▩ 연구논문

Nonparametric Estimation of Reliability in Strength-Stress Model for the Censored Data*

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Abstract

The strength-stress model has been widely used in a variety of areas including testing the reliability of the item or design procedures. This model was first introduced in 1950's and can be found on various applications in civil, aerospace engineering etc. This paper considers the strength-stress model in detail and proposes an estimator which deals with the reliability estimation problem based on censored observations in the strength variables.

1. Introduction

A physical system, whether it consists of a single component or not, is typically operating subject to some kind of environmental 'stress' which depends on many factors. Here, the withstanding power against the stress is named as the 'strength' of the system.

In strength-stress model, let X be the strength of the unit and Y the stress placed on the unit by the operating environment. Suppose X and Y are two random variables with cumulative distribution functions (cdfs) $F(x)$ and $G(y)$ respectively. Then the reliability, denoted by R, of a system is the probability that its strength exceeds the stress. That is,

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$$R = P(X>Y) = \int_0^\infty G(t)dF(t) = \int_0^\infty S(t)dG(t) \qquad (1.1)$$

where $S(t) = 1-F(t)$.

Parametric analyses are found in most literature: Church and Harris (1970) obtained the confidence interval for R under the assumption that X and Y are independently normally distributed and the distribution of Y is known. Beg (1980) derived estimator of R for exponential-family. Sathe and Shah (1981) derived minimum variance unbiased estimator for R when X and Y are independently exponentially distributed random variables.

However, when the parametric assumption is not realistic, a nonparametric approach is called for. Let the data consists of a random sample of size m of strengths $X_1, \cdots, X_m$ from $F(x)$ and an independent random sample of size n of stress $Y_1, \cdots, Y_n$ from $G(y)$. Birnbaum (1956) show that the Wilcoxon-Mann-Whitney statistic could be used as an estimator of R as follows:

$$\hat{R} = \frac{1}{mn}\sum_{i=1}^{m}\sum_{j=1}^{n} U_{ij} \quad \text{where} \quad U_{ij} = \begin{cases} 1, & X_i > Y_j \\ 0, & X_i \le Y_j \end{cases}$$

We can express $\hat{R}$ as

$$\hat{R} = \int_0^\infty (1-F_m(y))dG_n(y) \qquad (1.2)$$

where $F_m(y)$ and $G_n(y)$ are the empirical cdfs of the X's and Y's, respectively. Birnbaum and McCarty (1958) derived distribution-free upper confidence bound on R, which is based on independent samples of X and Y. Govindarajulu (1968) discussed the estimation of R using the Kolmogorov-Smirnov statistic when one of the distribution is known. They considered complete sample case.

For censored observations Delong and Sen (1981) dealt with the estimation of R based on progressively truncated version of the Wilcoxon-Mann-Whitney statistics. They considered the stochastic processes related to some generalized U-statistics under progressive right censoring for prediction purposes. McNichols and Padgett (1988) considered the situation in which censoring is performed to the strength of the item by some prespecified time.

Now we consider the following situations. Assume that there occur censored observations in strength variable. For example, there is a system which can not measure some characteristics of the strength of an item above a certain value. In this case, real strength may exceed the measured value, but we do not know the exact strength.

In this paper we consider estimation of the reliability in the case that there occur censored observation on the strength random variable but neither the distribution of strength nor stress is known.

In Section 2 we propose an estimator for R and derive some properties of the estimator. To see the finite sample performance of the estimator we give some simulation results in Section 3. In Section 4, we give some conclusions and remarks for further studies.

2. Estimation of the Reliability

Let $X_1^0, \cdots, X_m^0$ be real strength random variables from $F^0(x)$ and $Y_1, \cdots, Y_n$ be stress random variables from $G(y)$ where F^0 and G are continuous distributions. We will assume that $X_1^0, \cdots, X_m^0$ and $Y_1, \cdots, Y_n$ are independent.

