterioration rate, θ varies

A=256, p=1.67, r=0.56, H=10

Deterioration rate θ	Bahari-Kashani		Dave and Patel		Our Heuristic			
	Number of orders	Total relevant cost	Number of orders	Total relevant cost	Number of orders	Total relevant cost	Percentage of cost savings	
0.002	26	14868.26	30	15276.36	30	14632.19	1.59[a]	4.22[b]
0.004	26	14915.74	30	15323.10	30	14647.64	1.80	4.41
0.008	27	14983.30	30	15416.66	30	14692.69	1.94	4.70
0.016	27	15165.79	31	15601.24	31	15078.93	0.57	3.35
0.032	28	15505.99	31	15964.21	31	15194.82	2.01	4.82
0.064	29	16185.75	33	16666.35	33	16024.81	0.99	3.85
0.128	31	17477.17	36	17997.18	35	17117.76	2.06	4.89
0.256	36	19799.52	40	20414.26	40	19492.80	1.55	4.51
0.512	43	23875.72	49	24591.23	48	23566.01	1.30	4.17
1.024	54	30624.90	63	31437.68	60	29960.47	2.17	4.70

a : Percentage of cost saving by our heuristic compared to Bahari-Kashani's model

b : Percentage of cost saving by our heuristic compared to Dave-Patel's model

Table 5. Carrying cost, r varies

A=256, p=1.67, H=10, θ =0.003

Carrying cost r	Bahari-Kashani		Dave and Patel		Our Heuristic			
	Number of orders	Total relevant cost	Number of orders	Total relevant cost	Number of orders	Total relevant cost	Percentage of cost savings	
0.25	18	10057.57	20	10307.82	21	10035.72	0.22[a]	2.64[b]
0.50	25	14077.80	28	14470.94	28	13772.40	2.17	4.83
1.00	35	19784.77	40	20376.40	39	19351.99	2.19	5.03
2.00	50	27852.06	56	28745.33	55	27354.36	1.79	4.84
4.00	70	39298.88	79	40590.68	77	38565.15	1.87	4.99
8.00	99	55480.06	112	57348.97	108	54392.74	1.96	5.15
16.00	140	78369.83	158	81055.08	151	76590.54	2.27	5.51
32.00	198	110744.60	224	114583.43	213	108222.07	2.28	5.55
64.00	281	156512.66	316	162002.96	301	153090.57	2.19	5.50
128.00	397	221268.29	448	229065.67	424	216176.13	2.30	5.63

a : Percentage of cost saving by our heuristic compared to Bahari-Kashani's model

b : Percentage of cost saving by our heuristic compared to Dave-Patel's model

Table 6. Purchase cost, p varies

A=256, r=0.56, H=10, θ =0.003

Purchase cost p	Bahari-Kashani		Dave and Patel		Our Heuristic			
	Number of orders	Total relevant cost	Number of orders	Total relevant cost	Number of orders	Total relevant cost	Percentage of cost savings	
0.50	26	14843.55	30	15252.40	30	14623.08	1.49[a]	4.13[b]
1.00	26	14864.25	30	15272.62	30	14629.54	1.58	4.21
2.00	26	14905.66	30	15313.08	30	14644.62	1.75	4.37
4.00	27	14962.28	30	15393.99	30	14683.39	1.86	4.62
8.00	27	15121.23	31	15554.98	30	14795.25	2.16	4.88
16.00	27	15439.71	31	15868.09	31	15160.18	1.81	4.46
32.00	28	16031.44	32	16480.02	32	15705.38	2.03	4.70
64.00	30	17149.65	35	17638.63	34	16774.92	2.19	4.90
128.00	34	19179.37	39	19751.32	38	18781.89	2.07	4.91
256.00	51	28475.95	58	29388.76	56	27909.53	1.99	5.03

a : Percentage of cost saving by our heuristic compared to Bahari-Kashani's model

b : Percentage of cost saving by our heuristic compared to Dave-Patel's model

Table 7. Ordering cost, A varies

p=1.67, r=0.56, H=10, θ =0.003

Ordering cost A	Bahari-Kashani		Dave and Patel		Our Heuristic			
	Number of orders	Total relevant cost	Number of orders	Total relevant cost	Number of orders	Total relevant cost	Percentage of cost savings	
0.50	597	649.49	672	672.49	637	634.82	2.26[a]	5.60[b]
1.00	422	918.74	476	951.14	451	897.93	2.27	5.59
2.00	298	1299.78	337	1345.32	319	1269.87	2.30	5.61
4.00	211	1838.94	238	1902.97	227	1800.41	2.10	5.39
8.00	149	2602.50	168	2692.01	161	2546.92	2.14	5.39
16.00	105	3684.44	119	3808.67	114	3601.49	2.25	5.44
32.00	75	5215.57	84	5389.52	81	5100.69	2.20	5.36
64.00	53	7390.04	60	7628.29	58	7237.54	2.06	5.12
128.00	37	10485.24	42	10801.05	42	10352.04	1.27	4.16
256.00	26	14891.99	30	15299.73	30	14639.32	1.70	4.32

a : Percentage of cost saving by our heuristic compared to Bahari-Kashani's model

b : Percentage of cost saving by our heuristic compared to Dave-Patel's model

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