In the random censoring model, instead of observing real strength variables, we observe only censored observations $(X_1, \delta_1), \cdots, (X_m, \delta_m)$ of strength variables where

$$X_i = \min(X_i^0, C_i) \text{ and } \delta_i = \begin{cases} 1 & \text{if } X_i^0 \le C_i \text{ (uncensored)} \\ 0 & \text{if } X_i^0 > C_i \text{ (censored)} \end{cases} \quad i = 1, \cdots, m.$$

We assume that the censoring random variables $C_1, \cdots, C_m$ are independent and identically distributed (iid) according to the distribution H and that the X's and Y's are mutually independent. Hence the observed $X_1, \cdots, X_m$ constitute a random sample from the distribution F given by $1-F=(1-F^0)(1-H)$. The product-limit(PL) estimator S_m of $S=1-F_0$, introduced by Kaplan and Meier (1958), is

$$\hat{S}_m(t) = \prod_{\{i : X_{(i)} \le t\}} \left(\frac{n-i}{n-i+1}\right)^{\delta(i)}, \; 0 \le t < X_{(m)},$$

where $X_{(1)}, \cdots, X_{(m)}$ are ordered observations and $\delta_{(i)}$ is the censoring status corresponding to $X_{(i)}$, $i=1, \cdots, m$. Throughout this paper we treat $X_{(m)}$ as uncensored observation whether it is censored or not and define $\hat{S}_m(t)=0$ for $t \ge X_{(m)}$.

Now we propose an estimator of R in (1.1) using PL estimator

$$\hat{R}_{PL} = \int_0^\infty \hat{S}_m(t)\, dG_n(t) \qquad (2.1)$$

where $\hat{S}_m(t)$ and $G_n(t)$ are PL estimator of $S(t)$ and empirical distribution of $G(t)$, respectively. When there is no censoring, $\hat{R}_{PL}$ coincides with R in (1.2).

It is well known that the PL estimator $\hat{S}_m$ and empirical distribution G_n are uniformly consistent. Using this properties we can show the consistency of the proposed estimator.

Theorem 2.1 Let F^0, G and H be continuous distributions. If

$$\int_0^{F^{0-1}(1)} \frac{dF^0(t)}{1-H(t)} < \infty \qquad (2.2)$$

and

$$\int_0^\infty \{ S^2(t) \int_0^t \frac{dF^0}{S^2(1-H)} \}^{1/2} dt < \infty \qquad (2.3)$$

hold, then $\hat{R}_{PL}$ is a consistent estimator of R as $m, n \to \infty$.

Proof.

$$\begin{aligned} |\hat{R}_{PL} - R| &= \left| \int_0^\infty \widehat{S_m}(t)\, dG_n(t) - \int_0^\infty S(t)\, dG(t) \right| \\ &\le M_{1n} + M_{2n} \end{aligned}$$

where

$$M_{1n} = \left| \int_0^\infty (\widehat{S_m}(t) - S(t))\, dG_n(t) \right| \text{ and } M_{2n} = \left| \int_0^\infty S(t)(dG_n(t) - dG(t)) \right|.$$

We will show that M_{1n} and M_{2n} converge in probability to 0 as $m, n \to \infty$.

$$\begin{aligned} M_{1n} &\le \int_0^\infty | \widehat{S_m}(t) - S(t) |\, dG_n(t) \\ &\le \sup_t | \widehat{S_m}(t) - S(t) | \int_0^\infty dG_n(t). \end{aligned}$$

Since $\{\sqrt{m}\ \sup|\ \widehat{S}_m(t)-S(t)|\}$ is bounded in probability (see Joe and Proschan (1982)) by condition (2.2) and (2.3), M_{1n} converges in probability to 0. And by Helly's theorem, M_{2n} converges in probability to 0. Therefore we have that $\widehat{R}_{PL}$ converges in probability to R as $m, n \rightarrow \infty$. ◇

In order to study the limiting distribution of $\hat{R}_{PL}$ in terms of the joint limiting distribution of $\hat{S}_m$ and G_n, we define $\Lambda_m(t)$ and $\Lambda_{G_n}(t)$ by $\Lambda_m(t) = \sqrt{m}\ (\hat{S}_m(t)-S(t))$ and $\Lambda_{G_n}(t) = \sqrt{n}\,(G_n(t)-G(t))$, respectively. Then we have the expansion

$$\sqrt{N}\ (\widehat{R}_{PL}-R) = \sqrt{\frac{N}{m}}\,A_N + \sqrt{\frac{N}{n}}\,B_N + \sqrt{\frac{N}{m}}\,R_N$$

where $N = m + n$.

$$\begin{aligned} A_N &= \int_0^\infty \Lambda_m(t)\,dG(t), \\ B_N &= \int_0^\infty \Lambda_{G_n}(t)\,dF^0(t) \qquad (2.4) \\ \text{and} \quad R_N &= \int_0^\infty \Lambda_m(t)\,d(G_n(t)-G(t)). \end{aligned}$$

We will show that A_N and B_N converge weakly to A and B defined by $A = \int_0^\infty \Lambda(t)dG(t)$ and $B = \int_0^\infty \Lambda_G(t)dF^0(t)$, respectively. Likewise we will show that R_N converges in probability to 0. This will establish the following theorem.

Theorem 2.2 Suppose F^0, G and H are continuous distributions. Let $\lambda = \lim_{m\to\infty, n\to\infty} \frac{m}{N}$ and $0<\lambda<1$. In addition to (2.2), (2.3) suppose that

$$\int_0^\infty \{S^2(t)\int_0^t \frac{dF^0}{S^2(1-H)}\}^{1/2}\,dG(t) < \infty \qquad (2.5)$$

and

$$\int_0^\infty \{G(t)(1-G(t)\}^{1/2}\,dF^0(t) < \infty \qquad (2.6)$$

hold. Then

$$\sqrt{N}\ (\widehat{R}_{PL}-R) \xrightarrow{d} N(0, \sigma_2^2/\lambda + \sigma_2^2/(1-\lambda)) \quad \text{as} \quad m, n \rightarrow \infty$$

where

$$\sigma_1^2 = \int_0^\infty \frac{1}{S^2(t)(1-H(t))} \left[\int_t^\infty S(s)dG(s) \right]^2 dF^0(t)$$

and
$$\sigma_2^2 = \int_0^\infty (F^0(t))^2 dG(t) - \left(\int_0^\infty F^0(t)dG(t) \right)^2. \qquad (2.7)$$

Proof. To prove the limiting normality, it suffices to examine the convergence mentioned above. We use the fact, proved by Gill(1983), that $\{\Lambda_m(t),\ 0 \le t < X_{(m)}\}$ converges weakly to $\{\Lambda(t),\ 0 \le t \le \tau\}$ if $\int_0^\tau [1-H(t)]^{-1} dF^0(t) < \infty$, where $\tau = \min\{F^{0-1}(1), H^{-1}(1)\}$ and $\Lambda(t)$ is a Gaussian process with mean 0 and covariance

$$\mathrm{Cov}(\Lambda(t_1), \Lambda(t_2)) = S(t_1)S(t_2) \int_0^{\min(t_1, t_2)} \frac{dF^0(t)}{S^2(t)(1-H(t))}.$$

Condition (2.2) implies that $\tau = F^{0-1}(1)$ and $\int_0^\tau [1-H(t)]^{-1} dF^0(t) < \infty$. And it is well known (Billingsly (1968) Theorem 16.4) that $\Lambda_{G_n}(t)$ converges weakly to $\Lambda_G(t)$ where $\Lambda_G(t)$ is a Gaussian process with mean 0 and covariance

$$\mathrm{Cov}(\Lambda_G(t_1), \Lambda_G(t_2)) = (1-G(t_1))(1-G(t_2)) \int_0^{\min(t_1, t_2)} \frac{dG(t)}{(1-G(t))^2}.$$

Next note that $\int_0^\infty \Lambda(t)dG(t)$ is a proper random variable since

$$E\{\int_0^\infty |\Lambda(t)|dG(t)\} = \int_0^\infty E|\Lambda(t)|dG(t) \le \int_0^\tau [E\Lambda^2(t)]^{1/2} dG(t)$$
$$= \int_0^\tau \{S^2(t) \int_0^t \frac{dF^0}{S^2(1-H)}\}^{1/2} dG(t) < \infty$$

by condition (2.5) Similarly, $\int_0^\tau \Lambda_G(t)dF^0(t)$ is a proper random variable by condition (2.6). By the continuous mapping theorem (Billingsly (1968), p. 30), the leading terms in (2.4) A_N and B_N converges weakly to A and B, respectively. Turning to the remainder term, since $\sup|\Lambda(t)|$ is bounded in probability, R_N converges in probability to 0. And by Slusky's Theorem $\sqrt{N}(\widehat{R}_{PL} - R)$ converges weakly to

$$\frac{1}{\sqrt{\lambda}} \int_0^\infty \Lambda(t)dG(t) + \frac{1}{\sqrt{1-\lambda}} \int_0^\infty \Lambda_G(t)dF^0(t).$$

By the theory of stochastic integration (see Chapter 5 in cramér and Leadbetter (1967)), we can obtain that the limiting random variable is normal with mean 0 and the variance given by (2.7). ◇

3. Simulation Results

To see the finite sample performance of the proposed estimator we carry out the following Monte Carlo experiment. The simulation is performed on the subroutine FORTRAN of the package IMSL in CYBER 962-31 at Seoul National University.

The strength random numbers are generated from the Weibull distribution $W(\alpha_1, \beta)$, i.e.

$$F^0(t) = 1-\exp\{-(\frac{t}{\alpha_1})^\beta\}, \quad t \geq 0, \quad \alpha_1 > 0, \quad \beta > 0.$$

The stress random numbers are generated from the Weibull distribution $W(\alpha_2, \beta)$, i.e.

$$G(t) = 1-\exp\{-(\frac{t}{\alpha_2})^\beta\}, \quad t \geq 0, \quad \alpha_2 > 0, \quad \beta > 0.$$

In this case, the exact reliability is given by

$$R = Pr[X^0 > Y] = \frac{\alpha_1^\beta}{\alpha_1^\beta + \alpha_2^\beta}.$$

The censoring random numbers are generated from $1-H(t) = \{1-F^0(t)\}^\gamma$ for $\gamma = 1/9$ and $1/2$, here γ is viewed as a censoring parameter since the probability that an observation will be censored is $Pr(\delta_i = 0) = \frac{\gamma}{\gamma+1}$.

Tables 3.1~3.3 show that the results of the simulation with 1000 replications when $\beta = 1$ (i.e. exponential distribution), $(\alpha_1, \alpha_2) = (1, 1)$, $(1, 3/7)$ and $(1, 1/9)$ for m/N = 1/4, 2/4 and 3/4. Tables 3.4 ~ 3.6 show that the results of the simulation with 1000 replications when $\beta = 2$, $(\alpha_1, \alpha_2) = (1, 1)$, $(1, \sqrt{3}/\sqrt{7})$ and $(1, 1/3)$ for m/N = 1/4, 2/4 and 3/4.

〈 Table 3.1 〉 Results of the simulation with 1000 replications from $X^{\circ} \sim W(1, 1)$ and $Y \sim W(1, 1)$

$\frac{m}{N}$	Exact R	Sample size (m, n)	no censoring $\hat{R}$	S.E.	10% censoring $\hat{R}_{PL}$	S.E.	33% censoring $\hat{R}_{PL}$	S.E.
1/4	0.5	(5, 15)	.5033	.0050	.5023	.0051	.4888	.0055
		(10, 30)	.4978	.0034	.4956	.0035	.4913	.0037
		(15, 45)	.4990	.0027	.4984	.0027	.4952	.0030
		(20, 60)	.4969	.0024	.4969	.0024	.4945	.0025
		(25, 75)	.5026	.0021	.5027	.0021	.4996	.0023
1/2		(10, 10)	.4971	.0041	.4967	.0041	.4908	.0044
		(20, 20)	.5015	.0028	.5015	.0029	.4977	.0031
		(30, 30)	.5020	.0024	.5018	.0025	.5015	.0026
		(40, 40)	.4994	.0021	.5000	.0021	.4987	.0022
		(50, 50)	.5021	.0018	.5022	.0018	.5013	.0019
3/4		(15, 5)	.5041	.0047	.5036	.0047	.4997	.0050
		(30, 10)	.5016	.0034	.5008	.0034	.4988	.0035
		(45, 15)	.5018	.0028	.5022	.0028	.5001	.0029
		(60, 20)	.5027	.0023	.5028	.0024	.5020	.0024
		(75, 25)	.5005	.0021	.5005	.0021	.4998	.0021

〈 Table 3.2 〉 Results of the simulation with 1000 replications from $X^{\circ} \sim W(1, 1)$ and $Y \sim W(3/7,$ [illegible]

$\frac{m}{N}$	Exact R	Sample size (m, n)	no censoring $\hat{R}$	S.E.	10% censoring $\hat{R}_{PL}$	S.E.	33% censoring $\hat{R}_{PL}$	S.E.
1/4	0.7	(5, 15)	.7039	.0048	.7035	.0049	.6977	.0051
		(10, 30)	.6993	.0034	.6990	.0034	.6978	.0036
		(15, 45)	.7011	.0026	.7006	.0026	.7002	.0027
		(20, 60)	.7015	.0023	.7013	.0023	.7010	.0024
		(25, 75)	.6994	.0021	.6993	.0021	.6993	.0022
1/2		(10, 10)	.7010	.0039	.7001	.0039	.7026	.0040
		(20, 20)	.6956	.0026	.6950	.0026	.6958	.0027
		(30, 30)	.7011	.0021	.7011	.0021	.7011	.0022
		(40, 40)	.6994	.0019	.6996	.0019	.6990	.0020
		(50, 50)	.6991	.0017	.6990	.0017	.6989	.0018
3/4		(15, 5)	.7002	.0041	.7005	.0041	.7002	.0042
		(30, 10)	.6981	.0027	.6982	.0028	.6989	.0028
		(45, 15)	.6977	.0023	.6976	.0023	.6976	.0024
		(60, 20)	.6993	.0020	.6988	.0020	.6989	.0020
		(75, 25)	.7027	.0017	.7027	.0017	.7025	.0017

〈 Table 3.3 〉 Results of the simulation with 1000 replications from $X^{\circ} \sim W(1, 1)$ and $Y \sim W(1/9, 1)$

m/N	Exact R	Sample size (m, n)	no censoring $\hat{R}$	S.E.	10% censoring $\hat{R}_{PL}$	S.E.	33% censoring $\hat{R}_{PL}$	S.E.
1/4	0.9	(5, 15)	.9019	.0031	.9021	.0031	.9011	.0031
		(10, 30)	.8984	.0021	.8984	.0021	.8989	.0021
		(15, 45)	.9001	.0018	.9000	.0018	.8999	.0018
		(20, 60)	.9020	.0015	.9020	.0015	.9019	.0015
		(25, 75)	.9002	.0013	.9003	.0014	.9005	.0014
1/2		(10, 10)	.8959	.0025	.8960	.0025	.8953	.0025
		(20, 20)	.9030	.0017	.9032	.0017	.9036	.0017
		(30, 30)	.8994	.0013	.8994	.0013	.8995	.0013
		(40, 40)	.9012	.0011	.9013	.0011	.9012	.0011
		(50, 50)	.9014	.0010	.9013	.0010	.9015	.0010
3/4		(15, 5)	.9008	.0022	.9004	.0022	.9011	.0022
		(30, 10)	.9001	.0015	.9001	.0015	.9002	.0015
		(45, 15)	.8982	.0013	.8981	.0013	.8978	.0013
		(60, 20)	.9006	.0011	.9006	.0011	.9009	.0011
		(75, 25)	.8992	.0010	.8993	.0010	.8993	.0010

〈 Table 3.4 〉 Results of the simulation with 1000 replications from $X^{\circ} \sim W(1, 2)$ and $Y \sim W(1, 2)$

m/N	Exact R	Sample size (m, n)	no censoring $\hat{R}$	S.E.	10% censoring $\hat{R}_{PL}$	S.E.	33% censoring $\hat{R}_{PL}$	S.E.
1/4	0.5	(5, 15)	.5035	.0051	.4995	.0052	.4901	.0055
		(10, 30)	.5052	.0034	.5046	.0035	.4977	.0037
		(15, 45)	.4982	.0027	.4985	.0028	.4940	.0030
		(20, 60)	.4957	.0023	.4952	.0024	.4943	.0025
		(25, 75)	.4986	.0021	.4979	.0022	.4971	.0023
1/2		(10, 10)	.4967	.0042	.4960	.0043	.4888	.0045
		(20, 20)	.5012	.0029	.5008	.0030	.4967	.0031
		(30, 30)	.4980	.0024	.4972	.0024	.4954	.0026
		(40, 40)	.5024	.0021	.5024	.0021	.5024	.0022
		(50, 50)	.5033	.0019	.5039	.0019	.5027	.0019
3/4		(15, 5)	.4919	.0048	.4912	.0049	.4860	.0051
		(30, 10)	.5081	.0034	.5079	.0035	.5068	.0035
		(45, 15)	.4966	.0027	.4963	.0027	.4961	.0028
		(60, 20)	.5011	.0023	.5010	.0023	.5013	.0023
		(75, 25)	.4987	.0022	.4987	.0022	.4976	.0022

〈 Table 3.5 〉 Results of the simulation with 1000 replications from $X^{\circ} \sim W(1, 2)$ and $Y \sim W(\sqrt{3}\ \sqrt{7}, 2)$

$\frac{m}{N}$	Exact R	Sample size (m, n)	no censoring $\hat{R}$	S.E.	10% censoring $\hat{R}_{PL}$	S.E.	33% censoring $\hat{R}_{PL}$	S.E.
1/4	0.7	(5, 15)	.6899	.0047	.6898	.0048	.6822	.0051
		(10, 30)	.6979	.0034	.6981	.0034	.6942	.0035
		(15, 45)	.7008	.0026	.7015	.0026	.6998	.0028
		(20, 60)	.7028	.0022	.7036	.0022	.7032	.0023
		(25, 75)	.6976	.0020	.6972	.0020	.6964	.0021
1/2		(10, 10)	.6998	.0038	.6998	.0038	.7003	.0039
		(20, 20)	.7026	.0026	.7029	.0026	.7016	.0027
		(30, 30)	.6975	.0021	.6977	.0021	.6979	.0022
		(40, 40)	.7022	.0019	.7021	.0019	.7020	.0020
		(50, 50)	.7009	.0016	.7011	.0016	.7012	.0017
3/4		(15, 5)	.6986	.0040	.6981	.0040	.6969	.0042
		(30, 10)	.6995	.0029	.7000	.0029	.6982	.0029
		(45, 15)	.7009	.0023	.7006	.0023	.7013	.0023
		(60, 20)	.7024	.0020	.7029	.0020	.7022	.0020
		(75, 25)	.7006	.0018	.7001	.0018	.7007	.0018

〈 Table 3.6 〉 Results of the simulation with 1000 replications from $X^{\circ} \sim W(1, 2)$ and $Y \sim W(1/3, 2)$

$\frac{m}{N}$	Exact R	Sample size (m, n)	no censoring $\hat{R}$	S.E.	10% censoring $\hat{R}_{PL}$	S.E.	33% censoring $\hat{R}_{PL}$	S.E.
1/4	0.5	(5, 15)	.9064	.0030	.9061	.0030	.9061	.0031
		(10, 30)	.8983	.0021	.8983	.0021	.8985	.0022
		(15, 45)	.8998	.0018	.8997	.0018	.9002	.0018
		(20, 60)	.9024	.0015	.9023	.0015	.9026	.0015
		(25, 75)	.9003	.0013	.9003	.0013	.8999	.0014
1/2		(10, 10)	.9004	.0023	.9005	.0023	.9005	.0023
		(20, 20)	.8992	.0016	.8992	.0016	.8992	.0016
		(30, 30)	.9006	.0014	.9007	.0013	.9005	.0014
		(40, 40)	.9017	.0011	.9017	.0011	.9021	.0011
		(50, 50)	.9007	.0011	.9006	.0011	.9007	.0011
3/4		(15, 5)	.9036	.0022	.9036	.0022	.9047	.0022
		(30, 10)	.9036	.0015	.9036	.0015	.9039	.0015
		(45, 15)	.9004	.0012	9003	.0012	.9002	.0012
		(60, 20)	.8994	.0011	.8993	.0011	.8996	.0011
		(75, 25)	.9012	.0010	.9010	.0010	.9012	.0010

4. Conclusion and Remark

In this paper, We proposed an estimator of the system reliability based on the productlimit estimator in the strength-stress model when there occur censored observations on the strength variable. And we investigated the asymptotic behavior of the proposed estimator. We obstained the consistency and anymptotic normality for the proposed estimator. Finally, we performed Monte Carlo simulation to see the performance of the peoposed estimator via S.E.. From the simulation, we may conclude the following facts;

(1) The Standard Error (S.E.) increases as the censoring fraction increases.

(2) For all cases (no censoring, 10% censoring and 33% censoring case), the S.E. decreases as the exact R increases.

(3) When $m/N=1/2$, S.E.'s of estimators are smaller than others (i.e. $m/N=1/4$ and $3/4$) except in the case that the exact $R=0.9$.

(4) When $R=0.9$, the smallest S.E. is achieved when $m/N=3/4$.

(5) As N increases with fixed m/N, S.E. is nearly equal for all cases (no censoring, 10% censoring and 33% censoring case.)

An interesting subject for further study is estimation of the reliability R when the strength variable and stress variable are both censored.

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